TWOPORT NETWORK FUNCTION AND RESPONCES

INTRODUCTION

A network having two end ports is known as a two portnetwork. The ports may supply or consume electrical power. A complex network can be represented as a two port network constitutes two stations and a black box in between the station as below.

The study of the above network becomes complicated as the network present inside the black box is known so far the techniques has been developed , the two port networks are analyzed by using different parameters.

One can imagine the network inside the black box may be impedances or admittances connected in series or parallel randomly. Now applying KVL and KCL we can define the equations

As

$V_1 = Z_{11}I_1+Z_{12}I_2$

$V_2 = Z_{21}I_1+Z_{22}I_2$

Or

$I_1=Y_{11}V_1+Y_{12}V_2$

$I_2+Y_{21}V_1+Y_{22}V_2$

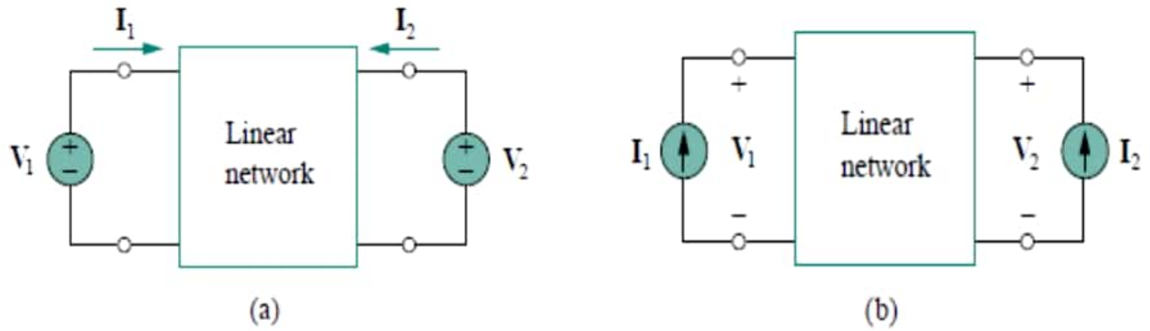
Z_{11} , Z_{12} , Z_{21} , & Z_{22}--→Z- Parameters

Y_{11} , Y_{12} . Y_{21} ,& Y_{22}--→Y-Parameters

IMPEDANCE PARAMETERS:

Impedance and admittance parameters are commonly used in the synthesisof filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks.

A two-port network may be voltage-drivenorcurrent-driven as shown in Fig.



The terminal voltages can be related to the terminal currents as,

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

or in matrix form as,

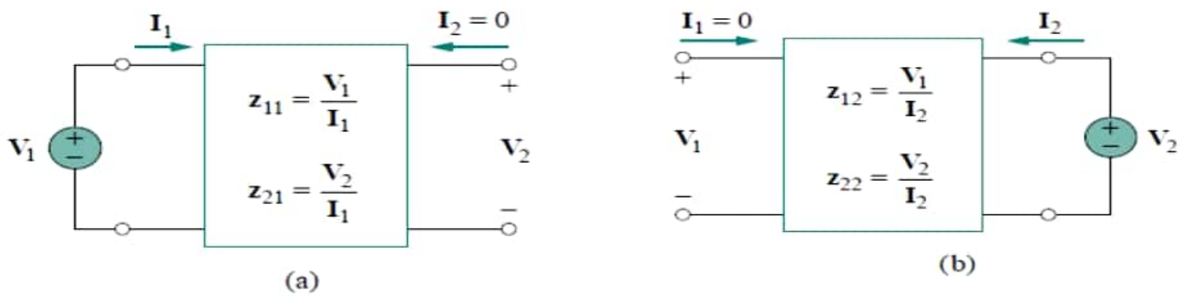
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where the **z** terms are called the *impedance parameters*, or simply *z- parameters*, and have units of ohms.

The values of the parameters can be evaluated by setting $I_1 = 0$ (input port open-circuited) or $I_2 = 0$ (output port open-circuited). Thus,

$$Z_{11} = \frac{V_1}{I_1} | I_2 = 0 \quad Z_{21} = \frac{V_2}{I_1} | I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2} | I_1 = 0 \quad Z_{22} = \frac{V_2}{I_2} | I_1 = 0$$



Since the z parameters are obtained by open-circuiting the input or outputport, they are also called the *open-circuit impedance parameters*.

Specifically,

Z_{11} = Open-circuit input impedance

Z_{12} = Open-circuit transfer impedance from port 1 to port 2

Z_{21} = Open-circuit transfer impedance from port 2 to port 1

Z_{22} = Open-circuit output impedance

Sometimes Z_{11} and Z_{22} are called *driving-point impedances*, while Z_{21} and Z_{12} are called *transfer impedances*.

ADMITTANCE PARAMETERS:

In general, for a two port network consisting of 2 loops,

$$I_1 = y_{11}V_1 + y_{12}\,V_2$$
$$I_2 = y_{21}V_1 + y_{22}\,V_2$$

or in matrix form as,

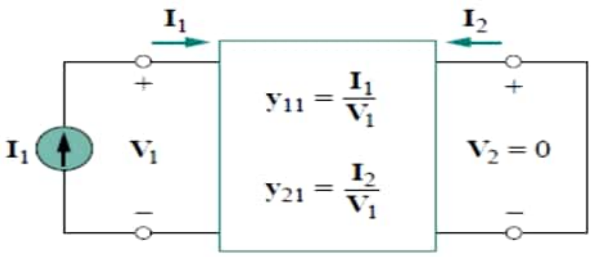
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y]\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Where, the **y**-terms are called the *admittance parameters*, or simply *y- parameters*, and have units of siemens.

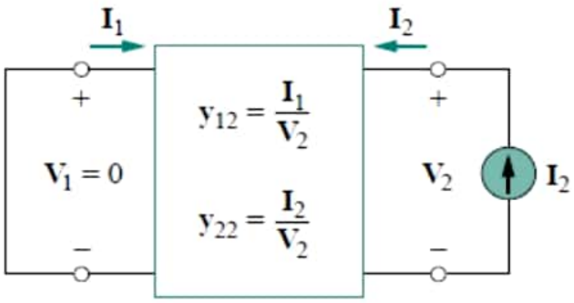
The values of the parameters can be determined by setting $V_1 = 0$ (input port short-circuited) or $V_2 = 0$ (output port short-circuited). Thus,

Now, $y_{11} = \frac{I_1}{V_1}\Big|V_2 = 0$ $y_{21} = \frac{I_2}{V_1}\Big|V_2 = 0$

$y_{12} = \frac{I_1}{V_2}\Big|V_1 = 0$ $y_{22} = \frac{I_2}{V_2}\Big|V_1 = 0$



Circuit to find Y_{11} and Y_{21}



Circuit to find Y_{12} and Y_{22}

Since the *y* parameters are obtained by short-circuiting the input or output port, they are also called the *short-circuit admittance parameters*.

Specifically,

$\mathbf{y}_{11}$= Short-circuit input admittance

$\mathbf{y}_{12}$= Short-circuit transfer admittance from port 2 to port 1

$\mathbf{y}_{21}$= Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{22}$= Short-circuit output admittance

HYBRID PARAMETERS:

This hybrid parameters is based on making V_1 and I_2 the dependent variables.

Thus,

$$V_1 = h_{11}I_1 + h_{12} V_2$$

$$I_2 = h_{21}I_1 + h_{22} V_2$$

or in matrix form as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The **h** terms are known as the *hybrid parameters* (or, simply, *h parameters*) because they are a hybrid combination of ratios. They are very useful for describing electronic devices such as transistors.

The values of the parameters are determined as,

$$h_{11} = \frac{V_1}{I_1} | V_2 = 0 \quad h_{12} = \frac{V_1}{V_2} | I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1} | V_2 = 0 \quad h_{22} = \frac{I_2}{V_2} | I_1 = 0$$

The parameters h_{11}, h_{12}, h_{21}, and h_{22} represent an impedance, a voltage gain, a current gain, and an admittance,respectively. This is why they are called the hybrid parameters.

To be specific,

h_{11}= Short-circuit input impedance

h_{12}= Open-circuit reverse voltage gain

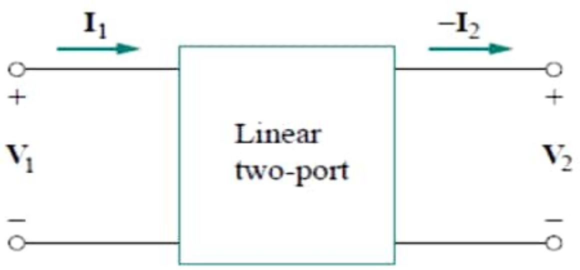
h_{21}= Short-circuit forward current gain

h_{22}= Open-circuit output admittance

The procedure for calculating the *h* parameters is similar to that used for the *z* or *y* parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis.

TRANSMISSION PARAMETERS:

Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port. Thus,



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

or

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V \\ -I_2 \end{bmatrix}$$

The two-port parameters provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber), because they express sending-end variables (V_1 and I_1) in terms of the receiving-end variables (V_2 and $-I_2$). For this reason, they are called *transmission parameters*. They are also known as **ABCD** parameters.

The transmission parameters are determined as,

$$A = \frac{V_1}{V_2} | I_2 = 0 \quad B = -\frac{V_1}{I_2} | V_2 = 0$$

$$C = \frac{I_1}{V_2} | I_2 = 0 \quad D = -\frac{I_1}{I_2} | V_2 = 0$$

Thus, the transmission parameters are called, specifically,

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

Aand**D**are dimensionless, **B** is in ohms, and **C** is in siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

Inter Relationship between parameters:

1. Z-parameters in terms of Y-parameters

   $[Z] = [Y]^{-1}$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \quad Z_{12} = \frac{-Y_{12}}{\Delta Y} \quad Z_{21} = \frac{-Y_{21}}{\Delta Y} \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

   Where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

2. Z-parameters in terms of h-parameters

$$Z_{11} = \frac{\Delta h}{h_{22}} \quad Z_{12} = \frac{h_{12}}{h_{22}} \quad Z_{21} = \frac{-h_{21}}{h_{22}} \quad Z_{22} = \frac{1}{h_{22}}$$

   Where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

3. Z-parameters in terms of ABCD-parameters

$$Z_{11} = \frac{A}{C} \quad Z_{12} = \frac{AD - BC}{C} \quad Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{D}{C}$$

4. Y-parameters in terms of Z-parameters

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \quad Y_{12} = \frac{-Z_{12}}{\Delta Z} \quad Y_{21} = \frac{-Z_{21}}{\Delta Z} \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

   Where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

5. Y-parameters in terms of ABCD-parameters

$$Y_{11} = \frac{D}{B} \quad Y_{12} = -\frac{AD - BC}{B} \quad Y_{21} = -\frac{1}{B} \quad Y_{22} = \frac{A}{B}$$

6. h-parameters in terms of Z-parameters

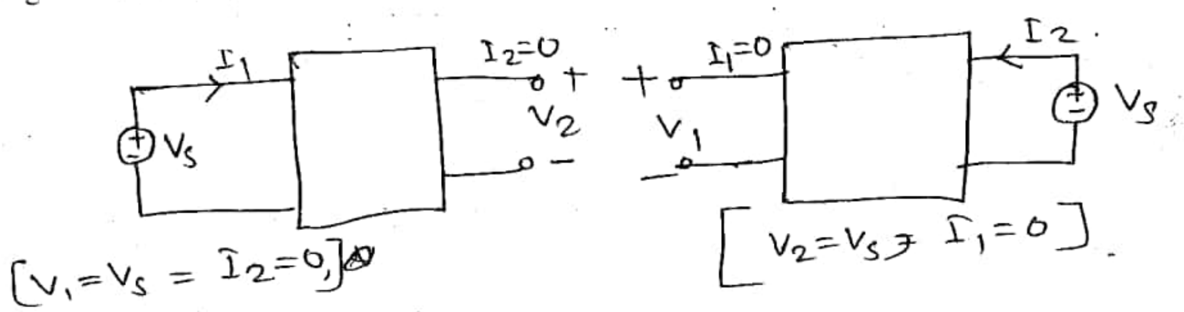
$$h_{11} = \frac{\Delta Z}{Z_{22}} \quad h_{12} = \frac{Z_{12}}{Z_{22}} \quad h_{21} = \frac{-Z_{21}}{Z_{22}} \quad h_{22} = \frac{1}{Z_{22}}$$

7. h-parameters in terms of Y-parameters

$$h_{11} = \frac{1}{Y_{11}} \quad h_{12} = -\frac{Y_{12}}{Y_{11}} \quad h_{21} = \frac{Y_{21}}{Y_{11}} \quad Z_{22} = \frac{\Delta Y}{Y_{11}}$$

Condition of symmetry:-

A two port network is said to be symmetrical if the ports can be interchanged without port voltages and currents.



1).In terms of Z-parameters:-

$$[V]_s/I_1|_{I_2=0} = Z_{11}$$

$$V_s/I_2|_{I_1=0} = Z_{22}$$

So, $Z_{11} = Z_{22}$

2). In terms of Y-parameters:-

$$I_1 = Y_{11}V_s + Y_{12}V_2$$
$$0 = Y_{21}V_s + Y_{22}V_2$$

So,

$$I_1 = Y_{11}V_s + Y_{12}\left\{\frac{-y_{21}}{y_{22}}\right\}V_s$$

$$\Rightarrow \frac{V_s}{I_1} = \frac{y_{22}}{y_{11}y_{22} - y_{21}y_{12}}$$

$$0 = y_{11}V_1 + y_{12}V_s$$

$$I_2 = y_{21}V_1 + y_{22}V_s$$

$$\frac{V_s}{I_s} = \frac{y_{11}}{y_{11}y_{22} - y_{21}y_{12}}$$

so, $y_{11} = y_{22}$

3). <u>In terms of ABCD- parameters:-</u>

$$V_s = AV_2$$

$$I_1 = CV_2$$

$$then, \frac{V_s}{I_1} = \frac{A}{C}$$

Again,

$$V_1 = AV_2 \quad BI_2$$

$$0 - CV_2 - DI_2$$

$$\frac{V_s}{I_2} = \frac{D}{C}$$

So, $\frac{A}{C} = \frac{D}{C}$

So, A=D

<u>Condition of reciprocity:-</u>

A two port network is said to be reciprocal, if the rate of excitation to response is invariant to an interchange of the position of the excitation and response in the network. Network containing resistors, capacitors and inductors are generally reciprocal.

1) <u>In terms of Z- parameters:-</u>

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Now. $V_s = Z_{11}I_1 - Z_{12}I_2'$

$$0 = Z_{21}I_1 + Z_{22}I_2'$$

$$I_2' = \frac{V_s Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

Similarly,

$0 = -Z_{11}I_1' + Z_{12}I_2$

$V_s = -Z_{21}I_1' + Z_{22}I_2$

$hence, I_1' = \frac{V_s Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$

Comparing I_2' and I_1' *we get,*

$Z_{12} - Z_{21}$

2) <u>In terms of Y- parameters:-</u>

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

So,

$I_2' = -Y_{21}V_S$

$I_1' = -Y_{12}V_s$

$so, Y_{21} = Y_{12}$

3) <u>In terms of ABCD-parameters:-</u>

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - Di_2$

So, $V_s = BI_2'$

$I_2' = \frac{V_s}{B}$

$I_1 = DI_2'$

Similarly,

$0 = AV_s - BI_2$

$-I_1' = CV_s - DI_2 = CV_s - D\frac{A}{B}V_s$

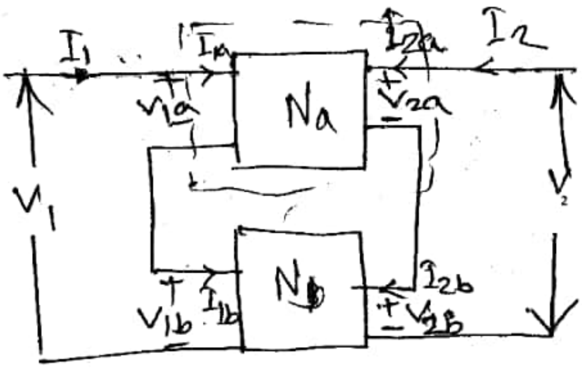
$$\Rightarrow I_1' = \frac{AD - BC}{B}V_s$$

$So, I_2' = I_1'$

$\Rightarrow AD - BC = 1$

Series Connection:

The fig. shows a series connection of two two-port networks Na and Nb with open circuit Z-parameters Za and Zb respectively.



For network Na,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

Similarly, for network Nb,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Then, their series connection requires that

$I_1 = I_{1a} = i_{1b} I_2 = i_{2a} = I_{2b}$

$V_1 = V_{1a} + V_{1b} V_2 = V_{2a} + V_{2b}$

Now, $V_1 = V_{1a} + V_{1b}$

$= (Z_{11a} I_{1a} + Z_{12a} I_{2a}) + (Z_{11b} I_{1b} + Z_{12b} I_{2b})$

$= (Z_{11a} + Z_{11b}) i_1 + (Z_{12a} + Z_{12b}) I_2$

Similarly, $V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$

So, in matrix form the Z-parameters of the series connected combined network can be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where, $Z_{11} = Z_{11a} + Z_{11b}$

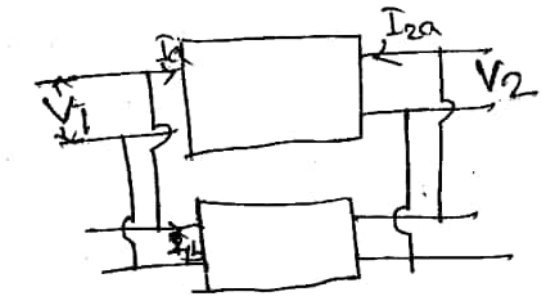
$Z_{12} = Z_{12a} + Z_{12b}$

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$

So, $[Z] = [Z_a] + [Z_b]$

Parallel Connection:



Here,

$V_1 = V_{1a} = V_{1b}$

$V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} + I_{1b}$

$- Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b}$

$I_2 = I_{2a} + I_{2b}$

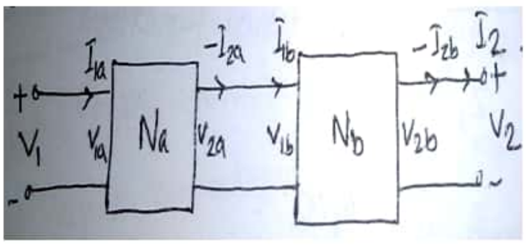
$= Y_{21a}V_{1a} + Y_{22a}V_{2a} + Y_{21b}V_{1b} + Y_{22b}V_{2b}$

So,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\Rightarrow [Y] = [Y_a] + [Y_b]$

Cascade Connection:



Now,

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Then, their cascade connection requires that

$I_1 - I_{1a} - I_{2a} - i_{1b}I_{2b} - I_2$

$V_1 = V_{1a}V_{2a} = V_{1b}V_{2b} = V_2$

So, $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$