Q.1 What are the various assumptions on which the design for the limit state of collapse in flexure is based?

[10 marks : 1995]

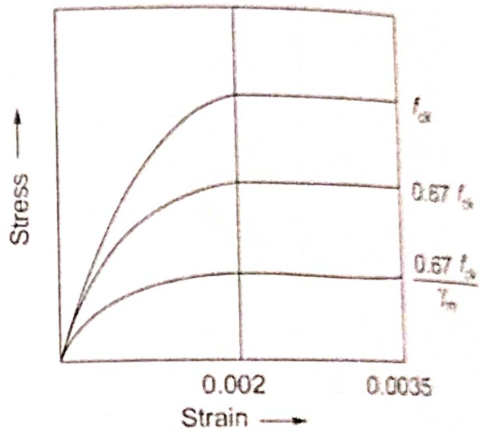
Solution:

As per IS : 456-2000, the design for the limit state of collapse in flexure shall be based on the assumptions given below:

(a) Plane sections normal to the longitudinal axis remain plane after bending.

(b) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.

(c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoidal, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve for concrete is given figure.



For design purposes, the compressive strength of the concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.

(d) The tensile strength of the concrete is ignored.

(e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. For design purposes, the partial safety factor $\gamma_m = 1.15$ shall be applied.

(f) The maximum strain in the tension reinforcement in the section at failure shall not be less than

$$\frac{f_y}{1.15E_s} + 0.002$$

where f_y = yield strength of steel

E_s = modulus of elasticity of steel

Q.2 A RC wall of 175 mm thickness and 3.2 m effective height is needed for a compressive load of 1000 kN/m. Design the wall using M15 grade concrete and mild steel reinforcement.

[15 marks : 1995]

Solution:

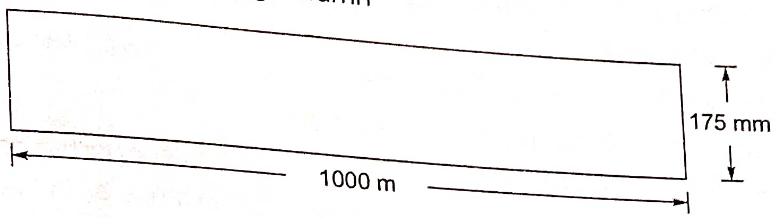
The wall should be designed for a length of 1 m.

Effective height, $H = 3.2 \text{ m} = 3200 \text{ mm}$

Thickness, $t = 175 \text{ mm}$

$$\text{Slenderness ratio} = \frac{H}{t} = \frac{3200}{175} = 18.29 > 12$$

Hence, it should be designed like a long column



$$C_R = 1.25 - \frac{H}{48t} = 1.25 - \frac{3200}{48 \times 175} = 0.869$$

We know that

$$P = C_R(\sigma_{cc} A_c + \sigma_{sc} A_{sc})$$

$$\Rightarrow \quad 1000 \times 10^3 = 0.869\,[(1000 \times 175 - A_{sc})\,4 + 130 \times A_{sc}]$$

$$\Rightarrow \quad A_{sc} = 3577.4 \text{ mm}^2$$

If thickness is more than 200 mm, then provide reinforcement on both sides but here thickness is less than 200 mm.

Reinforcement required on one face = 3577.4 mm^2

Assuming that 16 mm ϕ bars are used, then

$$\text{number of 16 mm } \phi \text{ bars} = \frac{3577.4}{\frac{\pi}{4} \times (16)^2} = 17.79 \approx 18 \text{ bars}$$

$$\text{Spacing of bars} = \frac{1000}{18} = 55.55 \text{ mm}$$

Hence, provide 9 No's 16 mm ϕ bars @ 55 mm c/c

Design of horizontal bars

$$A_{st} = \frac{0.25}{100} \times 1000 \times 175 = 437.5 \text{ mm}^2$$

$$\text{Number of 10 mm } \phi \text{ bars} = \frac{437.5}{\frac{\pi}{4} \times (10)^2} = 5.57 \approx 6 \text{ No's}$$

$$\text{Spacing of 10 mm } \phi \text{ bars} = \frac{1000}{437.5} \times \frac{\pi}{4} \times (10)^2 = 179.5 \text{ mm c/c}$$

Hence provide 6 No's of 10 mm ϕ bars at 170 mm c/c.

Q.3 What do you mean by a tread riser staircase? List out the steps for design. Draw a sectional elevation of this staircase showing the different reinforcements needed.

[15 marks : 1995]

Solution:

A stair may be defined as series of steps suitably arranged for the purpose of connecting different floors of a building at different elevations. It may also be defined as an arrangement of treads, risers, stringers, newel posts, hand rails and baluster, so designed and constructed as to provide an easy and quick access to the different floors rendering comfort and safety to the users. The enclosure containing the complete stair way is termed as staircase.

A staircase without any waist slab is called a tread riser staircase. In this type of staircase only riser and tread are provided so they are called tread riser staircase. They are also known as straight staircase.

Design steps for stairs:

(i) **Effective Span of Stairs:** Stair slab may be divided into two categories, depending upon the direction in which the stair slab spans.

(a) Stair slab spanning horizontally/transversely

(b) Stair slab spanning longitudinally(along the incline)

Stair slab spanning horizontally: In this category, the slab is supported on each side by side wall or stringer beam on one side and beam on the other side. Some times as in the case of straight stairs, the slab may also be supported on both sides by two side walls. In such a case the effective span L is the horizontal distance between centre to centre of supports.

Stair slab spanning longitudinally: In this category, the slab is supported at bottom and top of the flight and remain unsupported on the sides. The effective span of such stairs, without stringer beams, should be taken as follows:

(a) Where supported at top and bottom risers by beams spanning parallel with the risers, the distance between centre to centre of beams.

(b) Where spanning on the edge of a landing slab, which spans parallel, with the risers, a distance equal to the going of stairs plus at each end either half the width of the landing or one meter whichever is smaller.

(c) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as the distance c/c of the supporting beams or walls, the going being measured horizontally.

(ii) **Live load: IS : 875 - 1987 (Part-II)** give the loads for staircases. For stairs in residential buildings, office buildings, hospital wards, hostels, etc. where there is no possibility of over crowding, the live load may be taken to be 3000 N/m^2 subject to a minimum of 1300 N concentrated load at the unsupported end of each step for stairs constructed out of structurally independent cantilever step. For other public buildings liable to be over crowded, the live load may be taken by 5000 N/m^2.

(iii) **Distribution of loading on stairs:**

(a) In case of stairs with open walls, where spans partly crossing at right angles occur, the load on areas common to any two such spans may be taken as one-half in each direction.

(b) Where flights or landings are built into walls at a distance not less than 110 mm and are designed to span in the direction of the flight, a 150 mm strip may be deducted from the loaded area and the effective breadth of the section increased by 75 mm for the purposes of design.

(iv) **Estimation of dead weight:**

(a) **Dead weight of waist slab:** The dead weight w′ per unit area is first calculated at right angles to the slope. The corresponding load per unit horizontal area is then obtained by increasing w′ by the ratio $\frac{\sqrt{R^2+T^2}}{T}$ where R = rise and T = tread. Thus if t = thickness of waist slab in mm, then

$$w' = \frac{t \times 1 \times 1}{1000} \times 25000 = 25t \text{ N/m}^2 \text{ of inclined area}$$

Hence dead weight w_1 per unit horizontal area is given by

$$w_1 = w' \times \frac{\sqrt{R^2+T^2}}{T} = 25t\,\frac{\sqrt{R^2+T^2}}{T}$$

$$w_1 = 25t\sqrt{1+(R/T)^2}$$

(b) **Dead weight of steps:** The dead weight of steps is calculated by treating the steps to be equivalent horizontal slab of thickness equal to half the rise $\left(\frac{R}{2}\right)$. Thus if w_2 is the weight of step per unit horizontal area, we have

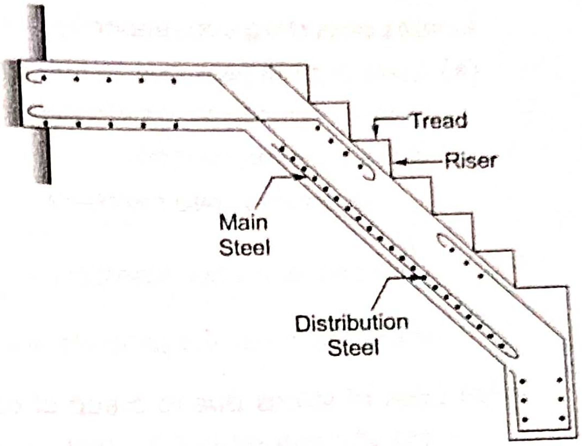
$$w_2 = \frac{R}{2\times 1000} \times 1 \times 1 \times 25000$$

$$= 12.5\ R\ \text{N/m}^2$$

Where R is rise in mm.

$$\text{Total } w = (w_1 + w_2)$$

per unit horizontal area



(v) **Depth of the Section:** The depth of the section shall be taken as the minimum thickness perpendicular to the soffit of the stair case. Once the depth of the section is finalized, area of reinforcement can be known too along with spacing.

Q.4 What are the different losses of prestress in pre-tensioned and post-tensioned beams. How are they estimated?

[10 marks : 1995]

Solution:

Loses of prestress take place in a prestressed concrete member due to many causes. In general these losses may be classified into the following:

(i) Loss of prestress during tensioning process

(ii) Loss of prestress at anchoring stage

(iii) Losses occurring subsequently

Loss of prestress during tensioning process: There always exist a certain amount of friction in the jacking and anchorage system and on the walls of the duct where the wires fan out at the anchorages with the result, the actual stress in the tendon is less than what is indicated by the pressure gauge. The losses due to friction in the jack and at the anchorage are different for different systems of prestressing. Considerable friction loss takes place due to friction between tendon and material surrounding it, namely the concrete or the sheathing. This loss due to friction may be classified into (a) loss due to length effect and (b) loss due to curvature effect. Thus the combined loss of prestress due to above two effects is given as

$$\text{Loss of prestress} = P_0\,(Kx + \mu\alpha)$$

where P_0 = Prestressing force in the prestressed steel at the tensioning end.

K = friction coefficient for wave effect whose value ranges between 15×10^{-4} per meter

μ = Coefficient of friction in curve

α = the cumulative angle in radians through which the tangents to the cable profile has turned between any two points under consideration.

x = distance of any point from jack

It may be noted that this loss occurs only in post tensioned members.

Loss of prestress at anchoring stage: This loss is due to the fact that the anchorage fixtures themselves are subjected to a stretch. It is also possible that the friction wedges holding the wires may slip a little. The loss of prestress due to anchorage slip is given by

$$\Delta f_s = \frac{\Delta a}{L} E_s$$

where Δa = anchorage slip

E_s = Young's modulus of elasticity

L = Length of the tendon

This loss is applicable to post tensioned members only.

Losses occurring subsequently: The losses which occur subsequent to prestressing are the following:

(a) Loss of prestress due to shrinkage of concrete

∴ loss of prestress = shrinkage strain × E_s

Value of shrinkage strain

for pretensioned members = 3×10^{-4}

for post tensioned members = $\dfrac{2 \times 10^{-4}}{\log_{10}(T+2)}$

where T is the age of concrete at transfer in days.

(b) Loss of stress due to creep of concrete

loss of prestress due to creep = $\phi m f_c$

where ϕ = creep coefficient = $\dfrac{\text{creep strain}}{\text{elastic strain}}$

m = modular ratio = $\dfrac{E_s}{E_c}$

f_c = original prestress in concrete

ϕ	age of loading
2.2	7
1.6	28
1.1	1 year

(c) Loss of stress due to creep of steel (stress relaxation)

Steel undergoes loss of prestress due to creep. Generally it is given as a percentage of initial stress. It varies from 1% – 5%. It may be normally adopted as 3%.

(d) Loss of prestress due to elastic shortening.

(i) For pretensioned member, loss of prestress is given by

$$\Delta f_c = m f_c$$

where m = modular ratio

f_c = stress in concrete at the level of steel

(ii) Post tensioned members:

The members get shortened due to compressive force applied over the surface at the time of tensioning. So if only one reinforcement is provided in post tensioned members, there will be no loss of prestress due to elastic shortening of the member because elastic shortening of concrete takes place before anchoring the steel. When there a number of bars tensioned simultaneously, then also loss of prestress due to elastic shortening is zero. But if they are tensioned one by one, the loss of prestress will occur in every bar except one which under goes tension in last.

.5 A post tensioned prestressed concrete beam 250 mm wide and has to be designed for a live load 10 kN/m across a span of 14 m. The stresses in concrete must not be exceed 17 MPa in compression and 1.4 MPa in tension. The loss of prestress may be assumed as 15%. Calculate

(a) minimum possible depth for the beam and
(b) for this depth, the minimum prestressing force and the corresponding eccentricity.

[15 marks : 1995]

Solution:

Imposed load, $q = 10 \text{ kN/m}, \eta = 1 - \frac{15}{100} = 0.85$

Width of section, $b = 250 \text{ mm}$

$f_{ct} = f_{cw} = 17 \text{ MPa}$

$f_{tt} = f_{tw} = -1.4 \text{ MPa}$

Span $= 14 \text{ m}$

Let overall depth of the section be h

Live load moment, $M_q = \frac{qL^2}{8} = \frac{10 \times 14^2}{8} = 245 \text{ kN-m}$

Dead load moment, $M_g = \frac{bh \times 25 \times 14^2}{8} = 612.5bh \text{ N-mm}$

Range of stress at bottom fibre, $f_{br} = \eta f_{ct} - f_{tw} = 0.85 \times 17 - (-1.4) = 15.85 \text{ MPa}$

(i) Minimum section modulus is given by

$$z_b = \frac{M_q + (1-\eta)M_g}{f_{br}}$$

$$\Rightarrow \quad \frac{bh^2}{6} = \frac{245 \times 10^6 + (1-0.85) \times 612.5bh}{15.85}$$

$$\Rightarrow \quad \frac{250 \times h^2}{6} = \frac{245 \times 10^6 + 0.15 \times 612.5 \times 250 \times h}{15.85}$$

$$\Rightarrow \quad 660.42\,h^2 = 245 \times 10^6 + 22968.75\,h$$

$$\Rightarrow 660.42\,h^2 - 22968.75h - 245 \times 10^6 = 0$$

$$\Rightarrow \quad h = 626.72 \text{ mm}$$

Take $h = 625$ mm

(ii) For the section provided $b = 250$ mm and $h = 625$ mm

Area of section, $A = 250 \times 625 = 156250 \text{ mm}^2$

$$z_b = z_t = \frac{bh^2}{6} = \frac{250 \times 625^2}{6} = 16.276 \times 10^6 \text{ mm}^3$$

Self weight moment, $M_g = 612.5\,bh = 612.5 \times 250 \times 625 = 95.7 \times 10^6 \text{ N-mm}$

$$f_{sup} = f_{tt} - \frac{M_g}{z_t} = -1.4 - \frac{95.7 \times 10^6}{16.276 \times 10^6} = -7.3 \text{ MPa}$$

$$f_{inf} = \frac{f_{tw}}{\eta} + \frac{M_g + M_q}{\eta z_b} = -\frac{1.4}{0.85} + \frac{(95.7 + 245) \times 10^6}{0.85 \times 16.276 \times 10^6} = 23 \text{ MPa}$$

Minimum prestressing force is given by

$$P = \frac{A\left(f_{inf} z_b + f_{sup} z_t\right)}{z_b + z_t} = \frac{156250 \times (23 - 7.3) \times 16.276 \times 10^6}{2 \times 16.276 \times 10^6}$$

$$= 1226562.5 \text{ N} = 1226.56 \text{ kN}$$

Corresponding eccentricity is given by

$$e = \frac{Z_t Z_b (f_{inf} - f_{sup})}{A(f_{inf} Z_b + f_{sup} Z_t)} = \frac{(16.276 \times 10^6)^2 \times (23 + 7.3)}{156250 \times 16.276 \times 10^6 \times (23 - 7.3)}$$

$$= 201.03 \text{ mm}$$

Q.6 What are the three assumptions made for design of reinforced concrete section for limit state of collapse in flexure that lead to the limiting value of depth of neutral axis? Calculate the limiting values of depth of neutral axis in terms of effective depth of section for two grades of steel having yield strength f_y = 250 and 415 N/mm².

[10 marks : 1996]

Solution:

The three assumptions that lead to the limiting depth of neutral axis are:

(i) Plane sections normal to the longitudinal axis remain plane after bending.

(ii) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.

(iii) The maximum strain in the tension reinforcement in the section at failure shall not be less than

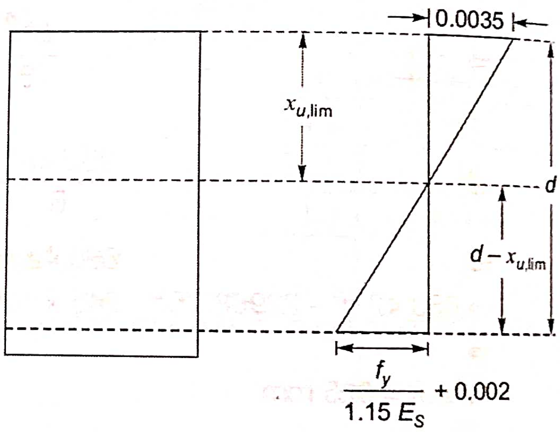
$$\frac{f_y}{1.15E_s} + 0.002$$

where f_y = yield strength of steel and

E_s = modulus of elasticity of steel

Derivation of limiting depth of neutral axis:

Limiting depth is calculated using strain diagram as suggested by **IS:456-2000**



Maximum strain in compression fibre at the topmost level in concrete = 0.0035

Maximum strain is steel = $\frac{f_y}{1.15E_s} + 0.002$

Since the strain diagram is linear, therefore from the property of similar triangles, we have

$$\frac{0.0035}{x_{u,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{d - x_{u,lim}}$$

$$\Rightarrow \quad \frac{d - x_{u,lim}}{x_{u,lim}} = \frac{\frac{f_y}{1.15E_s} + 0.002}{0.0035}$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} - 1 = \frac{\frac{f_y}{1.15E_s} + 0.002}{0.0035}$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} = 1 + \frac{\frac{0.87f_y}{E_s} + 0.002}{0.0035}$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} = \frac{0.0035 + \frac{0.87f_y}{E_s} + 0.002}{0.0035}$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} = \frac{0.0055E_s + 0.87 f_y}{0.0035 E_s} \qquad [E_s = 2 \times 10^5 \text{ N/mm}^2]$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} = \frac{0.0055 \times 2 \times 10^5 + 0.87 f_y}{0.0035 \times 2 \times 10^5}$$

$$\Rightarrow \quad \frac{d}{x_{u,lim}} = \frac{1100 + 0.87 f_y}{700}$$

$$\Rightarrow \quad \frac{x_{u,lim}}{d} = \frac{700}{1100 + 0.87 f_y}$$

$$\Rightarrow \quad x_{u,lim} = \left[\frac{700}{1100 + 0.87 f_y}\right]d$$

Now for

$$f_y = 250 \text{ N/mm}^2$$

$$x_{u,lim} = \left[\frac{700}{1100 + 0.87 \times 250}\right]d = 0.53d$$

and for

$$f_y = 415 \text{ N/mm}^2$$

$$x_{u,lim} = \left[\frac{700}{1100 + 0.87 \times 415}\right]d = 0.48d$$

Q.7 **A rectangular RC slab 2 m × 3 m is simply supported along shorter edges such that clear distance between the supporting wall is 2.7 m. The slab is 15 cm thick and reinforced with 16 mm diameter mild steel bars spaced at 25 cm c/c at effective cover of 25 mm along longer edges and with 10 mm diameter bars along shorter edges spaced at 25 cm c/c. Concrete used is M15 grade for which permissible stresses in bending, shear (nominal) and bond are 50, 3 and 6 kg/cm² respectively. Permissible tensile stresses in mild steel = 1400 kg/cm², m = 19. Calculate maximum safe intensity of load that the slab can carry in addition to its self weight.**

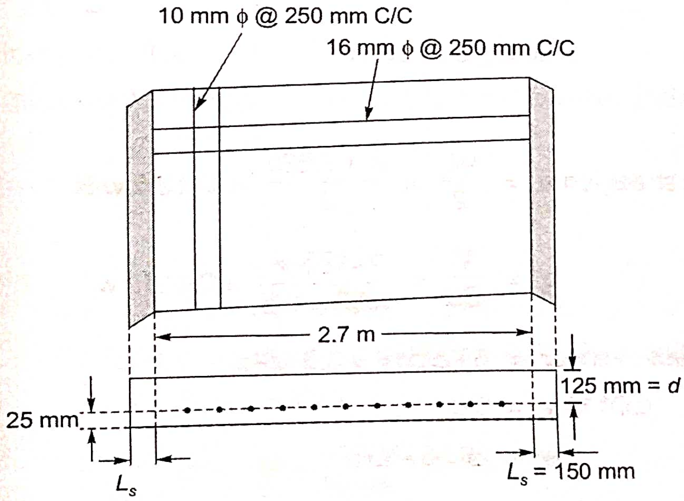
[15 marks : 1996]

Solution:

Assuming clear cover = 25 mm, width of support = 150 mm

Effective span for simply supported slab

(i) Clear span + effective depth = $L_0 + d = 2.7 + 0.125 = 2.825$ m



(ii) Centre to centre distance between supports = $L_0 + L_s = 2.7 + 0.15 = 2.850$ m
Lesser of (i) and (ii) is adopted.
$\therefore$ Effective span, $l_e = 2.825$ m
Let the total load including self weight of slab that can be carried by slab = w kN/m²

$$\text{Maximum bending moment} = \frac{wl_e^2}{8} = \frac{w \times (2.825)^2}{8} = 0.998 \text{ w kN-m} = 0.998 \times 10^6 \text{ w N-mm}$$

Now, m = 19, c = 50 kg/cm² = 5 MPa, t = 1400 kg/cm² = 140 MPa

$$x_c = \left(\frac{mc}{t+mc}\right)d = \left[\frac{19 \times 5}{140 + (19 \times 5)}\right] \times 125 = 50.53 \text{ mm}$$

$$A_{st} = \frac{1000}{250} \times \frac{\pi}{4} \times 16^2 = 804 \text{ mm}^2$$

Location of actual neutral axis

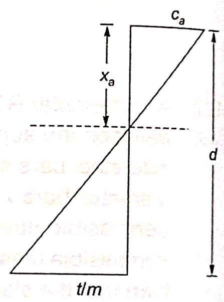
$$\frac{Bx_a^2}{2} = m A_{st}(d - x_a)$$

$$\Rightarrow \quad \frac{1000 \times x_a^2}{2} = 19 \times 804\,(125 - x_a)$$

$$\Rightarrow \quad 500\,x_a^2 + 15276\,x_a - 1909500 = 0$$

$$\Rightarrow \quad x_a = 48.38 \text{ mm}$$

$\because x_a < x_c$, section is under reinforced



Thus $\sigma_{st} = 140 = t$ and $c = \sigma_{cbc} > c_a$.

$$\therefore \quad \frac{c_a}{x_a} = \frac{t/m}{d - x_a}$$

$$\Rightarrow \quad c_a = \frac{140}{19} \times \frac{48.38}{(125 - 48.38)}$$

$$\Rightarrow \quad c_a = 4.65 \text{ N/mm}^2$$

Thus equating with MR formula, we have

$$0.998 \times 10^6\, w = B.x_a.\frac{c_a}{2}\left(d - \frac{x_a}{3}\right)$$

$$\Rightarrow \quad 0.998 \times 10^6 \times w = 1000 \times 48.38 \times \frac{4.65}{2}\left(125 - \frac{48.38}{3}\right)$$

$$\Rightarrow \quad w = 12.27 \text{ kN/m}^2$$

Checking for Shear

$$\text{Maximum shear force} = \frac{wl_e}{2} = \frac{w \times 2.825}{2} = 1412.5 \text{ w N}$$

$$\tau_v = \frac{V}{Bd} = \frac{1412.5\text{ w}}{1000 \times 125} = 0.0113 \text{ w}$$

Permissible stress in shear = 3 kg/cm² = 0.3 MPa

$$\therefore \quad 0.0113\text{ w} = 0.3$$

$$\Rightarrow \quad w = 26.55 \text{ kN/m}^2$$

$$k = \frac{mc}{t + mc} = \frac{19 \times 5}{140 + (19 \times 5)} = 0.404$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.404}{3} = 0.865$$

$$\text{Number of bars} = \frac{1000}{250} = 4 \text{ No's}$$

$$\text{Shear force, } V = 1412.5 \text{ w N}$$

ΣO = Sum of perimeters of steel bars

$$= 4 \times \pi \times 16 = 64\pi$$

Now,

$$\tau_{bd} = \frac{V}{\Sigma O . jd} = \frac{1412.5\text{ w}}{64\pi \times 0.865 \times 125}$$

Permissible value of bond stress = 6 kg/cm² = 0.6 MPa

$$\therefore \quad \frac{1412.5\text{ w}}{64\pi \times 0.865 \times 125} = 0.6$$

$$\Rightarrow \quad w = 9.24 \text{ kN/m}^2$$

The value of total load will be the lesser of the loads calculated in bending, shear and bond.

$$\therefore \quad w = 9.24 \text{ kN/m}^2$$

Maximum safe intensity of load excluding self weight

$$= 9.24 - (0.150 \times 1 \times 1 \times 25) = 5.49 \text{ kN/m}^2$$

Q.8 Design a square section column using M15 concrete and mild steel bars to carry an axial load (P) of 30,000 kg. Effective length of column (left) = 4 m. Assume permissible stresses in direct compression in M15 concrete (σ_{cc}) and in mild steel bars (σ_{sc}) as 40 and 1300 kg/cm², respectively. As per IS code

$P = C_R(\sigma_{cc} A_c + \sigma_{sc} A_s)$

where A_c and A_s are areas of cross-section of concrete and steel respectively.

C_R = Reduction Coefficient

$$= 1.25 - \frac{L_{eff}}{48B} \not> 1.0$$

B = lateral dimension of column.

Sketch the arrangement for longitudinal and lateral reinforcement. [15 marks : 1996]

Solution:

Assuming area of compression steel = 1% of gross area of column.

Let us design column as square shape and assume it as short column. Initially

$$P = \sigma_{cc} A_c + \sigma_{sc} A_{sc}$$

where symbols have their usual meaning

Given:

$$P = 30000 \text{ kg}$$

$$\sigma_{cc} = 40 \text{ kg/cm}^2$$

$$\sigma_{sc} = 1300 \text{ kg/cm}^2$$

$$30000 = 40 \times (B^2 - 0.01B^2) + 0.01\, B^2 \times 1300$$

$$B = 23.88 \text{ cm}$$

$$\Rightarrow \quad d = \frac{l_{eff}}{B} = \frac{400}{23.88} = 16.75 > 12$$

So, assumption failed as from above column is long

So we need to use, $C_R = 1.25 - \frac{l_{eff}}{48B}$

where, B = least lateral dimension, in case of rectangular/square shape column

$$C_R = \left(1.25 - \frac{400}{48B}\right)$$

$$P = C_R(\sigma_{cc}A_c + \sigma_{sc}A_{sc})$$

$$30000 = \left[1.25 - \frac{400}{48B}\right]\left[0.99B^2 \times 40 + 1300 \times 0.7B^2\right]$$

$$30000 = (52.6B^2)\left[1.25 - \frac{400}{48B}\right]$$

$$\Rightarrow \quad 30000 = \frac{(52.6B)\{(1.25 \times 48)B - 400\}}{48}$$

$$\Rightarrow \quad 1440000 = (52.6\,B)\,(60\,B - 400)$$

$$B = 24.95 \text{ cm}$$

So take 250 mm × 250 mm size square column. Provide 1% compression steel reinforcement

$$\Rightarrow \quad A_{sc} = 0.01 \times 250^2 = 625 \text{ mm}^2$$

Provide 4-Nos. 16 mm ϕ bars as spacing c/c between them

$$= B - 2\text{ (eff. cover)}$$

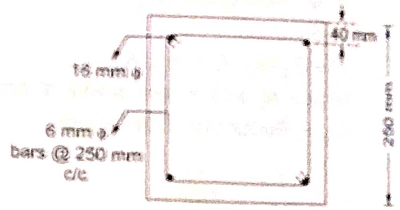
$$= 250 - 2 \times 40 = 170 \text{ mm} < 300 \text{ mm} \quad \text{(OK)}$$

Diameter of ties

$$\Rightarrow \quad \text{Maximum}\left(\frac{\phi_{mm}}{4}, 6 \text{ mm}\right)$$

$$\Rightarrow \quad \text{Maximum}\left(\frac{16}{4}, 6 \text{ mm}\right)$$

$$\Rightarrow \quad 6 \text{ mm}$$



Spacing of ties

$$\text{Minimum}\begin{bmatrix}\text{Least lateral} \\ \text{16 mm diameter of reinforcing bar} \\ \text{300 mm}\end{bmatrix}$$

$$\text{Minimum}\begin{bmatrix}250 \text{ mm} \\ 16 \times 16 = 256 \text{ mm} \\ 300 \text{ mm}\end{bmatrix}$$

∴ spacing @ 250 mm c/c.

Q.9 Explain the essential requirements of steel and concrete for prestressed concrete. What are the advantages of prestressed concrete over reinforced concrete?

[10 marks : 1996]

olution:

In order to get the maximum advantage of a prestressed concrete member, it is necessary to use both high strength concrete and high tensile steel wires.

High strength concrete is necessary for the following reasons:

(i) Since large prestressing forces are applied to the member by tendons, high bearing stresses are developed at the ends by anchoring devices. The anchorages are generally designed to be meant for use only for high strength concrete work.

(ii) Bursting stresses liable at the ends of the beam cannot be satisfactorily resisted by low strength concrete.

(iii) When the stress transfer to concrete has to take place by bond action, the concrete should have a high bond stress which can be offered only by high strength concrete.

(iv) Shrinkage cracks will be very little when high strength concrete is used.

(v) Due to high modulus of elasticity of high strength concrete, the elastic and creep strains are very small resulting in smaller loss of prestress in the steel reinforcement.

(vi) By using high strength concrete the cross sectional areas required for members will be reduced resulting in considerable reduction of dead load moments particularly in case of long span beams.

The mild steel used in ordinary reinforced concrete has a yield point of 200 MPa to 300 MPa. If such steel is used and if even it is subjected to a stress say 200 MPa at the stage of tensioning, we find that due to creep and shrinkage of concrete the net tensile stress left over will be extremely low. In the design of a prestressed concrete member, the estimated loss of prestress due to shrinkage and creep of concrete and steel is of the order of nearly 200 MPa. But high tension steel has an ultimate strength of 2100 MPa and if initially stressed to say 1000 MPa, there will still be large stresses in the reinforcement after making deduction for the loss of prestress.

The advantages of prestressed concrete over reinforced concrete are:

(i) In a RCC beam, the concrete in the compression side of the neutral axis alone is effective. The concrete in the tension side of the neutral axis is ineffective. But in a prestressed concrete beam the entire section is effective.

(ii) RCC sections are generally heavy. They always need shear reinforcements besides the longitudinal reinforcements for flexure. Prestressed concrete sections are lighter. By providing curved tendons and the precompression, a considerable part of the shear is resisted.

(iii) Prestressed concrete beams are suitable for heavy loads and long spans because for same load and span RCC beams are more heavy than PSC beams. They are slender and artistic treatments can be easily provided.

(iv) Cracks do not occur under working loads in PSC members. Even if minute cracks occur when overloaded, such cracks get closed when the overload is removed. The deflections of PSC beams are smaller than RCC beams.

(v) In RCC beams, there is no way of testing the steel and concrete. In PSC beams, testing of steel and concrete can be done while prestressing.

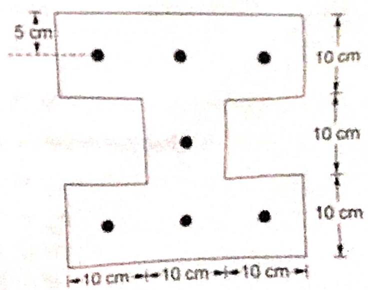
(vi) RCC construction does not involve many auxiliary units. But prestressed concrete constructions needs auxiliary units like prestressing equipment, anchoring etc.

Q.10 An I-section of concrete has following dimensions:

Flanges = 30 cm × 10 cm

Web = 10 cm × 10 cm

It is pretensioned by 7 steel wires of 8 mm diameter as shown in figure to an initial prestress of 10,000 kg/cm^2. Modular ratio m = 6. Calculate the stresses in concrete ad steel immediately on cutting the wires when the member is still supported in the prestressing bed. Assuming a loss of prestress of 2000 kg/cm^2 including loss due to elastic deformation, calculate the maximum moment to permit a maximum compressive stress of 120 kg/cm^2 and no tension in concrete. **[15 marks : 1996]**



Solution:

Area of cross section, $A = 300 \times 300 - 2 \times 100 \times 100 = 70{,}000 \text{ mm}^2$

Moment of inertia, $I = \frac{300 \times (300)^3}{12} - 2 \times \frac{100 \times (100)^3}{12} = 6.583 \times 10^8 \text{ mm}^4$

Initial prestressing force, $P = A_{st} f_0 = 7 \times \frac{\pi}{4} \times (8)^2 \times \frac{10{,}000 \times 1}{10 \times 10^3} = 351.858 \text{ kN}$

When the member is still supported in the prestressing bed the stress in

(i) concrete is, $f_c = \frac{P}{A} = \frac{351.858 \times 10^3}{70{,}000} = 5.03 \text{ MPa}$

(ii) steel is, $f_0 = 10{,}000 \text{ kg/cm}^2 = 1000 \text{ MPa}$

Loss of stress due to elastic shortening of concrete $= m f_c = 6 \times 5.03 = 30.18 \text{ MPa}$

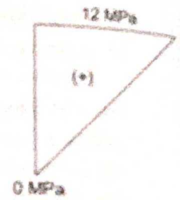
$\therefore f_s = f_0 - \text{Loss} = 1000 - 30.18 = 969.82 \text{ MPa}$

After considering a loss of prestress $= 2000 \text{ kg/cm}^2 = 200 \text{ MPa}$

we get, final stress in steel $= 1000 - 200 = 800 \text{ MPa}$

$\therefore$ Effective prestressing force,

$$P_e = 7 \times \frac{\pi}{4} \times (8)^2 \times 800 = 281486.70 \text{ N}$$



(+) shows compression

The final distribution of stress as per the problem should be

Let the maximum moment is M, then

$$f_{top} = \frac{P_e}{A} + \frac{My}{I}$$

and

$$f_{bot} = \frac{P_e}{A} - \frac{My}{I}$$

$\Rightarrow \quad 12 = \frac{281486.7}{70{,}000} + \frac{M \times 150}{6.583 \times 10^8}$ and $0 = \frac{281486.7}{70{,}000} - \frac{M \times 150}{6.583 \times 10^8}$

$\Rightarrow \quad M = 35.01 \text{ kN-m}$ and $M = 17.65 \text{ kN-m}$

Lower of the two will be adopted because if the maximum BM exceeds the lesser value i.e. 17.65 kN-m, the bottom fibre will be in tension which is undesirable. Hence $M = 17.65$ kN-m.

Q.11 A rectangular *RC* section 25 cm wide and 50 cm overall deep is reinforced with 3-16 mm diameter HYSD bars at an effective cover of 4 cm from bottom face. If permissible stresses in concrete in bending compression and steel are 50 kg/cm² and 2300 kg/cm² respectively, modular ratio m = 19, calculate the moment of resistance of the section using WSM.

[10 marks : 1997]

Solution:

Given data:

$B = 25 \text{ cm} = 250 \text{ mm}$

$D = 50 \text{ cm} = 500 \text{ mm}$

$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.19 \text{ mm}^2$

Effective cover $= 4 \text{ cm} = 40 \text{ mm}$

$\sigma_{cbc} = c = 50 \text{ kg/cm}^2 = 5 \text{ N/mm}^2$

$\sigma_{st} = t = 2300 \text{ kg/cm}^2 = 230 \text{ N/mm}^2$

$m = 19$

$d = D - 40 = 500 - 40 = 460 \text{ mm}$

(i) Calculating critical depth of NA

$$x_c = \left(\frac{mc}{t+mc}\right)d = \left(\frac{19\times5}{230+(19\times5)}\right)460 = 134.46 \text{ mm}$$

(ii) Calculating actual depth of NA by equating moment of area of compression side and tension side about NA

$$\Rightarrow \quad \frac{Bx_a^2}{2} = mA_{st}(d-x_a)$$

$$\Rightarrow \quad \frac{250}{2}x_a^2 = 19\times603.19\times(460-x_a)$$

$$\Rightarrow \quad x_a = 164.03 \text{ mm}$$

$\because x_a > x_c$, therefore section is over reinforced.

i.e.

$$c_a = \sigma_{cbc} = c$$

$$t_a < \sigma_{st} = t$$

$$\therefore \quad MR = B.x_a.\frac{c_a}{2}\left(d-\frac{x_a}{3}\right) = 250\times164.03\times\frac{5}{2}\times\left(460-\frac{164.03}{3}\right)$$

$$MR = 41.55 \text{ kN-m}$$

A rectangular *RC* beam simply supported at ends over an effective span of 5.0 m carries a UDL of 2000 kg/m including its own weight. If σ_{cbe} = 70 kg/cm², σ_{st} = 1900 kg/cm² and *m* = 13, design the beam section for flexure only by WSM. The size of the beam is restricted to 40 cm wide × 40 cm overall deep. Assume effective cover = 4.0 cm. Stress in compression reinforcement, if needed may be taken as 1.5 m times the stress in surrounding concrete.

[15 marks : 1997]

tion:

$$l_{eff} = 5 \text{ m}$$

$$w = 2000 \text{ kg/m} = 20 \text{ kN/m}$$

$$\sigma_{cbc} = c = 70 \text{ kg/cm}^2 = 7 \text{ N/mm}^2$$

$$t = \sigma_{st} = 1900 \text{ kg/cm}^2 = 190 \text{ N/mm}^2$$

$$m = 13$$

$$B = 40 \text{ cm} = 400 \text{ mm}$$

$$D = 40 \text{ cm} = 400 \text{ mm}$$

$$\text{Effective cover} = 4.0 \text{ cm} = 40 \text{ mm}$$

$$d = D-40 = 400-40 = 360 \text{ mm}$$

(i) Maximum $BM = \frac{w\,l_{eff}^2}{8} = \frac{20\times5\times5}{8} = 62.5$ kN-m

(ii) Design constants:

$$k = \left(\frac{mc}{t+mc}\right) = \frac{13\times7}{190+(13\times7)} = 0.3238$$

$$j = 1-\frac{k}{3} = 0.8920$$

$$Q = \frac{1}{2}cjk = \frac{1}{2}\times7\times0.8920\times0.3238 = 1.011$$

(iii) *MR* of the balanced section

$$M_1 = QBd^2 = 1.011\times400\times360\times360 = 52.41 \text{ kN-m}$$

(iv) BM > M_1, therefore a doubly reinforced section is required.

(v) Calculating area of steel for singly reinforced balanced section.

$$A_{st_1} = \frac{M_1}{\sigma_{st}\left(d-\frac{x_a}{3}\right)} = \frac{52.41\times10^6}{190\left(360-\frac{0.3238\times360}{3}\right)} \quad [\because x_a = x_c = kd]$$

$$A_{st_1} = 858.94 \text{ mm}^2$$

(vi) A_{st_2}, area of remaining tensile steel in the section with compression reinforcement

$$A_{st_2} = \frac{M_2}{\sigma_{st}(d-d_c)} = \frac{BM-M_1}{\sigma_{st}(d-d_c)} = \frac{(62.5-52.41)\times10^6}{190\times(360-40)}$$

$$A_{st_2} = 165.95 \text{ mm}^2$$

(vii) $$A_{st} = A_{st_1} + A_{st_2} = 858.94 + 165.95 = 1024.89 \text{ mm}^2.$$

(viii) A_{sc}, area of compression reinforcement

$$A_{sc} = \frac{m(d-x_a)}{(1.5m-1)(x_a-d_c)}\times A_{st_2} \quad [\because x_a = x_c = kd]$$

$$= \frac{13\times(360-116.57)}{(1.5\times13-1)(116.57-40)}\times 165.95$$

$$A_{sc} = 370.74 \text{ mm}^2$$

Q.13 Prove that the limiting moment of resistance 'M_u' of singly under reinforced rectangular concrete section using stress block parameters of code IS : 456-1978 is given by:

$$M_u = 0.87f_y\left(\frac{p_t}{100}\right)\left[1-1.005\frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)\right]bd^2$$

where b and d are width and effective depth of the beam, f_y and f_{ck} are characteristic strengths of steel and concrete.

$$p_t = \frac{A_s\times100}{bd}$$

where A_s = area of steel.

State the assumptions made.

Additional data required:

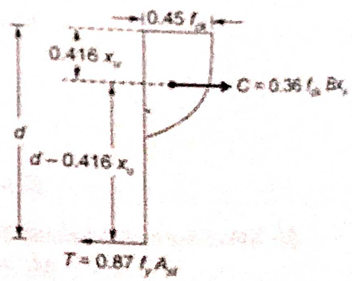
Area of stress block = $0.36f_{ck}x_u$

Depth of compressive force from the extreme in compression = $0.416x_u$

Total tension = $0.87f_yA_s$

Solution: **[15 marks : 1997]**

The given stress block



$$MR = \text{Tensile force}\times\text{Lever arm}$$

$$= 0.87\, f_y A_s (d-0.416\, x_u)$$

$$= 0.87\, f_y \frac{A_s}{Bd}(d-0.416\, x_u)\, Bd \quad \left[\because p_t = \frac{A_s}{Bd}\times100\right]$$

$$= 0.87\, f_y\left(\frac{p_t}{100}\right)d\left[1-0.416\frac{x_u}{d}\right]Bd$$

$$MR = 0.87 f_y \frac{p_t}{100}\left[1-0.416\frac{x_u}{d}\right]Bd^2 \quad \text{... (i)}$$

Now, in a singly reinforced section total compressive force = Total tensile force

$$0.36\, f_{ck} B x_u = 0.87\, f_y A_s$$

$\therefore$

$$x_u = \frac{0.87\, f_y A_s}{0.36\, f_{ck} B}$$

$\Rightarrow$

$$\frac{x_u}{d} = \frac{0.87\, f_y A_s}{0.36\, f_{ck} Bd} = 2.416\left(\frac{f_y}{f_{ck}}\right)\left(\frac{A_s}{Bd}\right) = 2.416\left(\frac{f_y}{f_{ck}}\right)\left(\frac{p_t}{100}\right)$$

$\Rightarrow$

Putting value of $\frac{x_u}{d}$ in (i), we get

$$MR = 0.87\, f_y \frac{p_t}{100}\left[1-0.416\times 2.416\left(\frac{f_y}{f_{ck}}\right)\left(\frac{p_t}{100}\right)\right]Bd^2$$

$\Rightarrow$

$$MR = 0.87\, f_y \left(\frac{p_t}{100}\right)\left[1-1.005\left(\frac{f_y}{f_{ck}}\right)\left(\frac{p_t}{100}\right)\right]Bd^2$$

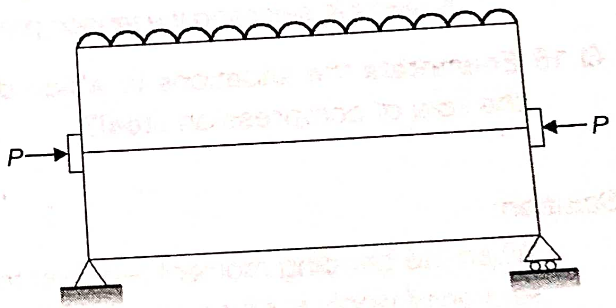
Q.14 Explain the different concepts as may be applied to explain and analyze the basic behaviour of prestressed concrete.

[10 marks : 1997]

Solution:

There are three different concepts which can be applied to analyse basic behaviour of prestressed concrete which are as follows:

(i) **Stress concept:** In this method stresses are calculated according to actual position of forces as per actual location of cable at different sections.



Above figure shows a simply supported beam of rectangular section prestressed by a tendon provided through its centroidal longitudinal axis. Let the beam be subjected to an external load system.

Let P be the prestressing force supplied by the tendon.

Due to this prestressing force, the compressive stress induced in concrete = $f_a = \frac{P}{A}$ where A is the cross sectional area of the member.

If due to external and dead loads, the bending moment at any section is M, then extreme stresses at that section due to bending moment alone will be

$$f_b = \pm\frac{M}{Z}$$

where Z is the section modulus of the beam section. Hence the final extreme stresses on the beam section are given by

$$f_{top} = \frac{P}{A} + \frac{M}{Z}$$

$$f_{bot} = \frac{P}{A} - \frac{M}{Z}$$

(ii) **Strength concept:** In contrast to the direct method of analysis of resultant stresses at a section of a PSC beam, the pressure line or thrust line concept can also be used to evaluate the stresses. In this method, the prestressed beam is analysed as a plain concrete elastic beam using basic principles of statics.

(a) The P-force which is tension in the tendon

(b) The C-force which is the compressive force acting on concrete.

In the absence of any moment, the C-force and P-force act at the same level. The line of action of the P-force is called P-line (cable line) which is nothing but the tendon line itself. The line of action of C-force is called C-line (pressure line). Hence in absence of any external bending moment the P-line and C-line coincide.

Suppose, the beam is subjected to a moment M, then the C-line will be shifted from the P-line by a distance 'a' called as the level arm which is given by

$$a = \frac{M}{P}$$

Now corresponding to the new position of the C-line and its eccentricity the stress distribution for concrete can be determined.

$$\text{Extreme stresses in concrete} = \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}$$

(iii) **Load balancing concept:** It is possible to select suitable cable profiles in a PSC member such that the transverse component of the cable force balances the given type of external loads.

The various types of reactions of a cable upon a concrete member depend upon the shape of the cable profile. Straight portions of the cable do not induce any reactions except at the ends, while curved cables results in UDLs. Sharp angles in a cable induce concentrated loads. The concept of load balancing is useful in selecting the tendon profile which can supply the most desirable system of forces in concrete.

Q.15 Enumerate the situations in which doubly reinforced concrete beams become necessary. What is the role of compression steel?

[10 marks : 1998]

Solution:

When the bending moment required to be resisted is more than the moment of resistance of a balanced section of singly reinforced beam of given size, there are two alternatives:

(i) To use an over-reinforced section.

(ii) To use doubly reinforced section.

An over reinforced section is always uneconomical and also undesirable because of sudden failure probability. Also the increase in the moment of resistance is not in proportion to the increase in the area of tensile reinforcement. The reason behind this is that the concrete, having reached maximum allowable stress, cannot take more additional load without adding compression steel. The other alternative is to provide reinforcement in the compression side of the beam and thus to increase the moment of resistance of the beam beyond that of a balanced section.

Doubly reinforced sections are also useful in following situations:

(i) Where the members are subjected to probable reversal of external loads and thereby the bending moment in the section reverses, such as in concrete piles etc.

(ii) When the members are subjected to loading, eccentric to either side of the axis, such as in columns subjected to wind loads.

(iii) When the members are subjected to accidental lateral loads, shock or impact.

The steel reinforcement provided in the compression zone is subjected to compressive stress. However, concrete undergoes creep strains due to continued compressive stress, with the result that the strain in concrete goes on increasing with time. This increases compressive strain in steel in addition to creep strain in compressive steel. Thus the total compressive strain in compressive steel will be much greater than the strain in surrounding concrete due to flexure alone. Thus, compressive steel takes up all the additional compressive stresses beyond the permissible compressive stress for concrete making the section safe against failure in flexure.

Q.16 State the assumptions made in the design for the limit state of collapse in flexure. Are these assumptions justified? Derive the stress block parameters for a rectangular cross-section.

[15 marks : 1998]

Solution:

The assumptions made in the design for the limit state of collapse in flexure are:

(i) Plane sections normal to the longitudinal axis remain plane after bending.

(ii) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.

(iii) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be a rectangle, trapezoid, parabola or any other shape which results in predictions of strength in substantial agreement with the results of the test. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.

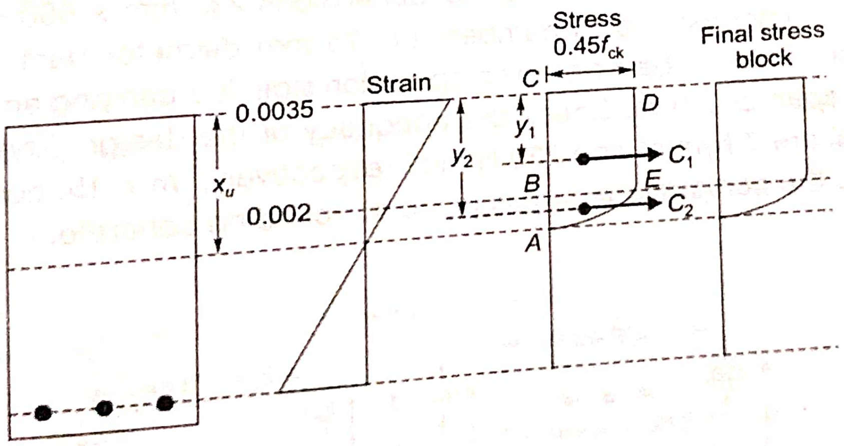
(iv) The tensile strength of the concrete is ignored.

(v) The stresses in reinforcement are derived from representative stress strain curve for the type of steel used. For design purposes, the partial safety factor $\gamma_m = 1.15$ shall be applied.

(vi) The maximum strain in the tension reinforcement in the section at failure shall not be less than

$$\frac{f_y}{1.15E_s} + 0.002$$

Derivation of stress block parameters:



Ratio of AB to AC i.e. $\frac{AB}{AC} = \frac{0.002}{0.0035}$ (from strain diagram)

$$\Rightarrow \quad \frac{AB}{AC} = \frac{4}{7}$$

$$\Rightarrow \quad AB = \frac{4}{7}AC$$

$$\Rightarrow \quad AB = \frac{4}{7}x_u$$

$$BC = AC - AB = x_u - \frac{4}{7}x_u = \frac{3}{7}x_u$$

Now, compressive force = width of section × Area of stress diagram

Taking rectangle *BCDE*

Compressive force $C_1 = B \times 0.45 f_{ck} \times \frac{3}{7}x_u = 0.1929 f_{ck} Bx_u$

Distance of line of action of compressive force C_1 from top

$$\Rightarrow \quad y_1 = \frac{1}{2} \times \frac{3}{7}x_u = \frac{3}{14}x_u$$

Taking parabola *ABEA*.

Compressive force, $C_2 = B \times \frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7}x_u = 0.1714 f_{ck} Bx_u$

Distance of line of action of compressive force C_2 from top

$$\Rightarrow \quad y_2 = \frac{3}{7}x_u + \frac{3}{8} \times \frac{4}{7}x_u = 0.64286x_u$$

Total compressive force $= C_1 + C_2 = 0.1929 f_{ck} Bx_u + 0.1714 f_{ck} Bx_u$

$$C = 0.36 f_{ck} Bx_u$$

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2} = \frac{0.1929 f_{ck} Bx_u \times \frac{3}{4}x_u + 0.1714 Bx_u \times 0.64286 x_u}{0.36 f_{ck} Bx_u}$$

$$\Rightarrow \quad \bar{y} = 0.416 x_u$$

Now total tensile force, $T = 0.87 f_y \times A_{st}$

Lever arm, $LA = d - 0.416 x_u$

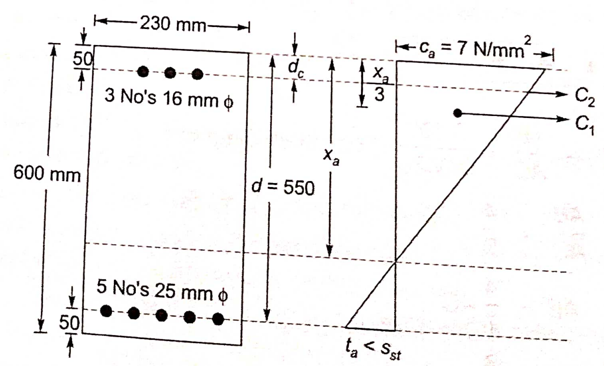
$$MR = 0.36 f_{ck} Bx_u (d - 0.416 x_u)$$

$$MR = 0.87 f_y A_{st} (d - 0.416 x_u)$$

Q.17 A rectangular beam of overall cross sectional dimensions 230 mm × 600 mm with 50 mm effective concrete cover is reinforced with 5 numbers of 25 mm diameter bars on the tension side and 3 numbers of 16 mm diameter bars on the compression side. It is carrying an imposed load of 50 kN/m over an effective span of 7.0 m. Check the adequacy of the design. The permissible stresses in concrete and steel are 7 N/mm² and 230 N/mm² respectively; m = 15; compressive stress in steel bars = 1.5m times the compressive stress in the surrounding concrete.

[15 marks : 1998]

Solution:



$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$$

$$A_{sc} = 3 \times \frac{\pi}{4} \times 16^2 = 603.19 \text{ mm}^2$$

$$M_{max} = \frac{w\, l_{eff}^2}{8}$$

(i)

Self weight of the beam = 0.230 × 0.600 × 1 × 25 = 3.45 kN/m

imposed live load = 50 kN/m

w = 53.45 kN/m

$$M_{max} = \frac{53.45 \times 7 \times 7}{8} = 327.38 \text{ kN-m}$$

(ii) Critical neutral axis,

$$x_c = \left[\frac{mc}{t + mc}\right] d = \left[\frac{15 \times 7}{230 + (15 \times 7)}\right] \times 550 = \left[\frac{105}{230 + 105}\right] \times 550$$

$$x_c = 172.39 \text{ mm}$$

(iii) Calculating actual depth of neutral axis:

Equating moment of area of compression side and tension side taken about actual NA i.e.

$$\frac{B\, x_a^2}{2} + (1.5\,m - 1)\, A_{sc} \times (x_a - d_c) = m\, A_{st}\,(d - x_a)$$

$$\Rightarrow \quad \frac{230}{2} x_a^2 + (1.5 \times 15 - 1) \times 603.18 \times (x_a - 50) = 15 \times 2454.37 \times (550 - x_a)$$

$$\Rightarrow \quad 115 x_a^2 + 12968.37\,(x_a - 50) - 36815.55\,(550 - x_a) = 0$$

$$\Rightarrow \quad x_a^2 + 112.77\,(x_a - 50) - 320.13\,(550 - x_a) = 0$$

$$\Rightarrow \quad x_a^2 + 432.90 x_a - 181710 = 0$$

$$\Rightarrow \quad x_a = 261.63 \text{ mm}$$

$\because$ $x_a > x_c$, hence the section is over reinforced

$c_a = \sigma_{cbc} = c = 7 \text{ N/mm}^2$

$t_a < \sigma_{st} = t = 230 \text{ N/mm}^2$.

(iv) Let c_a be the stress at the level of compression steel, then from similar triangles.

$$\frac{c_{a'}}{x_a - d_c} = \frac{c_a}{x_a}$$

$$\Rightarrow \quad c_{a'} = \frac{7}{261.63} \times (261.63 - 50)$$

$$\Rightarrow \quad c_{a'} = 5.66 \text{ N/mm}^2$$

(v) Moment of Resistance,

$$MR = B \times x_a \times \frac{c_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5\,m - 1)\, A_{sc} \times c_{a'}\,(d - d_c)$$

$$= 230 \times 261.63 \times \frac{7}{2} \times \left(550 - \frac{261.63}{3}\right) + (1.5 \times 15 - 1) \times (603.18 \times 5.66) \times (550 - 50)$$

$$= 134.17 \text{ kN-m}$$

$\because$ Moment of resistance of the given section is 134.17 kN-m which is less than maximum bendin moment applied (327.28 kN-m) on the section. Hence it is unsafe for the given loading.

Q.18 Comment on the following statements:

(a) In prestress concrete, dead load costs nothing.

(b) The shearing resistance of prestressed concrete is superior to that of reinforced concrete.

[8 marks : 1998]

Solution:

(a) The use of high strength concrete and steel in prestressed members results in lighter and slender members than is possible with reinforced concrete. The dead load moments are neutralized by the prestressing moment. The economy of prestressed concrete is well established for long span structures. Standard precast bridge beams between 10 m to 30 m long span and precast prestressed piles have proved to be more economical than steel and RCC. Thus, in the long span range, prestressed concrete is generally more economical than RCC. Also, the high strength concrete is very cheap to prepare as its cost does not increase in the same proportion as its strength does.

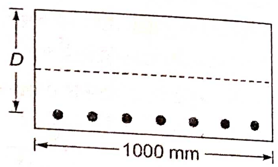
(b) Prestress concrete members possess improved resistance to shearing forces, due to the effect of compressive prestress which reduces the principal tensile stress. The use of curved cables particularly in long span members, helps to reduce the shear forces developed at the support sections.

In a prestressed concrete member, the shear stress is generally accompanied by a direct stress in the axial direction of the member and if transverse vertical prestressing is adopted, compressive stresses in the direction perpendicular to the axis of the member will be present in addition to the axial prestress. The direct stresses being compressive, the magnitude of the principal tensile stress is considerably reduced and in some cases even eliminated, so that under working loads both major and minor principal stresses are compressive thereby eliminating the risk of diagonal tension cracks in concrete.

Q.19 Design a prestressed concrete slab spanning 12 m carrying an imposed load of 20 kN/m². M40 concrete and steel with a ultimate tensile stress of 1600 N/mm² are used. The permissible stresses in concrete are 14 N/mm² in compression and zero in tension. Neglect the loss in prestress. Cables of 12 wires of 5 mm diameter capable of carrying an effective prestress of 225 kN are available. Indicate the zone in which resultant cable must lie.

[20 marks : 1998]

Solution:



Consider a 1 meter wide strip of the slab

Live load = 20 kN/m²

Moment due to live load, $M_1 = \frac{20 \times 12^2}{8} = 360$ kN-m

(i) Section modulus required,

$$Z = \frac{M_1}{f_c} = \frac{360 \times 10^6}{14} = 25714286 \text{ mm}^3$$

But

$$Z = \frac{BD^2}{6}$$

$$\Rightarrow \quad D^2 = \frac{25714286 \times 6}{1000}$$

$$\Rightarrow \quad D = 393 \text{ mm} \approx 400 \text{ mm}$$

(ii) Prestressing force

$$P = \frac{Af_c}{2} = \frac{1000 \times 400 \times 14}{2} = 2800 \text{ kN}$$

(iii) Bending moment due to dead load

$$\text{Dead load} = 0.4 \times 1 \times 1 \times 25 = 10 \text{ kN/m}^2$$

$$M_2 = \frac{10 \times 12^2}{8} = 180 \text{ kN-m}$$

(iv) Eccentricity

$$e = \frac{2M_2 + M_1}{2P} = \frac{(2 \times 180 + 360) \times 10^6}{2 \times 2800 \times 10^3} = 128.57 \text{ mm}$$

(v) Area of steel

Strength of 1 cable of 12 wires of 5 mm ϕ = 225 kN

Number of cables = $\frac{2800}{225} = 12.4 \approx 13$

$\therefore A_{st} = 13 \times 12 \times \frac{\pi}{4} \times (5)^2 = 3063.05 \text{ mm}^2$

Spacing of cables = $\frac{1000}{13} = 76.92 \approx 80$ mm

Hence provide 13 cables @ 80 mm c/c

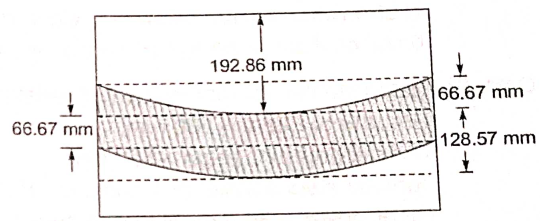
Now, kern distances $K_b = K_t = \frac{Z}{A} = \frac{D}{6} = \frac{400}{6}$

$= 66.67$ mm.

Distance from the top where the cable may lie

$$= \frac{M_2 + M_1}{P} = \frac{(180 + 360) \times 10^6}{2800 \times 10^3}$$

$= 192.86$.

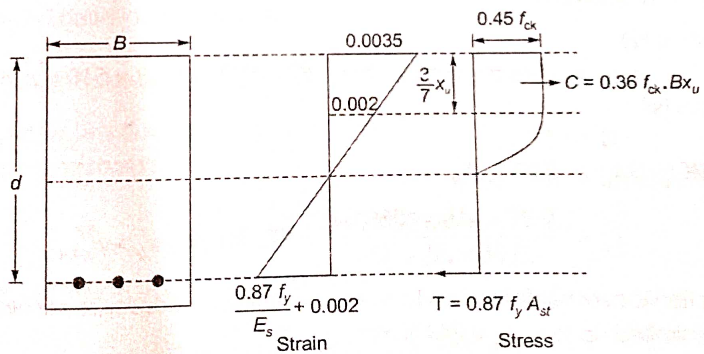


Shaded portion is the zone in which resultant cable must lie.

Q.20 Explain 'under-reinforced', 'balanced' and 'over-reinforced' sections in the ultimate load theory.
[10 marks : 1999]

Solution:

The stress block parameters are shown below:



Actual depth of neutral axis is given by

$$C = T$$

$$\Rightarrow \quad 0.36 f_{ck} Bx_u = 0.87 f_y A_{st}$$

$$\frac{x_u}{d} = \frac{0.87}{0.36} \cdot \frac{f_y}{f_{ck}} \cdot \frac{A_{st}}{Bd}$$

Also limiting value of neutral axis is given by.

$$\frac{x_{u,max}}{d} = \frac{700}{1100 + 0.87 f_y}$$

Now

(i) when $\frac{x_u}{d}$ is less than the limiting value, it means that the reinforcement provided is less than its limiting value. Thus steel reinforcement reaches its yield stress before ultimate strain is reached in concrete. Such a section is called a **under reinforced section.**

(ii) When $\frac{x_u}{d}$ is equal to the limiting value, the case corresponds to balanced section design in which the steel reinforcement reaches its yield stress at the same instant when ultimate strain is reached in concrete. Such a section is called **balanced section.**

(iii) When $\frac{x_u}{d}$ is greater than the limiting value, the case corresponds to the beam in which percentage of steel is sufficient to ensure that steel yield does not take place. Failure occurs when the strain in extreme fibres in concrete reaches its ultimate value. Thus failure takes place due to crushing of concrete while strain in steel remains below the yield strain. Such a section is called **over reinforced section.** It should be noted that compression failure is sudden and therefore not desirable.

Q.21 (i) Calculate the ultimate moment of resistance of an RC rectangular beam with the following data:
Breadth of beam = 230 mm,
Overall depth of beam = 550 mm
Tension steel consist of 4 numbers of 20 mm diameter bars of grade Fe 415
Clear cover = 30 mm, M20 concrete grade.

(ii) Hence determine the intensity of safe superimposed load (excluding self weight) this above beam can carry on a simply supported span of 5 m.

[25 + 5 = 30 marks : 1999]

Solution:

(i) $A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^4$

(a) Limiting depth of NA

$$x_{u,lim} = 0.48 \text{ d} = 0.48 \times (550 - 30 - 20/2) = 0.48 \times 510 = 244.8 \text{ mm}$$

(b) Actual depth of NA

$$C = T$$

$$0.36 f_{ck} B x_u = 0.87 f_y A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87 \times 415 \times 1256.64}{0.36 \times 20 \times 230} = 273.98 \text{ mm}; x_u > x_{u,\,lim}$$

Hence the section is over reinforced and since over reinforced section is not allowed in LSM, the depth of actual NA is limited up to $x_{u,\,lim}$ = 244.8 mm

(iii) Moment of Resistance,

$$MR = 0.36 f_{ck} Bx_{u,\,lim} (d - 0.42\, x_{u,\,lim})$$

$$= 0.36 \times 20 \times 230 \times 244.8 (510 - 0.42 \times 244.8)$$

$$MR = 165.06 \text{ kN-m.}$$

) *If 'w' is the net super imposed load including self weight of the beam, then the bending moment for the beam will be*

$$M_{max} = \frac{wl^2}{8} = MR$$

$$\Rightarrow \quad \frac{w \times 5^2}{8} = 165.06$$

$$\Rightarrow \quad w = \frac{165.06 \times 8}{25} = 52.82 \text{ kN/m}$$

But self weight of beam $= 0.230 \times 0.550 \times 1 \times 25 = 3.1625$ kN/m

Hence safe super imposed load $= \left(\frac{52.82}{1.5}\right) - 3.1625 = 32.05$ kN/m

Q.22 Use limit state method to design a *RC* rectangular beam having an effective simply supported span of 6 m. The beam is required to support live service and super imposed (dead) loads of 14 kN/m and 9.5 kN/m, respectively. The materials to be used are M20 grade concrete and HYSD bars of grade Fe 415. The unit weight of concrete is 25 kN/m³. Adopt *d*/*b* ratio as 2. For the given materials $p_{t, lim} = 0.955\%$.

[15 marks : 2000]

Solution:

Given data:

$f_{ck} = 20$ N/mm² — Live load = 14 kN/m

$f_y = 415$ N/mm² — Super imposed dead load = 9.5 kN/m

$\frac{d}{b} = 2$ — effective span, $l = 6$ m

(i) Assuming, overall depth of the beam as $\frac{1}{10}$ of effective span

i.e. $$D = \frac{1}{10} \times 6000 = 600 \text{ mm}$$

$$\therefore \quad b = \frac{600}{2} = 300 \text{ mm}$$

(ii) Load calculations

Live load = 14 kN/m

Super imposed dead load = 9.5 kN/m

$$\text{Self weight of the beam} = \frac{0.3 \times 0.6 \times 1 \times 25 = 4.5 \text{ kNm}}{\text{Total load w} = 28 \text{ kN/m}}$$

Factored load, $w_u = 1.5 \times w = 1.5 \times 28 = 42$ kN/m

(iii) Maximum bending moment,

$$M_{max} = \frac{w_u l^2}{8} = \frac{42 \times 6^2}{8} = 189 \text{ kN-m.}$$

(iv) Equating M_{max} with MR of the balanced section.

$$\frac{d}{b} = 2 \Rightarrow b = \frac{d}{2} \text{ and } x_{u, lim} = 0.48\, d$$

$$M_{max} = MR = 0.36 f_{ck} \times b \times x_{u, lim} (d - 0.42\, x_{u, lim})$$

$$\Rightarrow \quad 0.36 \times 20 \times \frac{d}{2} \times 0.48d\,(d - 0.42 \times 0.48 \times d) = 189 \times 10^6$$

$$\Rightarrow \quad d^2 (d - 0.2016\, d) = 109.375 \times 10^6$$

$$\Rightarrow \quad d^3 = 136.99 \times 10^6$$

$$\Rightarrow \quad d = 5.1550 \times 10^2 = 515.50 \text{ mm}$$

Adopting $d = 520$ mm

$$D = 520 + 50 = 570 \text{ mm} < 600 \text{ mm (assumed) Hence OK.}$$

Now, $b = \frac{d}{2} = \frac{520}{2} = 260$ mm

(v) Area of steel,

$$A_{st} = \frac{MR}{0.87 f_y (d - 0.42 x_{u,\lim})} = \frac{189 \times 10^6}{0.87 \times 415 \times (520 - 0.42 \times 0.48 \times 520)}$$

$$A_{st} = 1260.87 \text{ mm}^2$$

$$p_t = \frac{A_{st}}{bd} \times 100 = \frac{1260.87}{260 \times 520} \times 100 = 0.933\% < 0.955\% \text{ Hence OK.}$$

Q.23 List various methods used for post tensioning of concrete structures. Describe salient features of Magnel-Blaton system of post-tensioning.

[10 marks : 2000]

Solution:

Various methods used for post tensioning of concrete structures are:

(i) Freyssinet System
(ii) Magnel Blaton System
(iii) Gifford Udall System
(iv) PSC Monowire System
(v) CCL Standard System
(vi) Lee-Mccall System
(vii) Electrical Prestressing

Magnel Blaton System: In this system a cable of rectangular section is provided, which contains layers of wires 5 mm to 8 mm diameter. The wires are arranged with four wires per layer. The wires in the same layer and adjacent layers are separated with a clearance of 4 mm. The geometrical pattern of the wires is maintained in the same form throughout the length of the cable by providing grills or spacers at regular intervals. The grills do not offer any appreciable frictional resistance to the wires which can be moved relative to each other during the tensioning process.

The wires are anchored by wedging, two at a time into sandwich plates. The sandwich plates are about 25 mm thick and are provided with two wedge shaped grooves on its two faces. The wires are taken two in each groove and tightened. Then a steel wedge is driven between the tightened wires to anchor them against the plate. A complete anchorage unit may consist of one to eight sandwich plates, the number of wires depending on the number of wires in the cable. Each plate can anchor eight wires. The various sandwich plates forming a unit are arranged one above the other against a distribution plate. The wires are tightened by jacking two wires at a time.

Q.24 The size of a *RC* beam is restricted to 250 mm × 500 mm. It carries a super imposed load of 25 kN/m over a span of 6 m. Determine the reinforcements for the beam by LSD method. M20 concrete and Fe 415 steel are used. Effective cover to steel = 40 mm.

Salient points on design stress-strain curve for steel

[20 marks : 2001]

Stress	Strain
0.800 f_{yd}	0.00144
0.850 f_{yd}	0.00163
0.900 f_{yd}	0.00192
0.950 f_{yd}	0.00241
0.975 f_{yd}	0.00276
1.000 f_{yd}	0.00380

Solution:

(i) Load calculations

$$\text{Self weight of the beam} = 0.25 \times 0.5 \times 1 \times 25 = 3.125 \text{ kN/m}$$

$$\text{Super imposed load} = 25 \text{ kN/m}$$

$$\text{Total load, } w = 28.125 \text{ kN/m}$$

$$d = 500 - 40 = 460 \text{ mm}$$

(ii) Maximum bending moment

$$M_{max} = \frac{wl^2}{8} = \frac{28.125 \times 6^2}{8} = 126.56 \text{ kN-m}$$

$$\text{Factored } M_{max} = 1.5 \times 126.56 = 189.84 \text{ kN-m}$$

(iii) Calculating moment of resistance for balanced section:

$$M_{u,\,lim} = 0.36\, f_{ck}\, B\, x_{u,\,lim}\, (d - 0.42\, x_{u,\,lim})$$

$$= 0.36 \times 20 \times 250 \times 0.48 \times 460\,(460 - 0.42 \times 0.48 \times 460)$$

$$M_{u,\,lim} = 145.96 \text{ kN-m.}$$

$\because M_{max} > M_{u,\,lim}$, hence section is over reinforced. But over reinforced section is not allowed in LSD. Hence either the maximum bending moment is restricted to $M_{u,\,lim}$ or a doubly reinforced section is provided.

(iv) Now taking $M_{u,\,lim} = M_1$ and providing area of steel A_{st_1} as per the value of M_1

$$\therefore \quad A_{st_1} = \frac{M_1}{0.87\, f_y \left(d - 0.42\, x_{u,\,lim}\right)} = \frac{145.96 \times 10^6}{0.87 \times 415 \times (460 - 0.42 \times 0.48 \times 460)}$$

$$= 1100.789 \text{ mm}^2$$

(v) Now, remaining moment which is to be resisted is

$$M_2 = M_{max} - M_1 = 189.84 - 145.96 = 43.88 \text{ kN-m}$$

(vi) Providing tension steel as per the value of M_2

$$A_{st_2} = \frac{M_2}{0.87\, f_y\, (d - d_c)} = \frac{43.88 \times 10^6}{0.87 \times 415 \times (460 - 40)} = 289.33 \text{ mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 1100.789 + 289.33 = 1390.12 \text{ mm}^2$$

(vii) But as we are designing a doubly reinforced section. Hence the compression steel is also to be provided as per the value of M_2

$$A_{sc} = \frac{M_2}{(f_{sc} - 0.45\, f_{ck})\,(d - d_c)}$$

f_{sc} can be found with the help of strain in compression steel.

$$\varepsilon_{sc} = \left[\frac{x_{u,\,lim} - d_c}{x_{u,\,lim}}\right] \times 0.0035 = \left[\frac{0.48 \times 460 - 40}{0.48 \times 460}\right] \times 0.0035 = 0.00287$$

$$f_{sc} \text{ for } 0.00287 = 352.08 + \frac{(361.05 - 352.08)}{(0.00380 - 0.00276)} \times (0.00287 - 0.00276)$$

$$= 352.08 + 0.94875 = 353.03 \text{ N/mm}^2$$

$$\therefore \quad A_{sc} = \frac{43.88 \times 10^6}{(353.03 - 0.45 \times 20)\,(460 - 40)}$$

$$= \frac{43.88 \times 10^6}{144492.6} = 303.65 \text{ mm}^2$$

Q.25 Design a sloped square footing of a *RC* column 300 mm × 300 mm size carrying an axial load of 320 kN. The safe bearing capacity of soil is 150 kN/m². M20 concrete and Fe 415 steel are used. Use working stress method of design. Allowable shear stress data

p_t	τ_c, N/mm²
0.25	0.22
0.50	0.30
0.75	0.35
1.00	0.39
1.25	0.42
1.50	0.45

[20 marks : 2001]

Solution:

(i) Design constants

$$\therefore \quad k = \frac{mc}{t+mc} = \frac{13\times 7}{230+(13\times 7)} = 0.2835$$

$$j = 1-\frac{k}{3} = 1-\frac{0.2835}{3} = 0.9055$$

$$Q = \frac{1}{2}\times c\times j\times k = \frac{1}{2}\times 7\times 0.9055\times 0.2835 = 0.90$$

(ii) Design of size

$$\text{Load from column} = 320 \text{ kN}$$

$$\text{Foundation weight} = \frac{10}{100}\times 320 = 32 \text{ kN}$$

(Assuming 10% column load),

$$P_T = 352 \text{ kN}$$

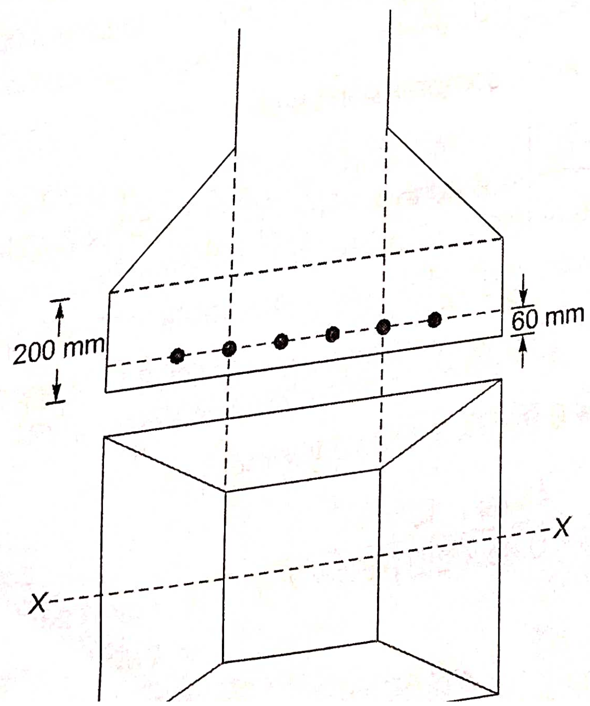
$$\therefore \quad \text{Area} = \frac{P_T}{q_0} = \frac{352}{150} = 2.35 \text{ m}^2$$

Since the footing is a square footing, therefore $B = \sqrt{A}$

$$\Rightarrow \quad B = \sqrt{2.35} = 1.53 \approx 1.55 \text{ m}$$

$$\therefore \quad A_p = 1.55\times 1.55 = 2.4045 \text{ m}^2$$

$$\therefore \text{ Net soil pressure } w = \frac{P}{A} = \frac{320}{2.4045} = 133.20 \text{ kN/m}^2$$



Assuming depth of slab = 200 mm and nominal cover = 60 mm

Q.25 Design a sloped square footing of a *RC* column 300 mm × 300 mm size carrying an axial load of 320 kN. The safe bearing capacity of soil is 150 kN/m². M20 concrete and Fe 415 steel are used. Use working stress method of design. Allowable shear stress data

p_t	τ_c, N/mm²
0.25	0.22
0.50	0.30
0.75	0.35
1.00	0.39
1.25	0.42
1.50	0.45

[20 marks : 2001]

Solution:

(i) Design constants

$$\therefore \quad k = \frac{mc}{t + mc} = \frac{13 \times 7}{230 + (13 \times 7)} = 0.2835$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2835}{3} = 0.9055$$

$$Q = \frac{1}{2} \times c \times j \times k = \frac{1}{2} \times 7 \times 0.9055 \times 0.2835 = 0.90$$

(ii) Design of size

Load from column = 320 kN

$$\text{Foundation weight} = \frac{10}{100} \times 320 = 32 \text{ kN}$$

(Assuming 10% column load),

$$P_T = 352 \text{ kN}$$

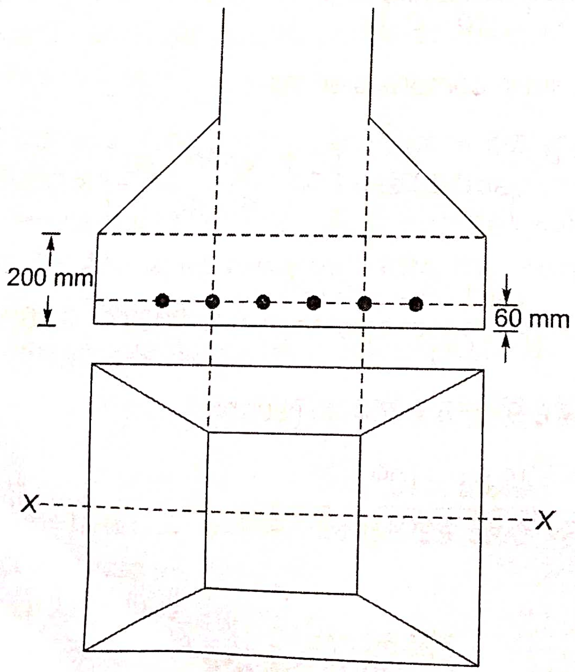
$$\therefore \quad \text{Area} = \frac{P_T}{q_0} = \frac{352}{150} = 2.35 \text{ m}^2$$

Since the footing is a square footing, therefore $B = \sqrt{A}$

$$\Rightarrow \quad B = \sqrt{2.35} = 1.53 \approx 1.55 \text{ m}$$

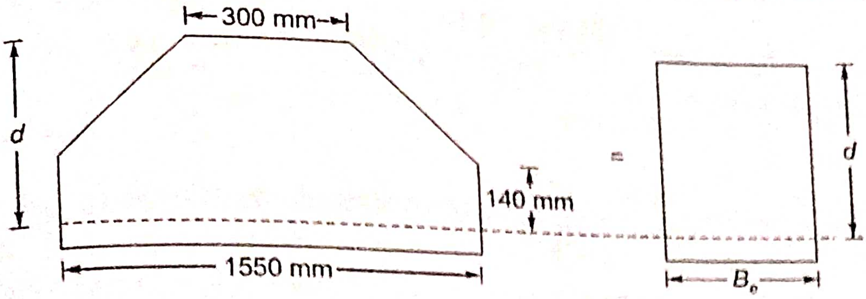
$$\therefore \quad A_p = 1.55 \times 1.55 = 2.4045 \text{ m}^2$$

$$\therefore \text{ Net soil pressure } w = \frac{P}{A} = \frac{320}{2.4045} = 133.20 \text{ kN/m}^2$$



Assuming depth of slab = 200 mm and nominal cover = 60 mm

(iii) Depth of foundation for bending moment critical section is at the face of column section



Here $b = 300$ mm, $B = 1550$ mm

Width of equivalent section,

$$B_e = b + \frac{1}{8}(B - b) = 300 + \frac{1}{8}(1550 - 300)$$

$$= 456.25 \text{ mm (It is used only for depth calculation)}$$

(iv) Bending Moment

Overhang, $O_x = \dfrac{1.55 - 0.3}{2} = 0.625$ m

Bending moment $= \text{W.B.}\dfrac{O_x^2}{2} = 133.2 \times 1.55 \times \dfrac{(0.625)^2}{2} = 40.32$ kN-m

$$\therefore \quad d = \sqrt{\frac{BM}{QB_e}} = \sqrt{\frac{40.32 \times 10^6}{0.9 \times 456.25}} = 313.40 \text{ mm}$$

Effective cover = 60 mm

∴ Total depth, $D = 313.40 + 60 = 373.40$ mm

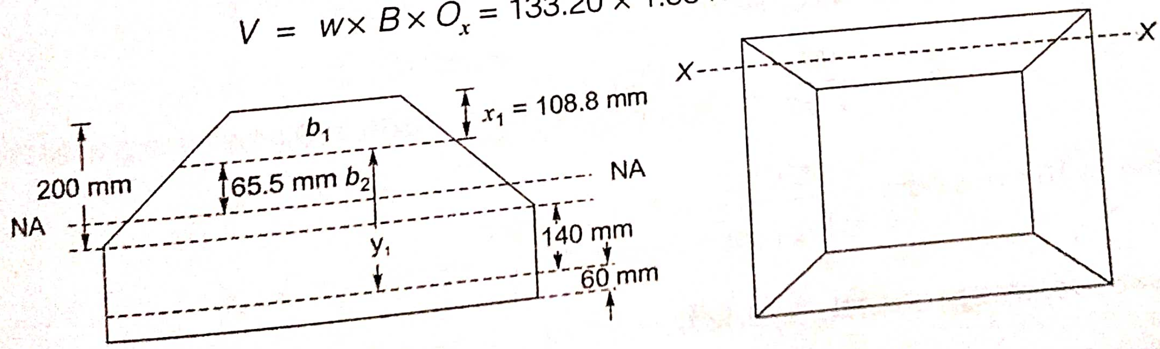
Adopting $D = 400$ mm

∴ Effective depth, $d = 400 - 60 = 340$ mm

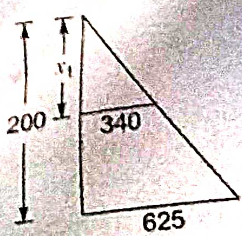
(v) One way shear

Overhang, $O_x = \dfrac{1.55 - 0.3}{2} - 0.34 = 0.285$ m

$V = w \times B \times O_x = 133.20 \times 1.55 \times 0.285 = 58.84$ kN



Shear stress, $\tau_v = \dfrac{V}{B \times d} = \dfrac{V}{b_2 \times d}$



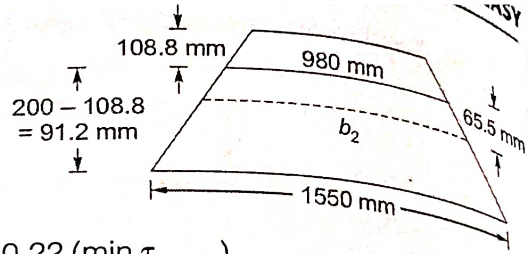
$b_1 = b + 2d = 300 + (2 \times 340) = 980$ mm

$x_1 = \dfrac{200}{625} \times 340 = 108.8$ mm

$y_1 = 340 - 108.8 = 231.2$ mm

Depth of neutral axis $= kd = ky_1 = 0.2835 \times 231.2 = 65.5$ mm

Now, width of the section of NA = b_2



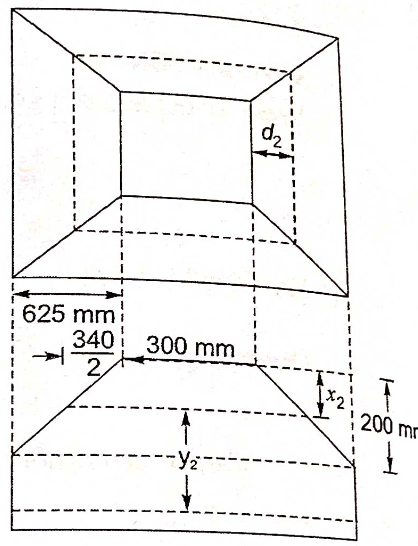
$$\therefore \quad b_2 = 980 + \left[\frac{1550-980}{91.2}\right] \times 65.5$$

$$b_2 = 1389.4 \text{ mm}$$

$$\text{Shear stress} = \frac{V}{b_2\, y_1} = \frac{58.84\times10^3}{1389.4\times231.2} = 0.18 \text{ N/mm}^2 < 0.22 \;(\min \tau_{c,\,min})$$

Hence the section is safe in single shear.

(vi) Check for punching shear (double shear)



$$\text{Net punching force} = P - w(a+d)(b+d)$$

$$= 320 - 133.2\,(0.3+0.34)\,(0.3+0.34)$$

$$P_{net} = 265.44 \text{ kN}$$

$$x_2 = \frac{200}{625}\times170 = 54.4 \text{ mm}$$

$$y_2 = 340 - x_2 = 340 - 54.4 = 285.6 \text{ mm}$$

$$\text{Punching shear stress} = \frac{\text{Net Punching Force}}{\text{Perimeter}\times\text{Depth}}$$

$$= \frac{265.44\times10^3}{2[(a+d)+(b+d)]\times y_2}$$

$$= \frac{265.44\times10^3}{2[(0.3+0.34)+(0.3+0.34)]\times285.6\times10^3}$$

$$= 0.36 \text{ N/mm}^2$$

$$\text{Permissible punching stress} = k_c\times\tau_c$$

$$= 1\times0.25\sqrt{f_{ck}} = 1\times0.25\sqrt{20}$$

$$= 1.118 \text{ N/mm}^2 > 0.36 \text{ N/mm}^2$$

Hence section is safe in punching shear.

(vii) Area of steel

$$M = 40.32 \text{ kN-m}$$

$$A_{st} = \frac{M}{t\,.\,jd} = \frac{40.32\times10^6}{230\times0.9050\times340} = 569.725 \text{ mm}^2$$

$$\text{Number of 10 mm } \phi \text{ bars} = \frac{569.725}{\frac{\pi}{4}\times(10)^2} = 7.25 \approx 8 \text{ bars}$$

$$\text{Minimum percentage of steel} = 0.12\%$$

$$= \frac{0.12}{100}\times B_e\times D = \frac{0.12}{100}\times456\times400$$

$$= 218.88 \text{ mm}^2 < 569 \text{ mm}^2$$

∴ *Provide* 8 No's 10 mm ϕ bars in both directions

(viii) Check for development length after providing 60 mm cover length available

$$= \frac{1}{2}(1550-300) - 60 = 565 \text{ mm}$$

τ_{bd} for M20 = 1.2 N/mm²,

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$L_d = \frac{\sigma_{st}}{4\,\tau_{bd}} = \frac{230}{4 \times 1.6 \times 1.2}\phi = 29.95\,\phi = 29.95 \times 10 = 299.5 \text{ mm} < 565 \text{ mm}$$

Hence ok.

Q.26 A simply supported post tensioned concrete beam of span 15 m has a rectangular cross section 300 mm × 600 mm. The prestress at ends is 1150 kN with zero eccentricity at the supports. The eccentricity at the mid span is 200 mm. The cable profile is parabolic. Assuming friction coefficient k_f = 0.15 per 100 m and μ = 0.35, determine the loss due to friction at the centre of the beam.

[10 marks : 2001]

Solution:

Given data:

$$\text{Span} = 15 \text{ m}$$
$$B = 300 \text{ mm}$$
$$D = 600 \text{ mm}$$

Initial prestressing force, P_0 = 1150 kN

Eccentricity at supports = 0

Eccentricity at mid span = 200 mm

$$k_f = 0.15 \text{ per } 100 \text{ m} = \frac{0.15}{100} \text{ per m} = 0.0015 \text{ per meter}$$

$$\mu = 0.35$$

Since the cable profile is parabolic, the equation of profile if

$$y = \frac{4h}{l^2} x\,(l - x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

$$\Rightarrow \quad \theta\,(x = 0) = \frac{4h}{l}$$

$$\Rightarrow \quad \theta = \frac{4 \times 200}{15000}$$

$$\Rightarrow \quad \theta = 0.0533 \text{ radians}$$

Since, prestressing is being done on the both ends, so $\alpha = \theta$

$$\therefore \quad \alpha = 0.0533 \text{ radians}$$

$$\therefore \quad P_x = P_0 e^{-(k_f x + \mu\alpha)} \qquad \left[\text{Distance of centre from one end, } x = \frac{15}{2} = 7.5 \text{ m}\right]$$

$$\Rightarrow \quad P_x = 1150\, e^{-(0.0015 \times 7.5 + 0.35 \times 0.0533)}$$

$$\Rightarrow \quad P_x = 1116.10 \text{ kN}$$

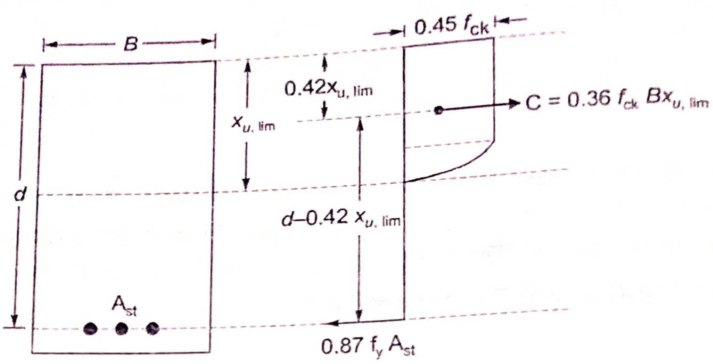
$$\therefore \quad \text{Loss of prestress} = P_0 - P_x = 1150 - 1116.10 = 33.89 \text{ kN}$$

$$\% \text{ loss of prestress} = \frac{33.89}{1150} \times 100 = 2.95\%$$

Q.27 Derive the expression for limiting percentage of tensile reinforcement in a flexural *RC* member.

[5 marks : 2002]

Solution:



Limiting percentage of tensile steel is calculated for a balanced section. Since, in flexure total compressive force is equal to total tensile force, we get

$$C = T$$

$$0.36 \times f_{ck} \times B\,x_{u,\,lim} = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_{u,\,lim} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, B}$$

$$\Rightarrow \quad \frac{x_{u,\,lim}}{d} = \frac{0.87\, f_y}{0.36\, f_{ck}} \times \frac{A_{st}}{Bd}$$

$$\Rightarrow \quad \frac{A_{st}}{Bd} = \frac{0.36 f_{ck}}{0.87 f_y} \times \frac{x_{u,\,lim}}{d}$$

$$\Rightarrow \quad \frac{A_{st}}{Bd} \times 100 = \frac{0.36}{0.87} \times 100 \times \frac{f_{ck}}{f_y} \times \frac{x_{u,\,lim}}{d}$$

$$\Rightarrow \quad p_t = 41.38 \times \frac{f_{ck}}{f_y} \times \frac{x_{u,\,lim}}{d}\,\%$$

If M 20 concrete and Fe 415 steel is used, then

$$p_t = 41.38 \times \frac{20}{415} \times \frac{0.48\, d}{d}$$

$$p_t = 0.957\%$$

Q.28 Design the *RC* floor slab for a room of internal dimensions of 4.0 m × 9.5 m. Assume the slab to be simply supported on 230 mm thick masonry walls. The slab is to support live load of 4.0 kN/m^2 and surface finish of 1 kN/m^2. Use M20 grade concrete, HYSD steel of Fe 415 grade. Draw reinforcement details.

[20 marks : 2002]

Solution:

(i) Calculation of design constants

For M 20 concrete, $m = 13.33$, $c = 7\text{ N/mm}^2 = \sigma_{cbc}$

For Fe 415 steel, $t = 230\text{ N/mm}^2 = \sigma_{st}$

$$\therefore \quad k = \frac{mc}{t + mc} = \frac{13.33 \times 7}{230 + (13.33 \times 7)} = 0.2886$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.2886}{3} = 0.9038$$

$$Q = \frac{1}{2}cjk = \frac{1}{2} \times 7 \times 0.9038 \times 0.2886 = 0.9129$$

(ii) As per the vertical deflection criterion, the span to effective depth ratio for spans upto 10 m for a simply supported slab is given by

$$\frac{l}{d} = 20$$

$$\Rightarrow \quad d = \frac{l}{20} = \frac{4000}{20} = 200 \text{ mm}$$

(iii) Effective span

(a) clear span + effective depth = 4.00 + 0.2 = 4.2 m

(b) centre to centre distance between supports = 4.0 + 0.23 = 4.23 m

Hence lesser of the above two will be adopted i.e. $l_e = 4.2$ m

(iv) Bending moment and shear force

Assuming a nominal cover of 20 mm, the total depth of the slab will become 200 + 20 = 220 mm.

(a) Load calculations:

Load due to self weight of slab	= 0.22 × 1 × 1 × 25	= 5.5 kN/m²
Superimposed live load		= 4.0 kN/m²
Load due to surface finishing		= 1.0 kN/m²
	Total	= 10.5 kN/m²

(b) Bending moment per meter run of slab

$$M = \frac{wl_e^2}{8} = \frac{10.5 \times (4.2)^2}{8} = 23.15 \text{ kN-m}$$

(c) Shear force

$$V = \frac{wl_e}{2} = \frac{10.5 \times 4.2}{2} = 22.05 \text{ kN}$$

(v) Design of section

$$d = \sqrt{\frac{M}{QB}} = \sqrt{\frac{23.15 \times 10^6}{0.9129 \times 1000}} \qquad [B = 1000 \text{ mm}]$$

$\Rightarrow \quad d = 159.24$ mm < 200 mm. Hence OK

Now taking $\quad d = 160$ mm and nominal cover = 20 mm

we get $\quad D = d + 20 = 160 + 20 = 180$ mm.

(vi) Main reinforcement

$$A_{st} = \frac{M}{\sigma_{st}\, jd} = \frac{23.15 \times 10^6}{230 \times 0.9038 \times 160} = 696 \text{ mm}^2$$

Minimum reinforcement should be 0.15% of total cross sectional area

$$= \frac{0.15}{100} \times 180 \times 1000 = 270 \text{ mm}^2$$

∴ A_{st} is more than minimum reinforcement.

$$\text{Spacing of bars, } s = \frac{1000 \times A_\phi}{A_{st}}$$

Adopting 10 mm ϕ bars, we get $A_\phi = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$

$$\therefore \quad s = \frac{1000 \times 78.54}{696} = 112.84 \text{ mm}$$

Adopting s = 110 mm c/c, we get actual

$$A_{st} = \frac{1000 \times 78.54}{120} = 654.5 \text{ mm}^2$$

Spacing is less than 3d and 300 mm both.

Bend every third bar at the support, at a distance of $\frac{l_e}{5} = \frac{4.2}{5} = 0.82$ m from the edge of support

A_{st} at support $= \frac{2}{3} \times 654.5 = 436.33 \text{ mm}^2$ which is more than minimum reinforcement (0.15%).

(vii) Check for development length at the support

The code stipulates that at simple supports, the diameter of the reinforcement be such that

$$L_d \leq 1.3 \frac{M_1}{V} + L_0$$

$$M_1 = \sigma_{st} \cdot j \cdot d \times A_{st} \text{ at support}$$
$$= 230 \times 0.9038 \times 160 \times 436.33$$
$$= 14.51 \times 10^6 \text{ N-mm}$$
$$V = 22.05 \text{ kN}$$

Assuming a clear cover of 25 mm is provided at the side (end) and providing a U-hook

Given width of support = 230 mm = l_s

$$\therefore \quad L_0 = \frac{l_s}{2} - x' + 13\,\phi = \frac{230}{2} - 25 + 13 \times 10 = 220 \text{ mm}$$

$$L_d = \frac{\phi\, \sigma_{st}}{4\, \tau_{bd}} = \frac{10 \times 230}{4 \times 1.6 \times 1.2} = 299.48 \text{ mm (60\% increase for HYSD bars)}$$

Now $$1.3 \frac{M_1}{V} + L_0 = \frac{1.3 \times 14.51 \times 10^6}{22.05 \times 10^3} + 220$$
$$= 1075.46 \text{ mm} > L_d \text{ Hence ok.}$$

The reinforcement should extend by a length equal to $\frac{L_d}{3} = \frac{479.17}{3} = 159.72 \text{ mm} \approx 160$ mm beyond the face of support.

(viii) Check for shear

p = % reinforcement at support $= \frac{436.33}{180 \times 1000} \times 100 = 0.24\%$

For $p = 0.15\%$, $\tau_c = 0.28$

For $p = 0.25\%$, $\tau_c = 0.36$

$$\therefore \text{ For } p = 0.24\%, \quad \tau_c = 0.28 + \left(\frac{0.36 - 0.28}{0.25 - 0.15}\right)(0.24 - 0.15) = 0.352 \text{ N/mm}^2$$

$$\text{Now } \tau_v = \frac{V}{Bd} = \frac{22.05 \times 10^3}{1000 \times 160} = 0.138 \text{ N/mm}^2$$

$\because \tau_v < \tau_c$ Hence safe.

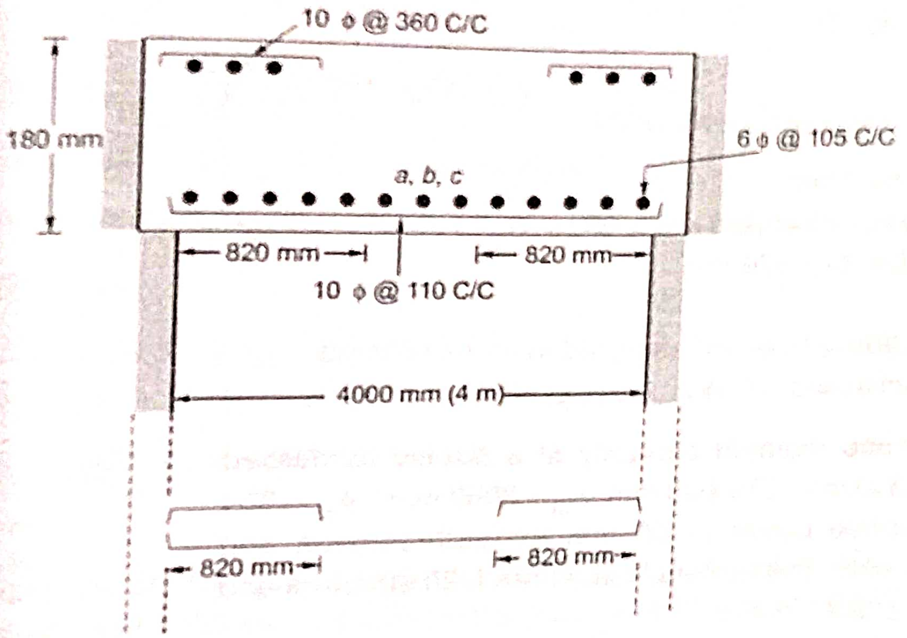
(ix) Distribution reinforcement

$$A_{st} = \frac{0.15\,BD}{100} = \frac{0.15 \times 1000 \times 180}{100} = 270 \text{ mm}^2$$

Using 6 mm ϕ bars, each having $A_\phi = \frac{\pi}{4} \times 6^2 = 28.27 \text{ mm}^2$

$$\text{Spacing of bars} = \frac{1000 \times A_\phi}{A_{sd}} = \frac{1000 \times 28.27}{270} = 104.7 \approx 105 \text{ mm}$$

This is less than 5d and 450 mm both. Hence provide 6 mmϕ bars @ 105 mm C/C. Near the edge of the support, the distribution reinforcement may be provided both at top as well as the bottom. The details of the reinforcement are shown below.



Q.29 A RC column of size 460 mm × 600 mm having effective length of 3.6 m is to be designed using LSM to support an axial service load of 2500 kN. Use M20 grade concrete and HYSD steel of Fe 415 grade.

[15 marks : 2002]

Solution:

(i) $$\frac{\text{Effective length}}{\text{Least lateral dimension}} = \frac{3600}{460} = 7.83 < 12$$

Hence, the column will be designed as short column.

(ii) Adopting $e_{min} = 20$ mm

But e_{min} shall not exceed $0.05\,B$ and $0.05\,D$

$\therefore$ $20 \le 0.05 \times 460$ and $20 \le 0.05 \times 600$

$20 \le 23$ and $20 \le 30$ Hence OK

Hence, the equation $P_u = 0.4\, f_{ck} A_c + 0.67\, f_y A_{sc}$ can be used

(iii) Service load = 2500 kN

Factored load, $P_u = 1.5 \times 2500 = 3750$ kN

$f_{ck} = 20 \text{ N/mm}^2$, where

$$A_c = A - A_{sc}$$
$$A = 460 \times 600 = 276000 \text{ mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$
$$\therefore \quad P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}$$
$$\Rightarrow \quad 3750 \times 10^3 = 0.4 \times 20 \times (276000 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$
$$\Rightarrow \quad A_{sc} = \frac{3750 \times 10^3 - 2208000}{270.05}$$
$$\Rightarrow \quad A_{sc} = 5710 \text{ mm}^2$$

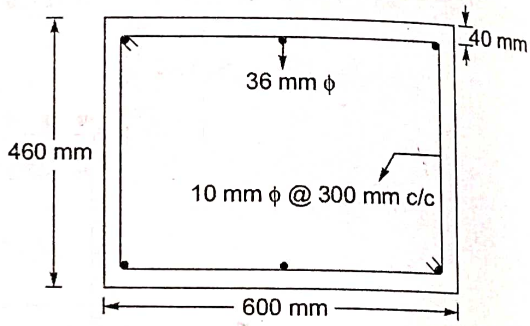
Adopting 36 mm bars, we get number of bars $= \dfrac{5710}{\frac{\pi}{4} \times (36)^2} = 5.61 \approx 6$ bars

(iv) Diameter of lateral ties

(a) $\frac{1}{4} \times 36 = 9$ mm

(b) 6 mm

$\therefore$ Provide 10 mm bars as lateral ties



(v) Spacing of lateral ties

(a) Least lateral dimension = B = 460 mm

(b) $16\,\phi = 16 \times 36 = 576$ mm

(c) 300 mm

lesser of the above three will be provided as the spacing.

Hence 10 mm ϕ bars will be provided at 300 mm c/c

Q.30 Determine ultimate moment capacity of a doubly reinforced beam with $B = 300$ mm, $D = 600$ mm, $A_{st} = 2060$ mm², $A_{sc} = 804$ mm² and effective cover of 50 mm for both tension and compression steels. The materials used are M20 concrete and HYSD steel of grade Fe 415.

The salient points on stress-strain curve are:

Stress, MPa	Strain
288	0.00144
306	0.00163
324	0.00192
342	0.00241
351	0.00276
360	0.00380

[15 marks : 2003]

Solution:

Given Data: $B = 300$ mm, $D = 600$ mm, $A_{st} = 2060$ mm², $A_{sc} = 804$ mm², $d_c = 50$ mm

$d = D - d_c = 600 - 50 = 550$ mm

$M20 \;/\; Fe415$

(i) Limiting depth of neutral axis

$$x_{u,\,lim} = 0.48\, d = 0.48 \times 550 = 264 \text{ mm}$$

***(ii)* Actual depth of neutral axis:**

Total compressive force = Total tensile force

$$0.36\, f_{ck}\, B\, x_u + A_{sc}(f_{sc} - 0.45\, f_{ck}) = 0.87\, f_y\, A_{st}$$
$$\Rightarrow \quad 0.36 \times 20 \times 300 \times x_u + 804\,(f_{sc} - 0.45 \times 20) = 0.87 \times 415 \times 2060$$
$$\Rightarrow \quad 2160\, x_u + 804\,(f_{sc} - 9) = 743763$$
$$\Rightarrow \quad 2160\, x_u + 804\, f_{sc} - 7236 = 743763$$
$$\Rightarrow \quad 2160\, x_u + 804\, f_{sc} = 750999$$

$$\Rightarrow \quad x_u = \frac{750999 - 804 f_{sc}}{2160}$$

Trial 1:

Assuming f_{sc} = 350 MPa.

we get

$$x_u = \frac{750999 - 804 \times 350}{2160} = 217.407 \text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{x_u}(x_u - d_c) = \frac{0.0035}{210.71}(217.407 - 50) = 0.002695$$

$$f_{sc} \text{ for } 0.002695 = 342 + \frac{(351 - 342)}{(0.00276 - 0.00241)}(0.002695 - 0.00241)$$

$$= 342 + 7.328 = 349.328 \text{ MPa}$$

Trial 2:

$$x_u = \frac{750999 - 804 \times 349.328}{2160} = 217.657 \text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{217.657}(217.657 - 50) = 0.002695$$

Hence f_{sc} = 349.328 MPa and x_u = 217.657 mm (adopted)

$\because x_u < x_{u,\,lim}$, hence section is under reinforced.

(iii) Moment of Resistance

$$MR = 0.36\, f_{ck}\, B\, x_u \times (d - 0.42\, x_u) + (f_{sc} - 0.45\, f_{ck})\, A_{sc}\,(d - d_c)$$

$$= [0.36 \times 20 \times 300 \times 217.657 \times (550 - 0.42 \times 217.657)]$$

$$+ [(349.328 - 0.45 \times 20) \times 804 \times (550 - 50)]$$

$$= 352.409 \text{ kN-m}$$

Q.31 Determine the thickness of the footing slab of uniform thickness for a 400 mm square column transmitting an axial load of 1100 kN. The safe bearing capacity of soil is 150 kN/m². The materials to be used are M20 concrete and HYSD steel of grade Fe 415. Use LSM.

[15 marks : 2003]

Solution:

(i) Size of foundation

Load from column = 1100 kN

$$\text{Foundation load} = \frac{10}{100} \times 1100 = 110 \text{ kN}$$

(Assume 10% of column load), P_T = 1210 kN

$$\therefore \quad \text{Area of foundation} = \frac{P_T}{q_0} = \frac{1210}{150} = 8.07 \text{ m}^2$$

Let us design a square footing

$$A = B^2$$

$$\therefore \quad B^2 = 8.07$$

$$\Rightarrow \quad B = \sqrt{8.07}$$

$$\Rightarrow \quad B = 2.84 \text{ m} \approx 2.9 \text{ m}$$

$$\Rightarrow$$

Now, final area selected $= B \times B = 2.9 \times 2.9 = 8.41 \text{ m}^2$

$\therefore$ Net soil pressure, $w = \frac{P}{A} = \frac{1100}{8.41} = 130.8$ kN/m^2

Factored soil pressure $= 1.5 \times 130.8 = 196.2$ kN/m^2

(ii) Design of depth as per bending moment

Overhang $O_x = \frac{B-a}{2} = \frac{2.9-0.4}{2} = 1.25$ m

$$M_{yy} = 196.2 \times 2.5 \times \frac{(1.25)^2}{2} = 383.20 \text{ kN-m}$$

Overhang, $O_y = \frac{B-a}{2} = 1.25$ m

$\therefore$ $M_{xx} = 383.20$ kNm

$$Q = 0.36\, f_{ck} \frac{x_{u,max}}{d}\left(1 - 0.42 \frac{x_{u,max}}{d}\right)$$

$$= 0.36 \times 20 \times 0.48\,(1 - 0.42 \times 0.43) = 2.76 \text{ N/mm}^2$$

Now, depth, $d = \sqrt{\frac{M_{xx} \text{ or } M_{yy}}{Q \times B}} = \sqrt{\frac{383.20 \times 10^3}{2.76 \times 2.9}} = 218.81$ mm

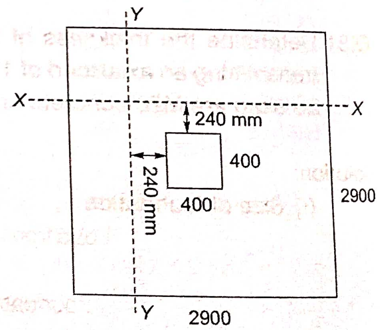
Assuming effective cover = 60 mm

$\therefore$ Total depth = 218.81 + 60 = 278.81 mm

Adopting total depth = 300 mm

$\therefore$ Effective depth, d = 300 – 60 = 240 mm

(iii) Check for shear



Overhang $O_{x_1} = O_{y_1} = \frac{2900-400}{2} - 24\text{m}$

$= 1010$ mm $= 1.01$ m

Shear force $V_{ux} = V_{uy} = 196.2 \times 2.9 \times 1.01 = 574.67$ kN

$$\tau_{VX} = \tau_{VY} = \frac{V_u}{B \times d} = \frac{574.67 \times 10^3}{2900 \times 240} = 0.83 \text{ N/mm}^2$$

But τ_c for minimum percentage of steel (0.15%) = 0.28

$\therefore$ Depth required, $d = \frac{V_u}{B\,\tau_c} = \frac{574.67 \times 10^3}{2900 \times 0.28} = 707.72$ mm

There should be a balance between overhang and shear force

Hence, using $d = 600$ mm

$$\text{Overhang} = \frac{2900-400}{2} - 600 = 0.650 \text{ m}$$

$$V_u = 196.2 \times 2.9 \times 0.65 = 369.84 \text{ kN}$$

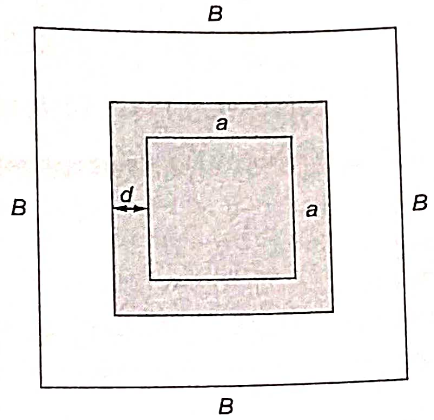
$\therefore$ $\tau_v = \frac{369.84 \times 10^3}{2900 \times 600} = 0.21 < 0.28$ Hence OK.

(iv) Check for punching shear

Net punching shear, $P_u = P_c\,(\text{design}) - w_u\,(a + d)\,(a + d)$

$$\Rightarrow \quad P_u = 1.5 \times 1100 - 196.2\,(0.4 + 0.6)^2$$

$$\Rightarrow \quad P_u = 1453.8 \text{ kN}$$



$$\text{Punching shear stress} = \frac{\text{Net Punching Force}}{2[(a+d)+(a+d)]\times d}$$

$$= \frac{1453.8\times 10^3}{2[(0.4+0.6)+(0.4+0.6)]\times 10^3\times 600}$$

$$= 0.61 \text{ N/mm}^2$$

Maximum permissible punching shear stress = $k_s \tau_c$

where $k_s = \left(0.5 + \frac{b}{a}\right) = 0.5 + \frac{a}{a} = 1.5$

but k_s shall not exceed 1.0

$$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.12 \text{ N/mm}^2$$

∴ Permissible punching shear stress = 1.0 × 1.12 = 1.12 > 0.61, hence OK.

∴ Effective depth provided as per all the three criteria = 600 mm

Effective cove = 60 mm

Total depth = 600 + 60 = 660 mm

(v) Area of steel

Since the final effective depth is far larger than that required from bending moment criteria, hence the foundation becomes under reinforced and thus A_{st} will be calculated as

$$A_{st} = 0.5\frac{f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6\,M_u}{f_{ck}\,Bd^2}}\right]Bd$$

$$= 0.5\times\frac{20}{415}\left[1-\sqrt{1-\frac{4.6\times 383.20\times 10^6}{20\times 2900\times 600^2}}\right]\times 2900\times 600$$

$$A_{st} = 1808.82 \text{ mm}^2$$

Adopting bar diameter = 12 mm

$$\therefore \text{ Number of bars required} = \frac{1808.82}{\frac{\pi}{4}\times(12)^2} = 15.99 \approx 16 \text{ bars}$$

$$\text{Spacing of bars} = \frac{2900}{16} = 181.25 \text{ mm}$$

16 bars will be provided in each direction at 180 mm c/c

$$\text{Minimum area of steel} = \frac{0.12}{100}\times 2900\times 660 = 2296.8 \text{ mm}^2 > 1808.82 \text{ mm}^2$$

Hence providing 12 mm ϕ bars

$$\text{Number of bars} = \frac{2296.8}{\frac{\pi}{4}\times(12)^2} = 20.31 \approx 21 \text{ bars}$$

$$\text{Spacing} = \frac{2900}{21} = 138.09 \text{ mm}$$

Provide 21 No's 12 mm ϕ bars @ 130 mm c/c.

(vi) Check for development length

$$L_d = \frac{\phi\, \sigma_{st}}{4\, \tau_{bd}} = \frac{230 \times 12}{4 \times 1.6 \times 1.2} = 359.375 \text{ mm (60\% increase for HYSD bars)}$$

Providing 60 mm side cover, length of bars available $= \frac{1}{2}(B - a) - 60$

$$= \frac{1}{2}(2900 - 400) - 60 = 1190 \text{ mm} > 359.375 \text{ mm (Hence OK)}$$