:- Foundation Engineering :-

Module-I

- → Lateral earth pressure & retaining structure
- → Concept of earth pressure
- ↳ Earth pressure at rest
- → Active earth pressure & passive earth pressure for cohesive soil & cohesionless soil
- → Earth pressure theories
- → Rankine's theory
- → Columb's wedge theory
- ↳ Graphical methods
  - (i) Rebhans Graphical method
  - (ii) Culmans Graphical method
- → Stability conditions for retaining wall

Definition :-

→ Foundation Engg. is a branch of science which deals with the soil structure or soil mechanics.

→ Basic on the soil parameters soil as classified as three types,

(i) Cohesive soil ($c-\phi$ soil) Sand + Clay
ϕ c

(ii) Cohesionless soil (ϕ - soil) Sand

(iii) Purely Cohesive soil (c-soil) clay

Shear strength of soil :-

→ When soil is loaded shearing stresses are induced in it.

→ When the shearing stresses such a limiting value shear deformation take place.

→ Leading to the failure of soil mass.

→ The failure may be in the form of sinking of a footing, or movement of a wedge of soil behind a retaining wall for sink it to move out or the slide in a earth embankment.

→ The shear strength of a soil is the resistance to deformation by continuous shear displacement of soil particles.

→ The shearing resistance of soil is constituted basically of the following component.

(i) The structural resistance to displacement of the soil because of the interlocking of the particles.

(ii) The frictional resistance to translocation between the individual soil particles at their contact points.

(iii) Cohesion & adhesion between the surface of the soil particles.

Lateral earth pressure of retaining wall / retaining structure:—

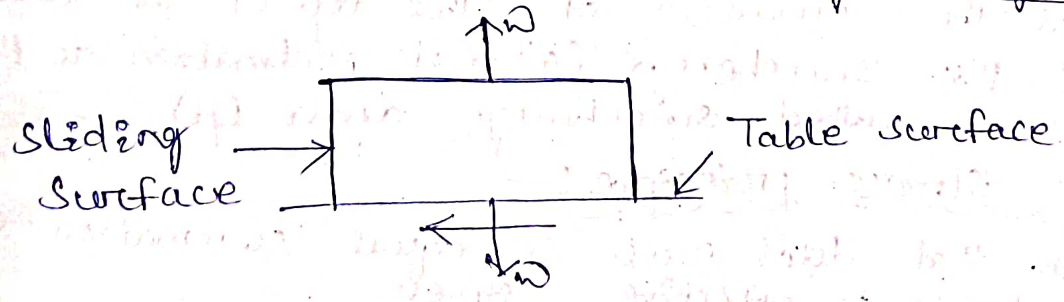
→ In the design of retaining wall, sheet pipe or other earth retaining structure. It is necessary to compute the lateral pressure exerted by the retained mass of the soil.

→ A retaining wall or retaining structure is used for maintaining the ground surface at different elevation on either side of it.

→ The material retained or supported by the structure is called back file.

→ The back file which may have its top surface horizontal or inclined.

→ The position of back file lying above a horizontal plane at the elevation of the top of a wall is called surcharge (q) & it's inclination to the horizontal is called surcharge angle (β).



Frictional resisting force i.e $W \times \mu$ which opposes sliding force.

We know that $\mu = \tan \phi'$

Using the cohesive shear strength

$$S = \tau_f = \sigma \tan \phi + C$$

Where,

S = Shear strength of soil

τ_f = Shear stress at failure

C = Cohesion = KN/m^2

For, cohesion less soil $c = 0$, so, $\tau_f = \sigma \tan\phi$

Relationship betn principle stress that is major principle stress (σ_1), minor principle stress (σ_3) at the state of plastic equilibrium or shear failure.

$$\boxed{\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha}$$

Where,

α = Angle of internal friction

$$\boxed{\alpha = 45^0 + \phi/2}$$

Plastic Equilibrium :-

At every point of the soil is at verge of failure then the soil is said to be plastic equilibrium.

$$\boxed{\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha}$$

$$\boxed{\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha}$$

→ Lateral pressure in retaining structure.

→ A retaining wall or retaining structure is used for maintening ground surfaces at different elevations on either side of it.

→ The material retained or supported by the structure is called backfile.

→ The position of the backfile lying about a horizontal plane at the elevation of the top of the wall is called the surcharge (q) & its inclination to the horizontal is called surcharge angle (β).

Effective Stress principle :-

We Know that total stress is equal summation of neutral stress & effective stress.

Total stress = Nutral stress or pore water pressure + effective stress

$$\sigma = U + \sigma' \quad (\sigma' = \sigma - U)$$

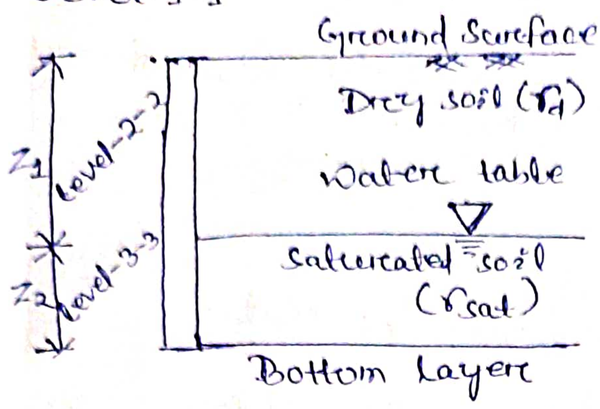
When,

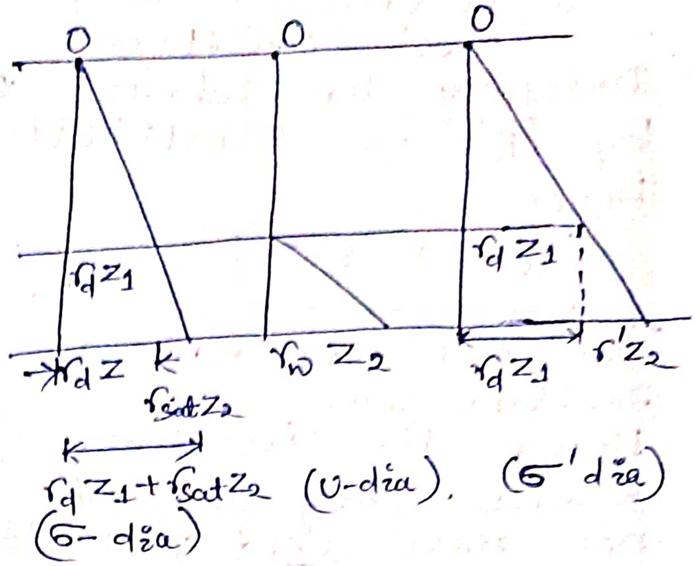
U = Pore water pressure = $\gamma_w h_p$

Where,

γ_w = Unit weight of water
h_p = Pressure head. It is equal to depth of water in the piezometer.

Level-1-1





$$\boxed{\begin{aligned}&\gamma_d Z_1 + \gamma_{sat} + Z_2 - \gamma_w Z_2\\ &\gamma_d Z_1 + (\gamma_{sat} - \gamma_w) Z_2\\ &= \gamma_d Z_1 + \gamma' Z_2\end{aligned}}$$

Types of earth pressure Condition :-

There are three types
(i) Earth pressure at rest condition.
(ii) Active earth pressure
(iii) Passive earth pressure

(i) Earth pressure at rest Condition :-

→ The earth pressure at rest exerted on the back of a rigid. on retaining structure can be calculated using theory of elasticity assuming the soil is semi-infinite, homogeneous, elastic & isotropic.

→ Consider an element of soil at depth (z) being acted upon by vertical stress (σ_v) in horizontal they are will be no shear stress. The stress (σ_h) lateral strength (ϵ_h) in the horizontal direction is given by

$$\boxed{\epsilon_h = \frac{1}{E}\left(\sigma_h - \mu(\sigma_h + \sigma_v)\right)}$$

The earth pressure at rest corresponding to the zero lateral strength

i.e. $\epsilon_h = 0$

Hence $\sigma_h = \mu(\sigma_h + \sigma_v)$

$$\frac{\sigma_h}{\sigma_v} = K_0 = \frac{\mu}{1-\mu}$$

Where,

K_0 = Co-efficient of lateral earth pressure at rest condition.

> Designing the lateral earth pressure (σ_h) at rest by (P_0) an substituting $\sigma_v = \gamma Z$

Hence,

$$\boxed{P_0 = K \cdot \gamma \cdot Z}$$

The soil mass will be in a state of static equilibrium.

Hence the retaining is at rest condition. Where they are no movement of wall i.e, does not move left & right side.

* For Cohesion less soil if M is not given then

$$\boxed{K_0 = 1 - \sin\phi}$$

* Total lateral earth pressure

$$\boxed{P_0 = \frac{1}{2} K_0 \gamma H^2}$$

* If the depth (Z) is given then

$$\boxed{P_0 = \frac{1}{2} K_0 \gamma Z^2}$$

Q A rigid retaining wall 6 m height is restrained from yielding the backfile consists of cohesionless soil having $\phi = 26^\circ$ & $\gamma = 19\ KN/m^3$ Compute the total earth pressure per metre length of the wall.

Soln Given data:—

$\phi = 26^\circ$

$\gamma = 19\ KN/m^3$

$H = 6\ m$

Since the wall is restrained from yielding.

The wall will be subjected to the lateral earth pressure (P_0) at rest condition is given by

$P_0 = \frac{1}{2} \times K_0 \times \gamma \times H \times H$

$= \frac{1}{2} \times K_0 \times \gamma H^2$

Here, K_0 can be calculated from this eqn i.e. given by Jacky.

$K_0 = 1 - \sin\phi$

$K_0 = 1 - \sin 26°$

$= 0.5616$

The value corresponding to send the total lateral earth pressure $P_0 = \frac{1}{2} \times K_0 \gamma \times H^2$

$P_0 = \frac{1}{2} \times 0.5616 \times 19 \times 6^2$

$= 192.6$ KN/metre length of the wall

(5) Active Earth Pressure (Rankine's theory):—

→ Rankine's theory of lateral earth pressure is applied to uniform cohesionless soil only.

→ This theory has also been extended to stratified partially immersed & submerged soil.

→ Following assumptions of the Rankine's theory are;

Assumptions :—

→ The soil mass is semi-infinite, homogeneous, dry & cohesionless.

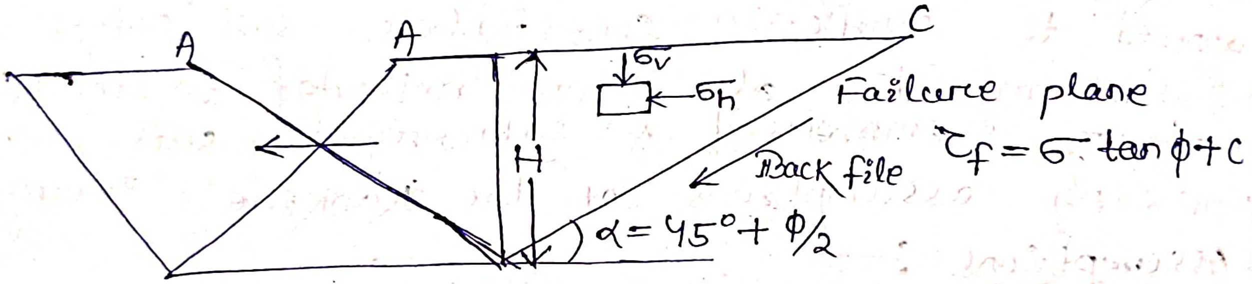
→ The ground surface is plane which may be horizontal or inclined.

→ The back of the wall is vertical & smooth. In other words there are no shearing stresses between the wall & the soil & the stress relationship for element adjacent to the wall & the same as for any other elements far away from the wall.

→ The wall yields about the base & thus satisfy the deformation condition for plastic equilibrium.

→ The retaining wall are constructed for masonary or concrete & hence the back of the wall is never smooth. Due to this frictional develop the Rankine's assumptions of no existance of frictional forces at the wall face.

→ The resultant pressure most be parallel to the surface of the backfill.
→ The existance of the friction makes the resultant force inclined to the normal to the wall at an angle that approaches the frictional angle between the soil & the wall.
→ The following cases of cohesionless backfile will now be considered;
(a) Dry or moist backfill with no surcharge.
(b) Submerged backfile.
(c) Backfile with uniform surcharge.
(d) Backfile with slopping surcharge.
(e) Inclined back & surcharge
The active earth pressure arises from the backfile.
→ When the wall moves away the internal resistance
→ Mobilisation of the wall the internal resistance of the soil.
Hence the earth pressure on the wall decrease.



→ When the wall rotates about its base sufficiently away from the backfill.
→ The triangular soil mass ABC comes to the state of plastic equilibrium & fails by sliding down the plane BC.
→ At that time σ_h is the active earth pressure & the failure plane inclination $\alpha = 45° + \phi/2$

i.e $K_a = \dfrac{\sigma_h}{\sigma_v}$

$\sigma_v = \sigma_1$ is major principle stress
$\sigma_h = \sigma_3$ is minor principle stress

i.e $K_a = \dfrac{\sigma_3}{\sigma_1}$

$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$

$\Rightarrow \sigma_1 = \sigma_3 \tan^2(45° + \phi/2) + \cancel{2c\tan\alpha}$ (c=0)

$\Rightarrow \sigma_1 = \sigma_3 \tan^2(45° + \phi/2)$

$$K_a = \frac{\sigma_3}{\sigma_1} = \frac{1}{\tan^2\alpha} = \cot^2\alpha$$

In active earth pressure condition the coefficient of lateral earth pressure

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} \quad \text{(For cohesionless soil)}$$

Total lateral active earth pressure per metre length of the wall

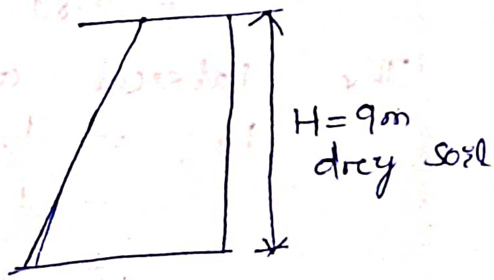
$$P_a = \frac{1}{2} K_a \gamma H \times H$$
$$= \frac{1}{2} K_a \gamma H^2$$

At the base active earth pressure

$$P_a = K_a \gamma H$$

Q. A retaining wall 9 m high retains dry sand with an angle of internal friction of 30° & unit weight of 18.2 KN/m³. Determine the earth pressure at rest if water table rises to top of the wall. Determine the increase in the thrust on the wall. Assume the submerged unit weight of sand 12 KN/m³.

Solⁿ Given data :-



$H = 9$ m

(cohesionless soil)

$\phi = 30°$

$\gamma = 18.2$ KN/m³

Coefficient of lateral earth pressure at rest condition

$$K_o = 1 - \sin 30°$$
$$= 0.5$$

Lateral earth pressure intensity of Lateral earth pressure at base

$$P_o = K_o \times \gamma \times H$$
$$= 0.5 \times 18.2 \times 9$$
$$= 81.9 \text{ KN/m}^2$$

Lateral earth pressure at rest condition

$$P_o = \frac{1}{2} \times K_o \times \gamma \times H \times H$$

$= \frac{1}{2} \times 0.5 \times 18.2 \times 9 \times 9$

$= 368.55$ KN/m length of the wall

The submerged unit weight of the soil

$\gamma_{sat} = 12 \text{ KN/m}^3$

Coefficient of lateral active earth pressure

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$= \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

The intensity of lateral active earth pressure at base

$P_1 = K_a \gamma H$

$= \frac{1}{3} \times 12 \times 9 = 36 \text{ KN/m}^2$

Total lateral active earth pressure

$P_{a_1} = \frac{1}{2} K_a \gamma H^2$

$= \frac{1}{2} \times \frac{1}{3} \times 12 \times 9^2$

$= 162 \text{ KN/m}^3$ length of the wall

At the water table position

$P_2 = \gamma_w \times H$

$= 9.81 \times 9 = 88.29 \text{ KN/m}^2$

The lateral active earth pressure

$P_{a_2} = \frac{1}{2} \times P_2 \times H$

$= \frac{1}{2} \times 88.29 \times 9$

$= 397.3 \text{ KN/m}^2$

Hence total active earth pressure

$P_a = P_{a_1} + P_{a_2}$

$= 162 + 397.3$

$= 559.3$ KN/m length of the wall

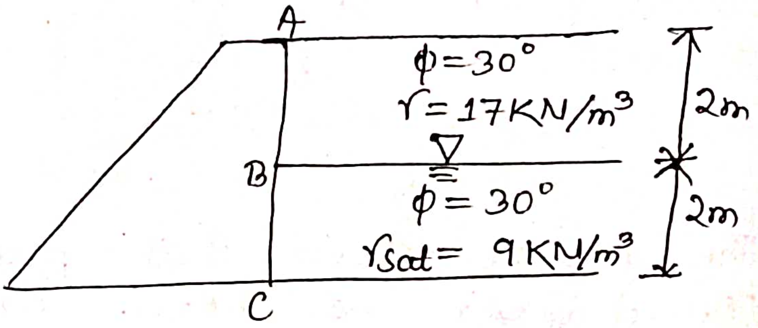
Increase in thrust on the wall

$= 559.3 - 368.55$

$= 190.75$ KN/m

Q- Determine the lateral earth pressure at rest per unit length of the wall as shown in figure. Also determine the location of the resultant earth pressure. Take $K_0 = 1 - \sin\phi$, $\gamma_w = 10\ KN/m^3$.

Sol Given data :-



$\gamma_w = 10\ KN/m^3$

$\phi = 30°$

$\gamma = 17\ KN/m^3$

$\phi' = 30°$

$\gamma_{sat} = 19\ KN/m^3$

The coefficient of lateral earth pressure at rest condition

$$K_0 = 1 - \sin\phi$$
$$= 1 - \sin 30°$$
$$= 0.5$$

At point 'B' $\sigma_z = \gamma \times H_1$
$$= 17 \times 2$$
$$= 34\ KN/m^2$$

$u = 0$

Total stress at base condition

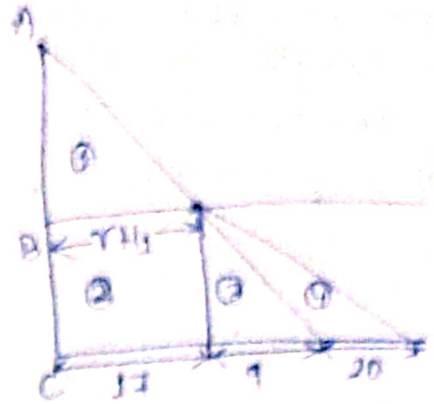
$$P_0 = K_0 \gamma \times H$$
$$= 0.5 \times 17 \times 2$$
$$= 17\ KN/m^2$$

At point 'C' the stress

$$\bar{\sigma_z} = \gamma H_1 + (\gamma_{sat} - \gamma_w) H_2$$
$$= 17 \times 2 + (9 - 10)2$$
$$= 32\ KN/m^2$$

$$P_0 = K_0 \times \bar{\sigma_z}$$
$$= 0.5 \times 32 = 16\ KN/m^2$$

Nutral stress, $U = \gamma_w \times H_2$
$$= 10 \times 2 = 20\ KN/m^2$$



The figure shows that pressure distribution diagram.
The diagram has been divided into 4 parts
part ①, ②, ③, ④ respectively.
Let P_1, P_2, P_3, P_4 are the lateral earth pressure due to these parts.
Total lateral earth pressure at part-1

$$P_1 = \frac{1}{2} \times \gamma H_1 \times H$$
$$= \frac{1}{2} \times 17 \times 2 = 17 \text{ KN/m}$$

At part-2

$$P_2 = 17 \times 2$$
$$= 34 \text{ KN/m}$$

At part-3

$$P_3 = \frac{1}{2} \times 9 \times 2 = 9 \text{ KN/m}$$

At part-4

$$P_4 = \frac{1}{2} \times 20 \times 2 = 20 \text{ KN/m}$$

Total pressure $= P_1 + P_2 + P_3 + P_4$
$= 17 + 34 + 9 + 20$
$= 80 \text{ KN/m}$

The line of action P is determine by taking a moment about 'C'

$$\bar{z} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A}$$

$$= \frac{17 \times 0.667 + 34 \times 1 + 9 \times 0.667 + 20 \times 0.667}{80}$$

$$= 0.8085$$

(Ans)

iii) Passive Earth Pressure :—

When the wall rotates about its base sufficiently towards the backfill the triangular soil mass ABC comes to the state of plastic equilibrium & fails by sliding of the plane BC.

At that time the horizontally effective stress σ_h is the passive earth pressure

$$K_p = \frac{\sigma_h}{\sigma_v},$$

$\sigma_h = \sigma_1$ = Major principal stress

$\sigma_v = \sigma_3$ = Minor principal stress

$$K_p = \frac{\sigma_1}{\sigma_3} = \frac{1}{\cot^2\alpha} = \tan^2\alpha = \tan^2(45^\circ + \phi/2)$$

For the cohesionless soil,

$$\boxed{K_p = \frac{1+\sin\phi}{1-\sin\phi}}$$

The total lateral passive earth pressure,

$$P_p = \frac{1}{2} \times K_p \cdot \gamma \cdot H \times H$$

$$\Rightarrow \boxed{P_p = \frac{1}{2} \times K_p \gamma H^2}$$

* Hence passive earth pressure is 9 times of active earth pressure.

Active earth pressure due to cohesive soil i.e $c-\phi$ soil :—

From the Bell's equation

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$$

$$(\alpha = 45^\circ + \phi/2)$$

$\sigma_1 = \sigma_v$, $\sigma_3 = \sigma_h = P_a$

$$\therefore \sigma_1 - 2c\tan\alpha = \sigma_3 \tan^2\alpha$$

$$\Rightarrow \frac{\sigma_1}{\tan^2\alpha} - \frac{2c\tan\alpha}{\tan^2\alpha} = \sigma_3$$

$$\Rightarrow \sigma_1 K_a - 2c\sqrt{K_a} = \sigma_3$$

$$\boxed{\begin{aligned} K_p &= \tan^2\alpha \\ K_a &= \cot^2\alpha \\ \sqrt{K_a} &= \cot\alpha \end{aligned}}$$

$$\boxed{P_a = \gamma \cdot z \cdot K_a - 2c\sqrt{K_a}} \leftarrow \text{at 'z' depth}$$

At $z = z_0$

$$P_a = \gamma z_0 K_a - 2c\sqrt{K_a}$$

At $P_a = 0$

$$\gamma \cdot z_0 \cdot K_a = 2c\sqrt{K_a}$$

$$z_0 = \frac{2c\sqrt{K_a}}{\gamma K_a} = \frac{2c}{\gamma\sqrt{K_a}}$$

$$z_0 = \frac{2c}{\gamma \cot\alpha}$$

Q. A cohesive soil has unit weight of $19.7\ KN/m^3$, unit cohesion as $12\ KN/m^2$ & angle of internal friction $11°$. Calculate the critical height of vertical excavation that can be made without lateral supports.

Sol^n Given data:-

$w = 19.7\ KN/m^3$

$\phi = 11°$

$c = 12\ KN/m^2$

$H_c = ?$

$$H_c = \frac{4c\tan\alpha}{\gamma}$$

$$\alpha = 45° + \frac{\phi}{2}$$

$$= 45° + \frac{11°}{2} = 50.5$$

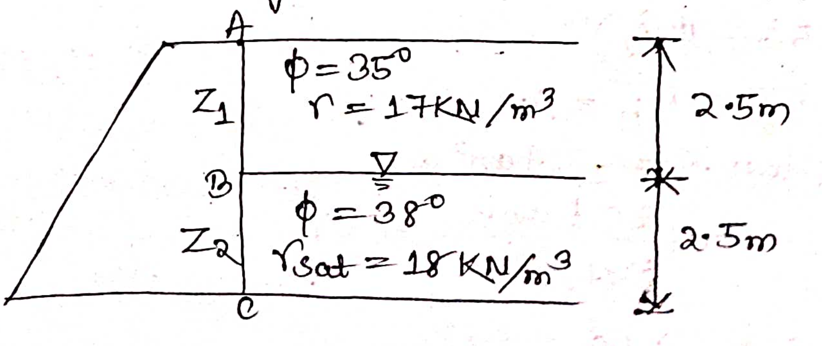
$\therefore H_c$ (critical height) $= \frac{4c\tan\alpha}{\gamma}$

$$= \frac{4 \times 12 \times \tan 50.5}{19.7}$$

$$= 2.95\ m$$

(Ans)

Q. Determine the active earth pressure on the retaining wall as shown in figure. Take $\gamma_w = 10\ KN/m^3$.



d" Given data:—

For upper layer,

$\phi = 35^0$

$\gamma = 17\ KN/m^3$

$Z_1 = 2.5\ m$

For bottom layer,

$\phi = 38^0$

$\gamma_{sat} = 18\ KN/m^3$

$Z_2 = 2.5\ m.$

Co-efficient of lateral earth pressure for upper layer,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{1-\sin 35^0}{1+\sin 35^0} = 0.271$$

Co-efficient of lateral earth pressure for lower layer,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{1-\sin 38^0}{1+\sin 38^0} = 0.238$$

At point 'B'

$$\sigma_Z = \gamma Z_1$$

$$= 17\times 2.5 = 42.5\ KN/m^2$$

Neutral axis, $U=0$

Active earth pressure at base

$$P_a = K_a \cdot \gamma \cdot Z$$

$$= 0.271 \times 42.5 = 11.5\ KN/m^2$$

Below the interface the active earth pressure,

$$P_a = K_a \cdot \gamma \cdot Z_1$$

$$= 0.238 \times 42.5 = 10.1\ KN/m^2$$

At point 'C'

$$\sigma_Z = \gamma \cdot Z_2 + \gamma' Z_2$$

$$= 17\times 2.5 + 18\times 2.5 = 87.5\ KN/m^2$$

The effective stress, $\sigma'_Z = \gamma Z_1 + \gamma' Z_2$

$$= \gamma Z_1 + (\gamma_{sat} - \gamma_w) Z_2$$

$$= 17\times 2.5 + (18-10)\times 2.5$$

$$= 17\times 2.5 + 8\times 2.5$$

$$= 62.5\ KN/m^2$$

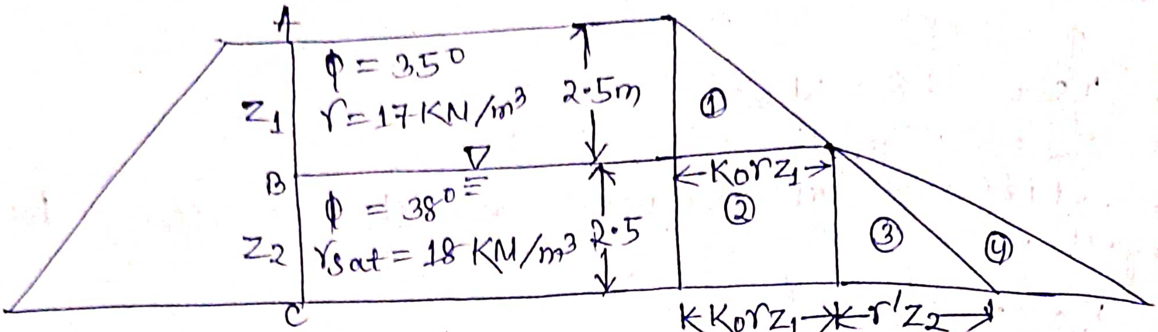
$U = \gamma_w \times h_p$

$= 10 \times 2{\cdot}5 = 25\ KN/m^2$

∴ The active earth pressure,

$P_a = K_a \times \sigma' z$

$= 0{\cdot}238 \times 62{\cdot}5 = 14{\cdot}875\ KN/m^2$



Pressure distribution diagram

The pressure distribution diagram divided into 4 parts. Part-1, part-2, part-3 & part-4 respectively. The forces P_1, P_2, P_3 & P_4 are determine from the pressure distribution diagram.

From Part-①

$P_1 = \frac{1}{2} \times (K_a \times \gamma \times Z_1) \times Z_1$

$= \frac{1}{2} \times (0{\cdot}271 \times 17 \times 2{\cdot}5) \times 2{\cdot}5 = 14{\cdot}4\ KN$

$1{\cdot}4$ KN act at $(2{\cdot}5 + H/3)$

$= \left(2{\cdot}5 + \frac{2{\cdot}5}{3}\right) = 3{\cdot}33$ m from base.

From Part-②

$P_2 = (K_o \cdot \gamma \cdot Z_1) \times Z_2$

$= (0{\cdot}238 \times 17 \times 2{\cdot}5) \times 2{\cdot}5 = 25{\cdot}3\ KN$

$2{\cdot}53$ KN act at $\left(\frac{H}{2}\right)$

$= \frac{2{\cdot}5}{2} = 1{\cdot}25$m from base.

From part-③

$P_3 = \frac{1}{2} \times (K_a \cdot \gamma' \cdot z_2) \times Z_2$

$= \frac{1}{2} \times K_a \times (\gamma_{sat} - \gamma_w) \times Z_2 \times Z_2$

$= \frac{1}{2} \times 0{\cdot}238 \times (18 - 10) \times 2{\cdot}5 \times 2{\cdot}5$

$= 5{\cdot}95 \simeq 6\ KN$

6 KN act at $(H/3)$

$= \left(\frac{2{\cdot}5}{3}\right) = 0{\cdot}833$ m from base

From part-(i)

$$P_4 = \frac{1}{2} \times (K_a \times \gamma_w \times Z_2) \times Z_2$$

$$= \frac{1}{2} \times (0.238 \times 10 \times 2.5) \times 2.5$$

$$= 7.43 \text{ KN}$$

7.43 act at $\left(\frac{H}{3}\right)$

$$= \frac{2.5}{3} = 0.833 \text{ m from base}$$

Total pressure,

$$A = P_1 + P_2 + P_3 + P_4$$

$$= 14.4 + 25.3 + 6 + 7.43 = 53.13$$

Taking moment about 'C'

$$\Sigma M_C = 0$$

$$\bar{z} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A}$$

$$= \frac{(14.4 \times 3.33) + (25.3 \times 1.25) + (6 \times 0.833) + (7.43 \times 0.833)}{53.13}$$

$$= 1.74 \text{ m from base}$$

Relationship betn Ka & Kp:—

$K_a \rightarrow$ Co-efficient of active earth pressure

$K_p \rightarrow$ Coefficient of passive earth pressure

$\phi \rightarrow$ Angle of internal friction

$\alpha \rightarrow$ Angle of shearing resistance

$\alpha = (45^\circ + \phi/2)$

$\tau_f = \sigma_1 \tan^2\alpha + 2C \tan\alpha$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$\phi = 30^\circ$

$K_a = 1/3$

$K_p = 3$

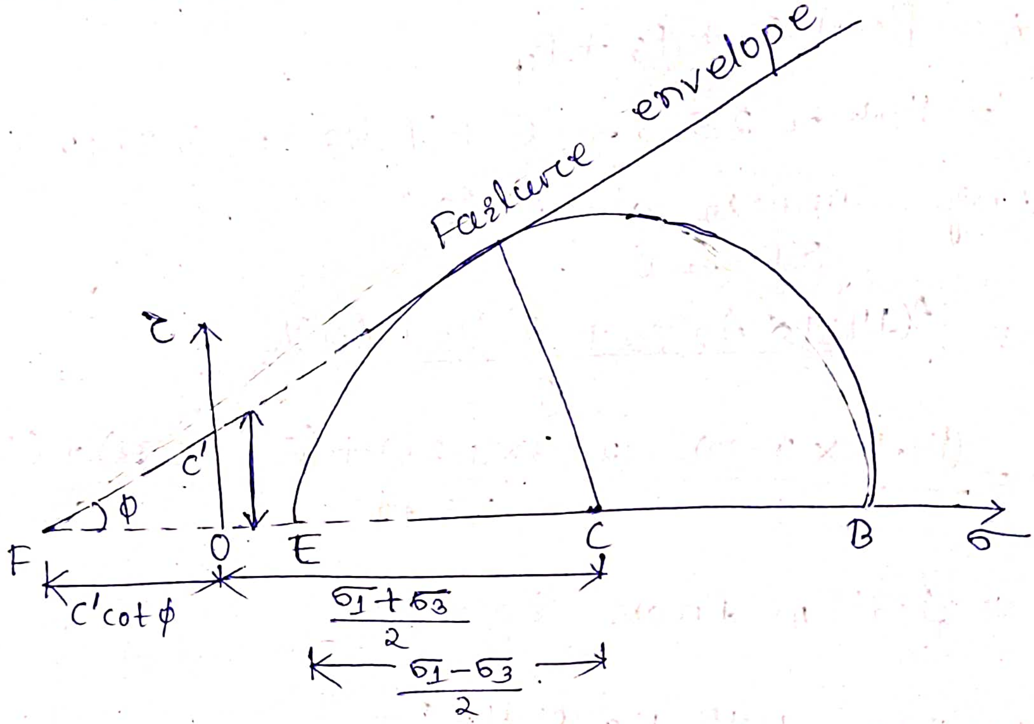
$$\frac{K_p}{K_a} = \frac{3}{(1/3)}$$

$$\frac{K_p}{K_a} = 9$$

$$\boxed{K_p = 9 K_a}$$

$$\boxed{K_a = \frac{1}{9} K_p}$$

Rankine's Earth Pressure in cohesive soil :-



The figure shows the Mohr's circle in which the point B indicates the vertical stresses & point E shows the active earth pressure.

σ_3 is called active earth pressure i.e. P_a

σ_1 = vertical stresses

$\sigma_v = \gamma \cdot Z$

$$P_a = K_a \cdot \gamma \cdot H - 2C'\sqrt{K_a}$$

Total active earth pressure,

$$\boxed{P_a = \frac{1}{2} K_a \gamma H^2 - 2C' \sqrt{K_a} \cdot H}$$

It is applicable before the formation of crack

$$P_a = \frac{1}{2} \cdot \gamma \cdot H^2 \cot^2\alpha - 2C H \cot\alpha$$

The active pressure after the tensile crack occurs,

Q. A retaining wall is subsaturated clay backfill is 7m high for the undrained condition ($\phi = 0$) of the backfill.

(i) Determine the maximum depth of tensile crack.

(ii) The active force before the tensile force occurs.

(iii) The active force after the tensile crack occurs.

Soln Given data :-

$\gamma = 16\ KN/m^3$

$C_u = 17\ KN/m^2$

$H = 7m$

Undrained condition, $\phi = 0$

Note

Using this Rankine's theory

$$P_a = \frac{1}{2} \cdot K_a \cdot \gamma \cdot H^2 - 2C\sqrt{K_a} \cdot H$$

$$P_a = K_a \cdot \gamma \cdot H - 2C'\sqrt{K_a}$$

(i) The maximum depth of the tensile crack

$$\frac{2C}{\gamma\sqrt{K_a}}$$

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{1-\sin 0}{1+\sin 0} = 1$$

$$z_0 = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2\times17}{16} = 2.125\,m$$

(ii) Active Force before the tensile crack occurs

$$P_a = \frac{1}{2}\times K_a\cdot\gamma\cdot H^2 - 2c'\sqrt{K_a}\cdot H$$

$$= \frac{1}{2}\times1\times16\times7^2 - 2\times17\times\sqrt{1}\times7$$

$$= 154\ KN/m$$ length of the wall

(iii) The active force after the tensile force occurs

$$P_a = \frac{1}{2}K_a\cdot\gamma\cdot H^2 - 2c'\sqrt{K_a}\cdot H + \frac{2c^2}{\gamma}$$

$$= \frac{1}{2}\times1\times16\times7^2 - 2\times17\times1\times7 + \frac{2\times17^2}{16}$$

$$= 190.125\ KN/m$$ length of the wall

$$\boxed{\text{Critical height, } h_c = 2\times Z_0}$$

Graphical method :—

(1) Rebhan's method
(2) Culmon's graphical method
(3) Columb wedge theory

(1) Rebhan's method :—

Rebhan's represented a graphical method for the location of slip plane & the total active earth pressure.

Steps :-

1/ Draw the ground line & ϕ-line at angles β & ϕ respectively with the horizontal to meet in point D.

2/ Draw a semicircle on BD as a diameter.

3/ Through B draw a line BH at an angle ψ with BD. The line BH is called the earth pressure line 'or' ψ line.

4/ Through 'A' draw line AG parallel to the ψ-line.
5/ Draw GJ perpendicular to BD to meet the semicircle in J.
6/ With 'B' as a centre & BJ as radius, draw an arc to cut BD in E.
7/ Through E, draw EC parallel to the ψ-line, BC then represents the slip plane.
8/ With 'E' as a centre & EC as radius, draw an arc to cut BD in K. Then joint CK.
9/ Calculate the total active earth pressure.

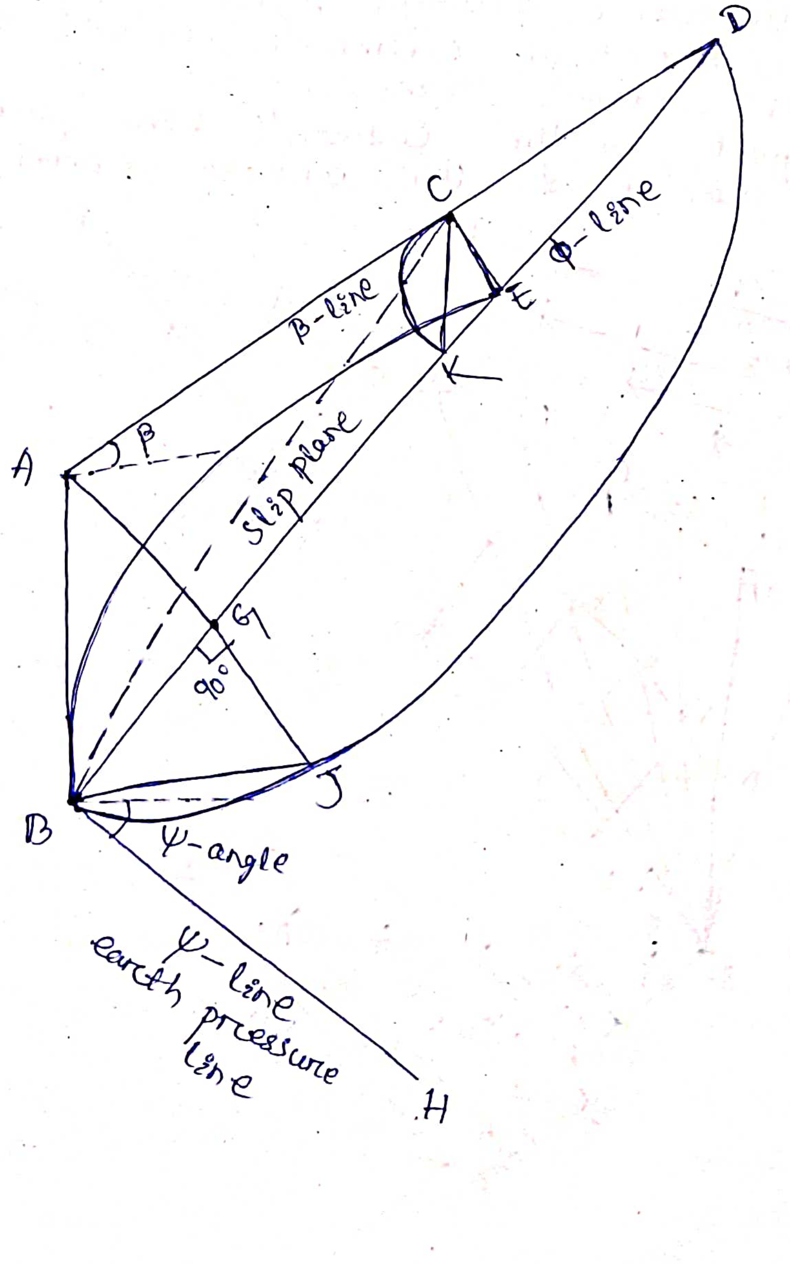
$$P_a = \gamma\ (\Delta KCE)$$

$$KE = CE$$

Let $x = CE \sin \psi$

$$PA = \frac{1}{2} K_a \gamma H^2$$

$$PA = \frac{1}{2} \gamma (CE)^2 \times \sin \psi$$

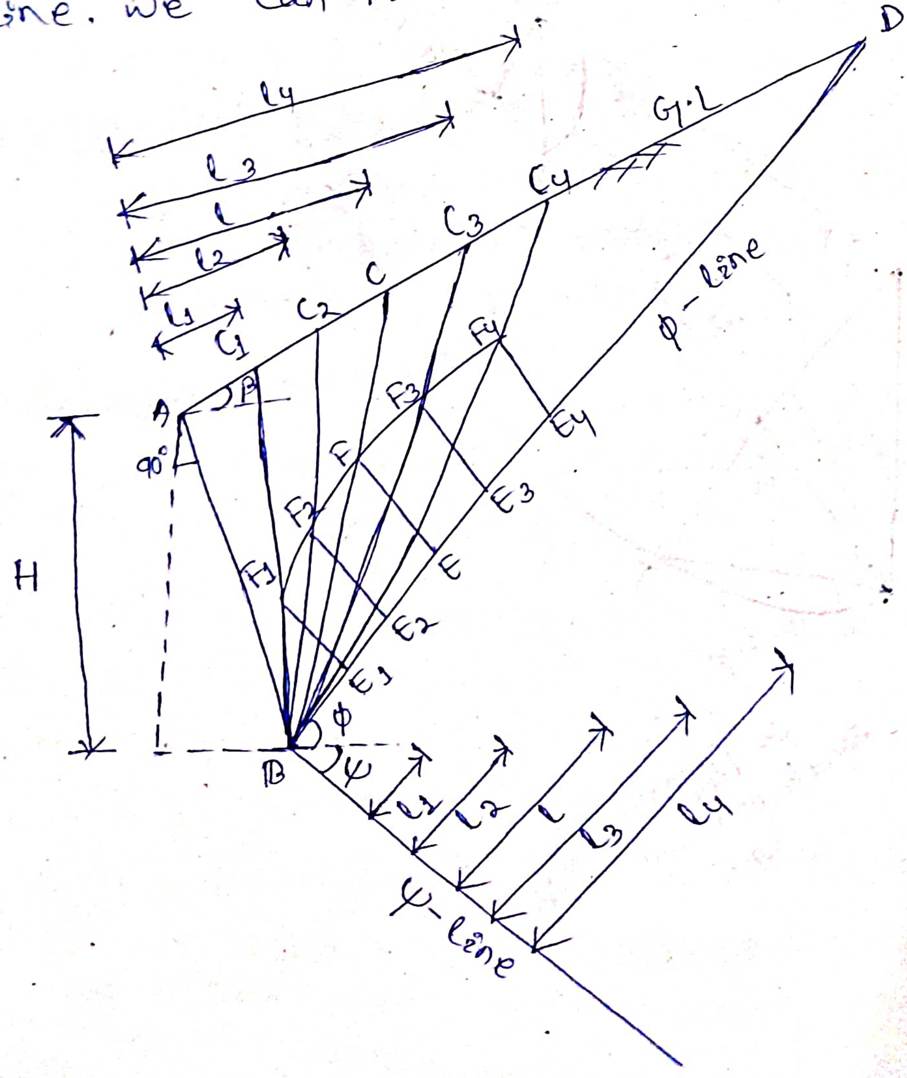


(2) Culmon's graphical methods for active earth pressure:

→ Culmon's theory is based on cohesionless soil.

Procedure:—

1/ Draw the wall profile closing a suitable scale.
2/ Draw groundline, ϕ-line & ψ-line.
3/ Take a slip plane BC, Calculate the weight of the wedge ABC. Plot it as BE, to some scale on the ϕ-line.
4/ Take another slip plane BC_1. Calculate the weight of wedge ABC_2 & plot it as BE_2 on the ϕ-line. Draw E_2F_2 parallel to the ψ-line cut the slip plane BC_2 in F_2.
5/ Take a number of such slip plate BC_3, BC_4. Plot the weight of the corresponding wedges of ϕ-line & obtained points F_3, F_4, through point 'B' - F_1, F_2, F_3, F_4, etc.
6/ Draw a smooth curve This curve is known as culmon's line. is known as culmon's curves & the tangent line Draw a tangent to the culmon's line parallel to the ϕ-line. We can find the active earth pressure.



(3) Coulomb's wedge Theory :-

→ The wedge theory of earth pressure is based on the concept of a sliding wedge which is torn up from the rest of the backfill on movement of the wall.

→ In the case of active earth pressure the sliding wedge moves downward & outwards on a slip surface relative to the in act of backfill.

→ In the case of passive earth pressure the sliding wedge moves upward & inward.

Assumptions of the wedge theory :-

1/ The backfill is dry, cohesionless, homogeneous, isotropic & elastically undeformable but breakable.

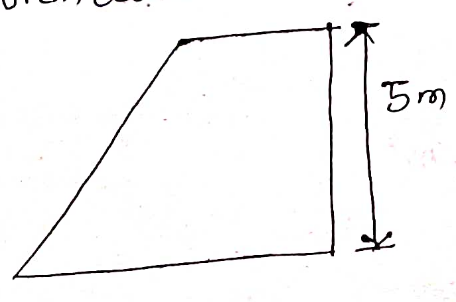
2/ The slip surface is plain which pass through the hill of the wall.

3/ The position & direction of the resultant earth pressure are known. The resultant pressure acts on the back of the wall at one-third the height of the wall from the base & is inclined at an angle Δ to the normal to the back.

4/ The sliding wedge itself acts as a rigid body & the value of earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.

5/ The forces acting on a wedge of soil are :
(i) Its weight (W)
(ii) Reaction
(iii) Active thrust (P_A)

Q. 5 m high retaining wall as shown in figure. Determine the Rankine active earth pressure on the wall.
(i) Before the formation of crack
(ii) After the formation of crack



$\phi = 30°$

$c = 50 \, KN/m^2$

$\gamma = 17.5 \, KN/m^2$

$H = 5 \, m$

Sol^n Given data:—

$\phi = 30°$

$C = 50\ KN/m^2$

$\gamma = 17.5\ KN/m^2$

$H = 5m$

Co-efficient of lateral earth pressure at rest condition

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

(i) Before tensile crack occurs

$$P_a = \frac{1}{2} K_a \cdot \gamma \cdot H^2 - 2 \cdot C \cdot H \cdot \sqrt{K_a}$$

$$= \frac{1}{2} \times \frac{1}{3} \times 17.5 \times (5)^2 - 2 \times 50 \times 5 \times \sqrt{\frac{1}{3}}$$

$= -215.75$ KN/m Length of the wall

(ii) After tensile crack occurs

$$P_a = \frac{1}{2} K_a \gamma H^2 - 2 \cdot C H \sqrt{K_a} + \frac{2C^2}{\gamma}$$

$$= \frac{1}{2} \times \frac{1}{3} \times 17.5 \times (5)^2 - 2 \times 50 \times 5 \times \sqrt{\frac{1}{3}} + \frac{2 \times 50^2}{17.5}$$

$= 69.95$ KN/m Length of the wall (Ans)

Q A vertical excavation has made in a clay deposite having a weight of 20 KN/m³. It caved in after the depth of digging reached 4m. Taking the angle of internal friction to be zero. Calculate the value of cohesion. If the same clay is used as a backfill against a retaining wall upto a height of 8m.

(i) Calculate active earth pressure

(ii) Calculate total passive earth pressure

Assume that the wall yields for enough to allow Rankine deformation condition to established.

Sol^n Given data:—

weight $= 20\ KN/m^3$

Height, $H_1 = 4m$

$\phi = 0$

$C = ?$

Height, $H_2 = 8\,m$

$P_a = ?$

$P_p = ?$

The critical height, H_c of an unsupported vertical cut in cohesive soil is given by a

$$H_c = \frac{4c \tan\alpha}{\gamma}$$

At $\phi = 0$

$$\tan\alpha = \tan\left(45^\circ + \frac{\phi}{2}\right)$$

$$= 1$$

From above formula

$$H_c = \frac{4C \tan\alpha}{\gamma}$$

$$\Rightarrow C = \frac{H_c \gamma}{4 \tan\alpha}$$

$$= \frac{4 \times 20}{4 \times 1} = 20\,KN/m^2$$

(i) Total active earth pressure,

$$P_a = \frac{1}{2} \gamma H^2 \cot^2\alpha - 2 \cdot C \cdot H \cdot \cot\alpha$$

$$= \frac{1}{2} \times 20 \times (8)^2 \times \cot^2(1) - 2 \times 20 \times 8 \times \cot(1)$$

$$= \frac{1}{2} \times 20 \times 8^2 \times \frac{1}{1} - 2 \times 20 \times 8 \times \frac{1}{1}$$

$$= 320\ KN/m$$

(ii) Total passive earth pressure,

$$P_p = \frac{1}{2} \times \gamma \times H^2 \times \tan^2\alpha + 2C \times H \times \tan\alpha$$

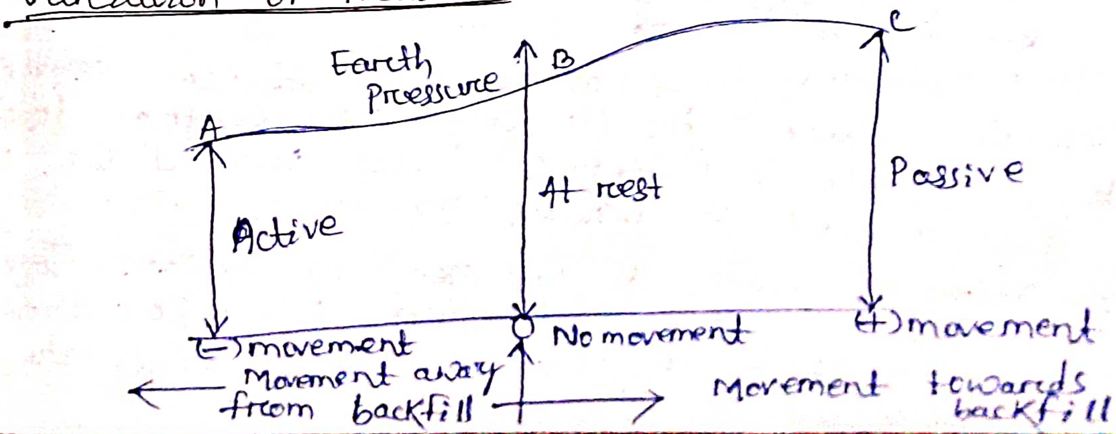
$$= \frac{1}{2} \times 20 \times (8)^2 \times \tan^2(1) + 2 \times 20 \times 8 \times \tan(1)$$

$$= \frac{1}{2} \times 20 \times (8)^2 \times \frac{1}{1} + 2 \times 20 \times 8 \times \frac{1}{1}$$

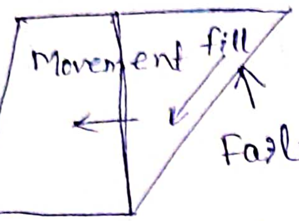
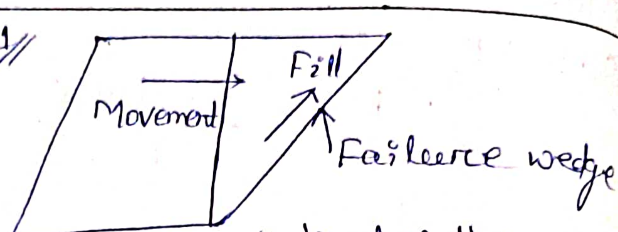
(Ans)

$$= 960\ KN/m$$

Variation of Pressure :—

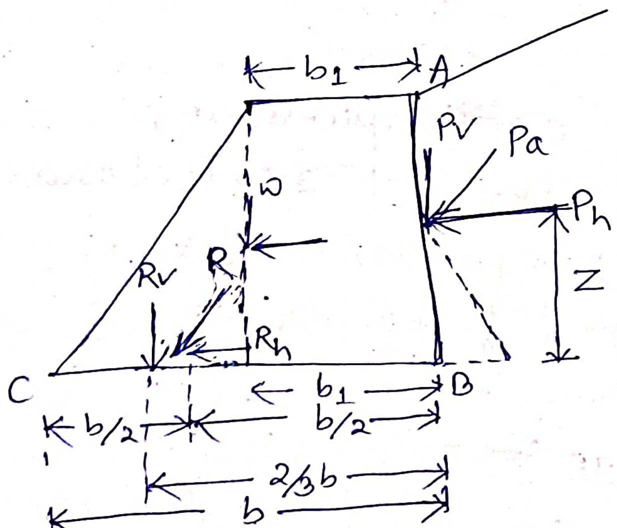


Differentiate bet^n Active earth pressure & passive earth Pressure :-

Active earth Pressure	Passive earth Pressure
1/  The figure indicates the active earth pressure, when the wall moves away from backfill. Some portion of backfill located immediately behind the wall tries to away from the rest of the soil mass.	1/  The figure indicates the passive earth pressure when the wall moves towards the backfill. The earth pressure increases. The failure wedge moves upward & inward.
2/ The lateral earth pressure exerted on the wall is a minimum in this case.	2/ The maximum value of the earth pressure is the passive earth pressure.
3/ The soil is at the verge of failure due to decrease in the lateral stresses.	3/ The soil is at verge of failure due to an increase in the lateral pressures.

Stability of Retaining Wall :-

Design of gravity retaining wall :-



Where,
R = Resultant force
b = bottom width
b_1 = Top width
w = self height of
R = Total Resultant force
R_h = Resultant component in horizontal direction
R_v = Resultant component in vertical direction

→ A gravity retaining wall is the wall which resist the lateral earth pressure by its weight in contract to the Cantilever & counterfort retaining wall in which the pressure is resisted by bending action.

→ A gravity retaining wall is thickner in section. They are constructed of mass of concrete brick or stone masonry. The criteria of gravity retaining wall are ;

1/ The base width of the wall must be such that the max^m pressure exerted on the foundation soil doesn't exceed the safe bearing capacity of the soil.
2/ The wall must be safe against overturning.
3/ The wall must be safe against sliding.
4/ Non - tension should be develop anywhere in the wall.

Let P be the resultant active earth pressure acting on the wall space.
P_h & P_v are its component in horizontal & vertical direct.
'w' is the weight of the wall acting its centroid.
The resultant R will pass through the point of intersection of w & Pa & it can be resolve into vertical & horizontal component R_v & R_h.

$$\boxed{R_v = w + P_v}$$

$$\boxed{R_h = P_h}$$

To find the distance of the point of application are,
Taking moment at B = 0

$$\Sigma M_B = 0$$

$$\Rightarrow R_v \bar{x} = w x_1 + P_v x_2 + P_h z$$

$$\Rightarrow \bar{x} = \frac{w x_1 + P_v x_2 + P_h z}{R_v}$$

$$= \frac{w x_1 + P_v x_2 + P_h z}{w + P_v}$$

$$\bar{x} = \frac{\Sigma M}{\Sigma V}$$

Where, ΣM = Sum of moments of all forces about B
i.e. $w x_1 + P_v x_2 + P_h \cdot z$

ΣV = Sum of the vertical force i.e $w + P_v$.

The resultant vertical force $R_v = \Sigma V$ acts eccentrically on the base.
The bearing pressure on the soil beneath the base are the combination of direct & bending stress.

$$f_1 = \frac{R_v}{b}\left(1 + \frac{6e}{b}\right)$$

$$f_2 = \frac{R_v}{b}\left(1 - \frac{6e}{b}\right)$$

where, b = Base width of wall

e = Eccentricity or the distance from the mid point of the base to the point of application of 'R' i.e. $\bar{x} - \frac{b}{2}$.

If $e = \frac{b}{6}$, then $f_1 = \frac{2R_V}{b}$

$f_2 = 0$

When $e > b/6$, tension is developed at B.
Since the soil is generally considered incapable to resist tension.

If $e < b/6$, f_1 & f_2 both are compressive.

* The following are the criteria of design of gravity retaining wall :-

1/ For no tension develop, $e < b/6$.

2/ For no sliding to occur, $R_h < R_v \mu$.

3/ The maximum pressure f_1 should not exceed the bearing capacity of the soil.

Hence the factor of safety again sliding,

$$F = \frac{R_v \cdot \mu}{R_h}$$

The minimum factor of safety $= 1.5$

4/ For the wall to be stable against overturning are must be pass within the base width.

Q. Design a gravity retaining wall 5m high with vertical back to retain dry cohesionless backfill of unit weight 18 KN/m³ & angle of shearing resistance is 30°. Find also the factor of safety against sliding assuming angle of friction between base of the wall & the foundation of the soil as 30°. The wall into be 1m wide at top & to be constructed of brick masonry having unit weight of 20 KN/m³ using Rankine's theory.

Solⁿ Given data :-

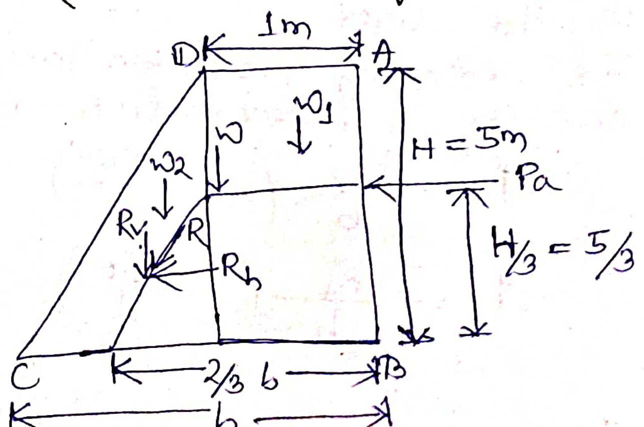
$H = 5$ m

$\gamma = 18$ KN/m³

Cohesionless soil, $\phi = 30°$

Top width $= 1$m

Unit weight $= 20$ KN/m³



Co-efficient of lateral earth pressure

$$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$$P_A = \frac{1}{2} \times K_a \times r \times H \times H$$

$$= \frac{1}{2} \times \frac{1}{3} \times 18 \times 5 \times 5$$

$$= 75 \text{ KN/m. Length of the wall}$$

Let b_1 width of the wall at it's top

$b_1 = 1m$

b_2 width of the wall at it's bottom

$b_2 = w$

The max^m lateral earth pressure occures at a height of $\frac{h}{3} = \frac{5}{3}$ m from the base of the wall.

Here, w = self weight of the wall.

w_1 = Self weight of the rectangular

w_2 = self weight of the triangular

The moment of the forces

w & P_a must be equal to the moment of the resulting force.

$\Sigma M_B = 0$

$$R_v \times \frac{2}{3} b = \left(w_1 \times \frac{b_1}{2}\right) + w_2 \left(b_1 + \frac{b-b_1}{3}\right) + P_a (H/3)$$

$$R_v = w = \left(\frac{b+b_1}{2}\right) \rho \times H = \left(\frac{b+1}{2}\right) \times 20 \times 5 = 50(b+1)$$

$$w_1 = (b_1 \times H)\rho = 1 \times 5 \times 20 = 100 \text{ KN}$$

$$w_2 = \frac{1}{2}(b-1) \times H \times \rho = \frac{1}{2}(b-1) \times 5 \times 20 = 50(b-1)$$

$$R_v \times \frac{2}{3} \times b = \left(w_1 \times \frac{b_1}{2}\right) + w_2\left(b_1 + \frac{b-b_1}{3}\right) + P_a \times (H/3)$$

$$50(b+1) \times \frac{2}{3} \times b = \left(100 \times \frac{1}{2}\right) + 50(b-1) \times \left(1 + \frac{b-1}{3}\right) + 75 \times (5/3)$$

$$\Rightarrow (50b+50) \times \frac{2}{3} b = 50 + (50b-50)\left(\frac{3+b-1}{3}\right) + 125$$

$$\Rightarrow 50b \times \frac{2}{3} b + 50 \times \frac{2}{3} b = 50 + (50b-50) \times \left(\frac{2+b}{3}\right) \times 125$$

Factor of safety $= \frac{\mu W}{P_a}$

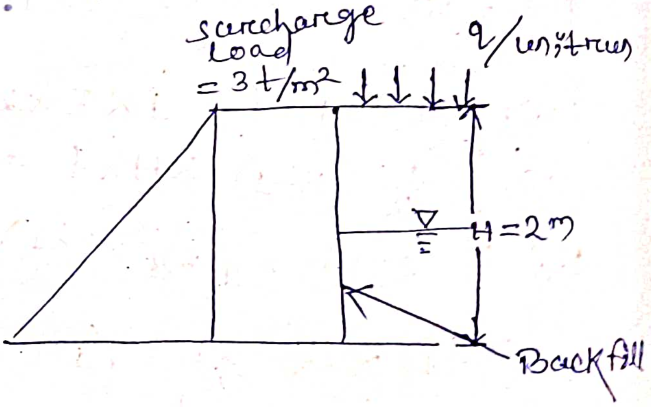
$= \frac{175 \tan\phi}{75} = 1.35$

Q. A retaining wall 2 m high has a smooth vertical surface the backfill has a horizontal levelled surface with the top of the retaining wall. The density of the backfill is 1.8 tone/m³, shearing resistance angle of 30° & cohesion is 0. A uniformly distributed surcharge load of 3t/m² Intensity is acting on the backfill.

(a) Calculate the magnitude & point of application of active earth pressure per metre length of the retaining wall.

(b) If during rainy season water table raises behind the wall to a height of 1 m from the base of the wall. Work out the effect on the value of active earth pressure. If there is no change of the internal angle of shearing resistance surcharge unit weight of the backfill 1.25 t/m³.

Soln:- Given data:-



Height $= H = 2\,m$

Density $(\rho) = 1.8\ t/m^3$

Angle of shearing resistance $= 30° = \phi$

Cohesion $(C) = 0$

Surcharge load $= 3\ t/m^2$

(a) Magnitude = ?

Point of application (Active pressure) = ?

Water table position = 1 m from the base

Active earth pressure = ?

Surcharge unit weight of backfill = $1.25\ t/m^3$

Co-efficient of lateral earth pressure,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30}{1+\sin 30}$$

$$= \frac{1}{3}$$

$$P_{a1} = K_a \cdot r$$
$$= \frac{1}{3} \times 3 = 1\ t/m^2$$

$$P_{a2} = K_a \cdot r \cdot H$$
$$= \frac{1}{3} \times 3 \times 2 = 2\ t/m^2$$

Active earth pressure,

$P_1 = K_a \cdot r \cdot H$

$= 1 \times 2 = 2\ t/m$ length of the wall

$P_2 = \frac{1}{2} \times K_a \cdot r \cdot H^2$

$= \frac{1}{2} \times \frac{1}{3} \times 3 \times 2^2$

$= 2\ t/m$ length of the wall

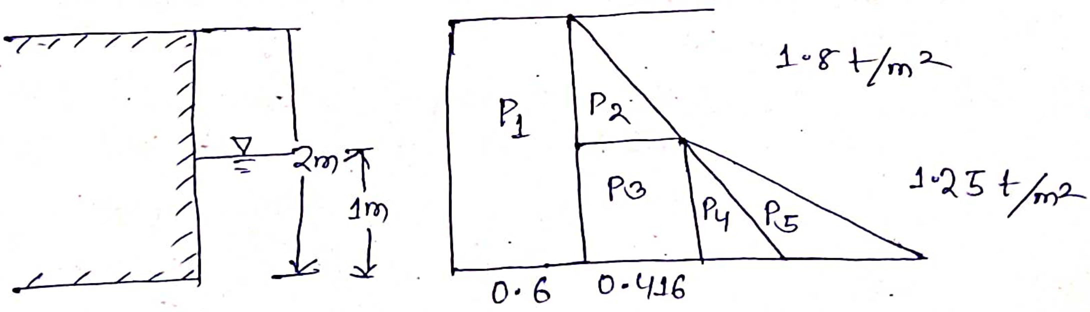
Acting at $\frac{H}{2} = \frac{2}{2} = 1\ m$ from the base of the wall

Acting at $\frac{H}{3} = \frac{2}{3} = 0.66\ m$ from the base of the wall

$P_a = P_1 + P_2$

$= 2 + 2 = 4$

(b) Water table increases 1 m from the base of the wall :-



P_{a1} (due to surcharge load) $= \frac{1}{3} \times 3 = 1\ t/m^2$

$P_{a2} = K_a \cdot r \cdot H = \frac{1}{3} \times 1.8 \times 1 = 0.6\ t/m^2$

$P_{a3} =$ same as $P_{a2} = 0.6\ t/m^2$

$P_{a4} = K_a \cdot r \cdot H = \frac{1}{3} \times 1.25 \times 1 = 0.416\ t/m^2$

$P_{a5} = \gamma_w \times H = 1 \times 1 = 1\ t/m^2$

Total active earth pressure are

$P_1 = 1 \times 2 = 2\ t/m$ length of the wall acting at $\frac{H}{2} = \frac{2}{2} = 1\ m$ from the base

$P_2 = \frac{1}{2} \times 0.6 \times 1 = 0.3\ t/m$ length of the wall acting at $\frac{H}{3} = \frac{1}{3} = 0.333\ m$ from the base

$P_3 = 0.6 \times 1 = 0.6\ t/m$ length of the wall acting at $\frac{H}{2} = \frac{1}{2} = 0.5\ m$ from the base

$P_4 = \frac{1}{2} \times 0.416 \times 1 = 0.208\ t/m$ length of the wall acting at $\frac{H}{3} = \frac{1}{3} = 0.333\ m$ from the base

$P_5 = \frac{1}{2} \times 1 \times 1 = 0.5\ t/m$ length of the wall acting at $\frac{H}{3} = \frac{1}{3} = 0.333\ m$ from the base

$$A = P_1 + P_2 + P_3 + P_4 + P_5$$
$$= 2 + 0.3 + 0.6 + 0.208 + 0.5$$
$$= 3.608$$

$$\bar{Z} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 + A_5\bar{y}_5}{A}$$

$$= \frac{2 \times 1 + 0.3 \times 0.33 + 0.6 \times 0.5 + 0.208 \times 0.33 + 0.5 \times 0.333}{3.608}$$

$$= 0.813\ m$$