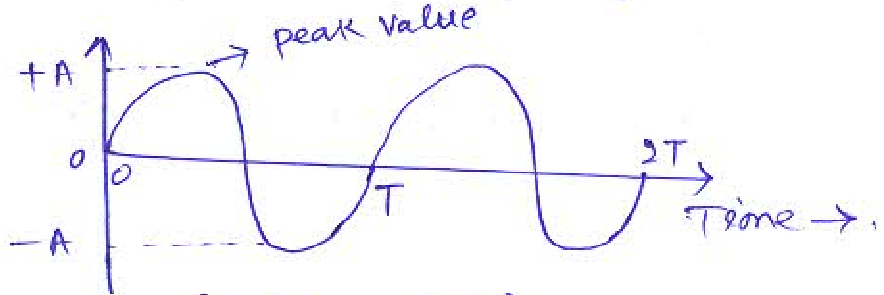
NT Module - III ①

Representation of sine functions as Rotating phasor.

→ The phase of an alternating quantity at any instant in time can be represented by a phasor diagram, so phasor diagrams can be thought of as "functions of time".

→ A complete sine wave can be constructed by a single vector rotating at an angular velocity of $\omega = 2\pi f$, where 'f' is the frequency of the waveform.



(sine wave.)

→ A generalized sinusoid is defined as follows

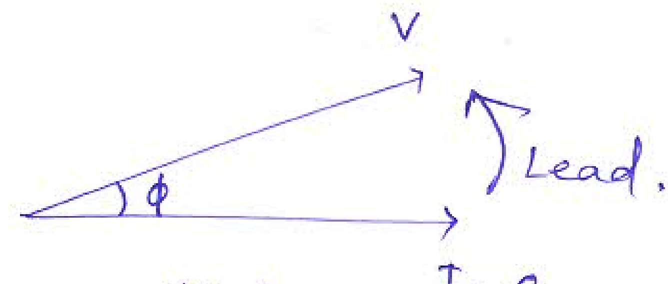
$$x(t) = A \sin \omega t = A \sin 2\pi f t.$$

where A is amplitude, ω is radian frequency

f = natural frequency.

$\omega = 2\pi f$ (radian/sec).

Phasor Diagrams :→



→ phasor diagrams are a graphical way of representing the magnitude and directional relationship between two or more alternating quantities.

→ sinusoidal waveforms of the same frequency can have a phase Difference between themselves which represents the angular difference of the two sinusoidal waveforms.

Impedance and Admittances :→

→ Admittance is defind as, where Y is the admittance, measured in siemens, Z is the impedance, measured in ohms.

→ Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (Known as Reactance).

$$\boxed{Y = \frac{1}{Z}}$$

In R-L Series circuit.

$$Z = \sqrt{R^2 + X_L^2}$$

$$X_L = \omega L = 2\pi f L$$

In R-C Series circuit.

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

In R-L-C Series circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

AC Circuit Analysis.

→ An a.c. circuit is one in which the magnitude of the current changes periodically with time.

→ The a.c. is produced by an alternating voltage supply.

→ The pattern of the a.c voltage is sinusoidal in nature, that is it varies like the sine curve with constant amplitude and frequency.

→ Average value = $\dfrac{\text{Area under the curve}}{\text{length of the base of the curve}}$

→ RMS value = $\sqrt{\dfrac{\text{Area of half cycle wave squared}}{\text{Half cycle base}}}$

→ Form factor :→ The ratio of rms value to average value of an alternating quantity is known as form factor. It is represented by K_f.

$$K_f = \frac{\text{R.M.S. Value}}{\text{Average value}}$$

→ Peak factor :→ The ratio of maximum value to the rms value of an alternating quantity is known as peak factor or amplitude factor. It is represented by K_a.

$$K_a = \frac{\text{Maximum value}}{\text{R.M.S. Value}}$$

MODULE-III. AC Circuit Analyses.

FUNDAMENTALS OF POWER

Power

* The ability of work done is known as power.
* The Rate of consumption of energy is known as power.

Power In DC Ckt :-

→ Power is product of voltage & current.

$$\boxed{P = VI}$$

Power IN AC Ckt :-

→ The Rate of cage of energy w.r.t time in terms of voltage & current.

→ In an AC ckt the voltage & current are continiously changing. Hence power value changes w.r.t time.

→ The average value of vareing power is

$$P_{av} = \frac{1}{T}\int_0^T p \cdot dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} p \, dt$$

→ The average power is also called as active power or true power.

Euler Identity :-

$$\boxed{E^{j\theta} = e^{j\theta} = \cos\theta + j\sin\theta}$$

$$I = 5e^{j0} = 5\angle 0^\circ = 5 + j0$$

Power System:-

- Generation, transmittion & distribution system are the main component of an electrical power system.
- Generating station & distribution system are connected through a transmission line.
- Electric power system is the combination of components that transforms other types of energy into electrical energy and transmits this energy to the consumer.

POLAR FORM

- $A \angle \theta$

Ex $10 \angle 30^\circ$

- Multiplication & Division.

RECTANGULAR FORM

- $Z = R + jX$

Ex $2 + j5$

- Addition & substraction.

Addition

* $(2+j3) + (4+j5)$
  $= 6+j8$
* $(2+j3) + 3\angle 30^\circ$
  $= 2+j3 + 2.59 + j1.5$
  $= 4.59 + j4.5$

Multiplication

$2\angle 30^\circ \times 5\angle 40^\circ$
$= 10\angle 30+40$
$= 10\angle 70^\circ$

* $(3-j6) \times 5\angle 10^\circ$
  $= (6.70\angle -63.43) \times 5\angle 10$
  $= 33.5\angle(-63.43+10)$
  $= 33.5\angle -53.43$

Division

* $\dfrac{10\angle 30^\circ}{2\angle 50^\circ}$
  $= 5\angle 30^\circ - 50^\circ$
  $= 5\angle -20^\circ$
* $(6+j8) \div 10\angle 5$
  $= (10\angle 53.13) \div (10\angle 5)$
  $= 1\angle 53.13 - 5$

PROBLEM

Two impedances $Z_A = (25 - j16)$, $Z_B = (3 + j9)$ are connected in parallel in a ckt. calculate the equivalent impedance for the given ckt in rectangular form.

Soln $Z_A = 25 - j16$

$Z_B = 3 + j9$

$$Z_{eq} = \frac{Z_A Z_B}{Z_A + Z_B}$$

$$= \frac{(25 - j16)(3 + j9)}{(25 - j16) + (3 + j9)}$$

$$= \frac{(29.68 \angle -32.61)(9.48 \angle 71.5)}{28 - j7}$$

$$= \frac{281.366 \angle 38.89}{28 - j7}$$

$$= \frac{281.366 \angle 38.89}{28.86 \angle -14.03}$$

$$= 9.74 \angle 52.9$$

$$= 5.875 + j7.76$$ (ANS)

TYPES of POWER

1. Active power / True power / Real power
2. Reactive / Pulsating power
3. Apparent power = $|V| |I|$
4. complex power = VI^*

(I^* = complex Congugate of I)

1. INSTANTNEOUS & AVERAGE POWER

→ In an AC ckt the power at any instant is called so.

→ It is equal to the product of the values of voltage & current at any instant.

→ In an ac ckt let the instantaneous value of the voltage & current.

$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t - \alpha)$$

Where, α = phase angle betⁿ V&I.

→ Instantaneous power $p = VI$.

$$= V_m \sin \omega t \times I_m \sin(\omega t - \alpha)$$

$$= V_m I_m \sin \omega t \cdot \sin(\omega t - \alpha)$$

$$= \frac{1}{2} V_m I_m \left[\cos \alpha - \cos(2\omega t - \alpha)\right]$$

$$\left[\because \cos A \cdot \cos B = 2\sin\left(\frac{A+B}{2}\right) - \sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{1}{2} V_m I_m \cos \alpha - \frac{1}{2} V_m I_m \cos(2\omega t - \alpha)$$

→ The second term of right hand side of the above eqⁿ contains a double frequency 2ω. so the magnitude of the average value of the 2nd term is zero. It is because average of a sinusodial quantity of double frequency over a complete cycle is zero. Hence the 2nd term is avoided..

$$P_{av} = \frac{1}{2} V_m I_m \cos \alpha$$

→ The instantneous power is the average power in AC ckt.

$$P_{av} = \frac{1}{2} V_m I_m \cos \alpha$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \alpha$$

$P_{av} = VI\cos\theta$

Where, $V = \frac{V_m}{\sqrt{2}}$ = rms value of the voltage in AC ckt.

$I = \frac{I_m}{\sqrt{2}}$ = rms value of current in AC ckt.

$$\boxed{P = VI\cos\theta}$$

P = Active power.

2. Reactive power :- (VAR)

→ Reactive power generates from reactive element (inductance & capacitance).

→ The product of RMS value of voltage & current with the sine of the angle betn them is called the reactive power in AC ckt.

$$\boxed{Q = VI\sin\theta.}$$

→ Reactive power for purely inductive ckt.

$$Q_L = V_L I = I^2 X_L = \frac{V_L^2}{X_L}$$

Where, X_L = Inductive Reactance.

$$\boxed{X_L = \omega L = 2\pi f L}$$

→ Reactive power for purely capacitive ckt.

$$Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C}$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

3. Complex power :-

→ The product of RMS value of the voltage & current in a ckt is called complex power.

→ It is represented by 'S' & its unit is VA.

→ In complex from

$S = P + jQ$ (for inductive ckt) or $S = P - jQ$ (for capacitive ckt)

→ Magnitude of the complex power of
$S = \sqrt{P^2 + Q^2}$

→ If the voltage across & the current in a certain load is expressed as

$$\boxed{S = VI^*}$$

$V = |V| \angle \alpha$

$I = |I| \angle \beta$

$I^* = |I| \angle -\beta$

$= |V| \angle \alpha \cdot |I| \angle -\beta$

$= |V||I| \angle (\alpha - \beta)$

(In polar form).

→ In Rectangular form, complex power

$$S = |V||I| \cos(\alpha - \beta) - j|V||I| \sin(\alpha - \beta)$$

(for capacitive ckt)

$$S = |V||I| \cos(\alpha - \beta) + j|V||I| \sin(\alpha - \beta)$$

(for inductive ckt)

→ Reactive power Q will be +ve when the angle bet^n V & I will be +ve ($\alpha > \beta$), which means the current is lagging to the voltage.

→ similarly Reactive power Q will be +ve for ($\beta > \alpha$). which indicates that current is leading to the voltage.

POWER TRIANGLE :-

→ It is the geometrical representation of the apparent power, Active power & Reactive power.

→ In an Inductive load the impedance triangle ~~by 'I^2', we get the power triangle which is shown in fig-2.~~ are shown in fig.

→ Multiplying each side of the impedance triangle by 'I^2', we get the power triangle which is shown in fig-2.