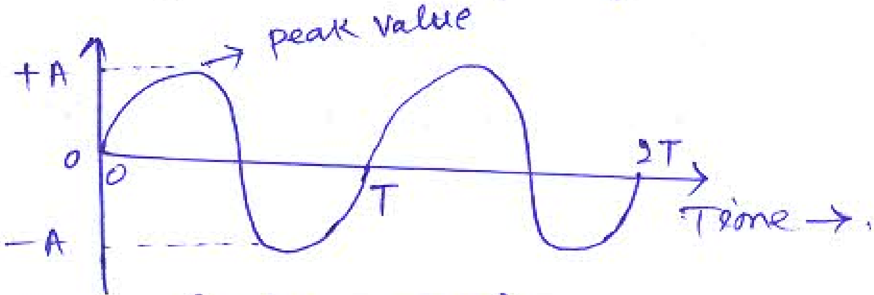
Representation of sine functions as Rotating phasor.

→ The phase of an alternating quantity at any instant in time can be represented by a phasor diagram, so phasor diagrams can be thought of as "functions of time."

→ A complete sine wave can be constructed by a single vector rotating at an angular velocity of $\omega = 2\pi f$, where 'f' is the frequency of the waveform.



(sine wave.)

→ A generalized sinusoid is defind as follows

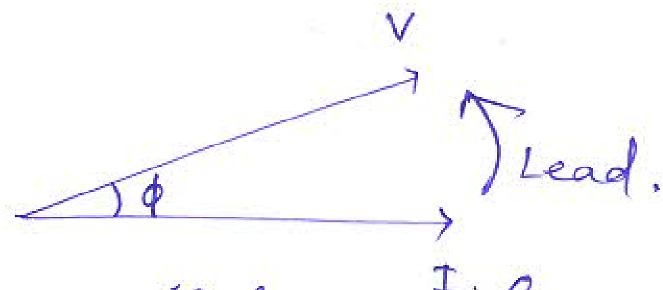
$$x(t) = A \sin \omega t = A \sin 2\pi f t.$$

where A is amplitude, ω is radian frequency

f = natural frequency.

$\omega = 2\pi f$ (radian/sec).

Phasor Diagrams :→



→ phasor diagrams are a graphical way of representing the magnitude and directional relationship between two or more alternating quantities.

→ sinusoidal waveforms of the same frequency can have a phase Difference between themselves which represents the angular difference of the two sinusoidal waveforms.

Impedance and Admittances :→

→ Admittance is defind as, where Y is the admittance, measured in siemens, Z is the impedance, measured in ohms.

→ Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (Known as Reactance).

$$\boxed{Y = \frac{1}{Z}}$$

In R-L Series circuit.

$$Z = \sqrt{R^2 + X_L^2}$$

$$X_L = \omega L = 2\pi f L$$

In R-C Series circuit.

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

In R-L-C Series circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

AC Circuit Analysis.

→ An a.c. circuit is one in which the magnitude of the current changes periodically with time.

→ The a.c. is produced by an alternating voltage supply.

→ The pattern of the a.c voltage is sinusoidal in nature, that is it varies like the sine curve with constant amplitude and frequency.

→ Average value $= \dfrac{\text{Area under the curve.}}{\text{length of the base of the curve.}}$

→ RMS value $= \sqrt{\dfrac{\text{Area of half cycle wave squared.}}{\text{Half cycle base.}}}$

→ <u>Form factor</u> :→ The ratio of rms value to average value of an alternating quantity is known as form factor. It is represented by K_f.

$$K_f = \frac{\text{R.M.S. Value}}{\text{Average value}}$$

→ <u>peak factor</u> :→ The ratio of maximum value to the rms value of an alternating quantity is known as peak factor or amplitude factor. It is represented by Ka.

$$K_a = \frac{\text{Maximum value}}{\text{R.M.S. Value}}$$

FUNDAMENTALS OF POWER

Power

* The ability of work done is known as power.
or. * The Rate of consumption of energy is known as power.

Power In DC Ckt :-

→ power is product of voltage & current.

$$\boxed{P = VI}$$

Power IN AC Ckt :-

→ The Rate of cage of energy w.r.t time in terms of voltage & current.
→ In an AC ckt the voltage & current are continiously changing. Hence power value changes w.r.t time.
→ The average value of vareing power is

$$P_{av} = \frac{1}{T}\int_0^T p \cdot dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} p\,dt$$

→ The average power is also called as active power or true power.

Euler Identity :-

$$\boxed{E^{j\theta} = e^{j\theta} = \cos\theta + j\sin\theta}$$

$$I = 5e^{j0} = 5\angle 0 = 5 + j0$$

5

Power System:-

Generation, transmission & distribution.

→ Generation, transmiffion & distribution system are the main component of an electrical power system.

→ Generating station & distribution system are connected through a transmission line.

→ Electric power system is the combination of components that transforms other types of energy into electrical energy and transmits this energy to the consumer.

POLAR FORM	RECTANGULAR FORM
→ $A \angle \theta$	→ $Z = R + jX$
Ex $10 \angle 30^\circ$	Ex $2 + j5$
→ Multiplication & Division.	→ Addition & substraction.

Addition

* $(2+j3) + (4+j5)$
$= 6 + j8$

* $(2+j3) + 3\angle 30^\circ$
$= 2 + j3 + 2.59 + j1.5$
$= 4.59 + j4.5$

Multiplication

$2\angle 30^\circ \times 5\angle 40^\circ$
$= 10\angle 30 + 40$
$= 10\angle 70^\circ$

* $(3 - j6) \times 5\angle 10^\circ$
$= (6.70\angle -63.43) \times 5\angle 10$
$= 33.5\angle(-63.43 + 10)$
$= 33.5\angle -53.43$

Division

* $\dfrac{10\angle 30^\circ}{2\angle 50^\circ}$
$= 5\angle 30^\circ - 50^\circ$
$= 5\angle -20^\circ$

* $(6+j8) \div 10\angle 5$
$= (10\angle 53.13) \div (10\angle 5)$
$= 1\angle 53.13 - 5$

PROBLEM

Two impedances $Z_A = (25 - j16)$, $Z_B = (3 + j9)$ are connected in parallel in a ckt. Calculate the equivalent impedance for the given ckt in rectangular form.

Sol) $Z_A = 25 - j16$

$Z_B = 3 + j9$

$$Z_{eq} = \frac{Z_A Z_B}{Z_A + Z_B}$$

$$= \frac{(25 - j16)(3 + j9)}{(25 - j16) + (3 + j9)}$$

$$= \frac{(29.68 \angle -32.61)(9.48 \angle 71.5)}{28 - j7}$$

$$= \frac{281.366 \angle 38.89}{28 - j7}$$

$$= \frac{281.366 \angle 38.89}{28.86 \angle -14.03}$$

$$= 9.74 \angle 52.9$$

$$= 5.875 + j7.76 \text{ (Ans)}$$

TYPES of POWER

1. Active power / True power / Real power
2. Reactive / Pulsating power
3. Apparent power = $|V||I|$
4. Complex power = VI^*

(I^* = complex Congugate of I)

1. INSTANTNEOUS & AVERAGE POWER

→ In an AC ckt the power at any instant is called so.

→ It is equal to the product of the values of voltage & current at any instant.

→ In an ac ckt let the instantaneous value of the voltage & current.

$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t - \alpha)$$

Where, α = phase angle betn V & I.

→ Instantaneous power $p = VI$.

$$= V_m \sin \omega t \times I_m \sin(\omega t - \alpha)$$

$$= V_m I_m \sin \omega t \cdot \sin(\omega t - \alpha)$$

$$= \frac{1}{2} V_m I_m \left[\cos\alpha - \cos(2\omega t - \alpha)\right]$$

$$\left[\because \cos A \cdot \cos B = 2\sin\left(\frac{A+B}{2}\right) - \sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{1}{2} V_m I_m \cos\alpha - \frac{1}{2} V_m I_m \cos(2\omega t - \alpha)$$

→ The second term of right hand side of the above eqn contains a double frequency 2ω. so the magnitude of the average value of the 2nd term is zero. It is because average of a sinusodial quantity of double frequency over a complete cycle is zero. Hence the 2nd term is avoided..

$$P_{av} = \frac{1}{2} V_m I_m \cos\alpha$$

→ The instantneous power is the average power in AC ckt.

$$P_{av} = \frac{1}{2} V_m I_m \cos\alpha$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$P_{av} = VI\cos\theta$

Where, $V = \frac{V_m}{\sqrt{2}}$ = rms value of the voltage in AC ckt.

$I = \frac{I_m}{\sqrt{2}}$ = rms value of current in AC ckt.

$$\boxed{P = VI\cos\theta}$$

P = Active power.

2. Reactive power :- (VAR)

→ Reactive power generates from reactive element (inductance & capacitance).

→ The product of RMS value of voltage & current with the sine of the angle betn them is called the reactive power in AC ckt.

$$\boxed{Q = VI\sin\theta.}$$

→ Reactive power for purely inductive ckt.

$$Q_L = V_L I = I^2 X_L = \frac{V_L^2}{X_L}$$

Where, X_L = Inductive Reactance.

$$\boxed{X_L = \omega L = 2\pi f L}$$

→ Reactive power for purely capacitive ckt.

$$Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C}$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

3. Complex power :-

→ The product of RMS value of the voltage & current in a ckt is called complex power.

→ It is represented by 'S' & its unit is VA.

→ In complex from

$S = P + jQ$ (for inductive ckt) or $S = P - jQ$ (for capacitive ckt)

→ Magnitude of the complex power of
$S = \sqrt{P^2 + Q^2}$

→ If the voltage across & the current in a certain load is expressed as

$$\boxed{S = VI^*}$$

$V = |V| \angle \alpha$

$I = |I| \angle \beta$

$I^* = |I| \angle -\beta$

$$= |V| \angle \alpha \cdot |I| \angle -\beta$$
$$= |V||I| \angle(\alpha - \beta)$$

(In polar form).

→ In Rectangular form, complex power

$$S = |V||I| \cos(\alpha - \beta) - j|V||I| \sin(\alpha - \beta)$$

(for capacitive ckt)

$$S = |V||I| \cos(\alpha - \beta) + j|V||I| \sin(\alpha - \beta)$$

(for inductive ckt)

→ Reactive power Q will be +ve when the angle betn V & I will be +ve ($\alpha > \beta$), which means the current is lagging to the voltage

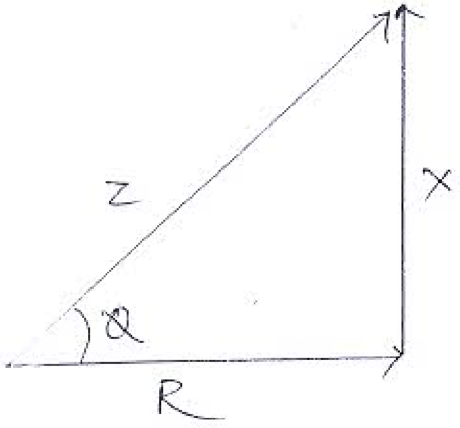
→ similarly Reactive power Q will be +ve for ($\beta > \alpha$). which indicates that current is leading to the voltage.

POWER TRIANGLE :-

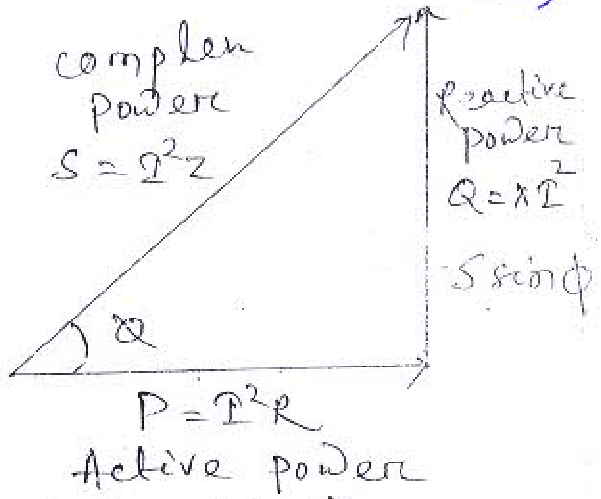
→ It is the geometrical representation of the apparent power, Active power & Reactive power.

→ In an Inductive load the impedance triangle ~~by 'I²', we get the power triangle which is shown in fig-2.~~ are shown in fig.

→ Multiplying each side of the impedance triangle by 'I²', we get the power triangle which is shown in fig-2.



[fig(1)]



$P = S\cos\theta$

$Q = S\sin\theta$

$\tan\theta = \left(\frac{Q}{P}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{Q}{P}\right)$

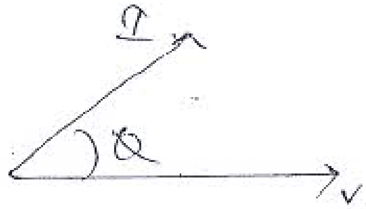
POWER FACTOR

→ It is the ratio of active power to apparent power in an AC ckt.

→ power factor $= \frac{VI\cos\theta}{VI} = \cos\theta = \frac{R}{Z}$

so, unit less.

→ The power factor of an AC ckt is also equal to the co-sine of the phase angle between the applied voltage and the ckt current.



→ From impedance triangle, powerfactor is also defined as the ratio of resistance (R) to the impedance (Z) of the ckt.

→ power factor is unitless.

→ For a purely resistive ckt, power factor is 1 i.e. $\cos 0 = 1$.

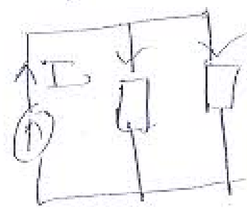
→ For a purely inductive ckt, angle $\cos 90 = 0$, so p.f = 0.

→ For a purely capacitive ckt, p.f = 0 angle $\cos(-90) = 0$.

Problem

1. Two impedances Z_a and Z_b take the following currents, $I_a = 10\angle 65^\circ$ A.
$I_b = 8\angle -25^\circ$ A.
Supply voltage being 24V taking it to be the reference phasor, determine the amount of complex power drawn from the supply.

Ans Complex power (S) $= VI^*$



$V = 24$ volt i.e $24\angle 0^\circ$

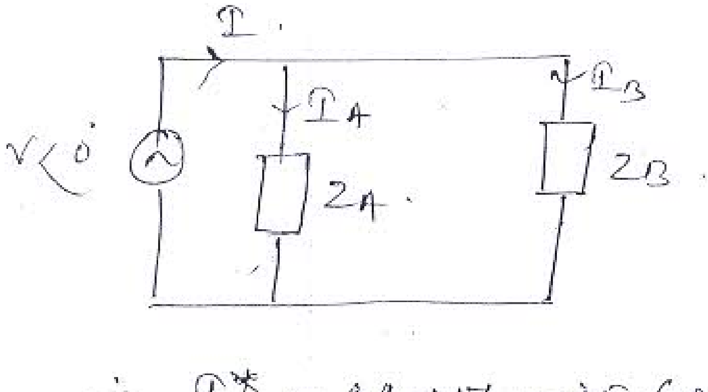
$I^* = ?$

$I_{tot} = I_a + I_b$ (as there are 2 current value given, Z_a & Z_b must be parallel)

$= 10\angle 65^\circ + 8\angle -25^\circ$

$= (4.22 + j9.06) + (7.25 - j3.38)$

$= 11.47 + j5.68$



$\therefore I^* = 11.47 - j5.68$

$I = 12.79\angle 26.34$

$I^* = 12.79\angle -26.34$

$VI^* = 24\angle 0^\circ \times 12.79\angle -26.34 = 306.96\angle -26.34^\circ$

→ For a purely ...

POWER IN BALANCED 3-Φ CKT (12)

→ If the magnitude of voltages to neutral "Vp" for a star connected load is

$$|V_p| = |V_{an}| = |V_{bn}| = |V_{cn}|$$

→ If the magnitude of phase current 'Ip' for a star connected load.

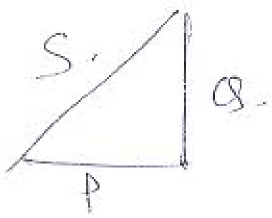
$$|I_p| = |I_{an}| = |I_{bn}| = |I_{cn}|$$

→ The total 3-Φ power is

$$P = 3|V_p||I_p|\cos\theta_p \quad \text{——} (1)$$

Where, θ_p = phase angle by which phase current 'Ip' lags the phase voltage 'Vp' i.e the angle of impedance in each phase.

→ If magnitude of $|V_L|$ & $|I_L|$ are the magnitudes of line to line voltage & line to line current respectively.



$$\left\{\begin{array}{l} |V_p| = \dfrac{|V_L|}{\sqrt{3}} \\ |I_p| = |I_L| \quad \text{——} (2) \end{array}\right.$$

substituting this value in eqn (1) we get

$$P = \sqrt{3}|V_L||I_L|\cos\theta_p \quad \text{——} (3)$$

→ The total power VAR's are

$$\left.\begin{array}{l} Q = 3|V_p||I_p|\sin\theta_p \\ Q = \sqrt{3}|V_L||I_L|\sin\theta_p \end{array}\right\} \text{——} (4)$$

→ The volt ampers of the load are

$$|S| = \sqrt{P^2+Q^2} = \sqrt{3}|V_L||I_L| \quad \text{——} (5)$$

→ If the load is connected in delta the voltage across each impedance is line to line $= \frac{|I_L|}{\sqrt{3}}$

$$\left.\begin{array}{l} |V_p| = |V_L| \\ |I_p| = \dfrac{|I_L|}{\sqrt{3}} \end{array}\right] \text{—} (6)$$

Coupled circuit

Coupling :→ coupling is an electric/magnetic phenomenon in which direct ~~or~~ mutual interaction takes place between two or more passive elements of a Network such that the changes in one element or circuit affect the performance of other elements in same circuit or neighbouring circuit.

→ Coupled circuits is basically of two types.

(1) conductively coupled circuit.

(2) Magnetically coupled circuit.

(1) conductively coupled circuit :→

→ In conductively coupled circuit, the variation in one loop of a given circuit being changes in the neighbouring loops of the same circuit through the current.

(2) Magnetically coupled circuit :→

→ In magnetically coupled circuit, the variation in one loop of a given circuit may affect the performance of other loops in the same circuit or in the neighbouring circuit though the circuits are electrically isolated.

→ It can also be understood as the interconnected loops of an electric network through magnetic field.

Self inductance :→

→ Self inductance of a coil is defined as the property of a coil which opposes any change in the flux linkages of the coil or any change in the current flowing through the coil, by inducing an emf across the coil. This is known as Faraday's laws of Electromagnetic induction. The opposition is given i.e represented by a '−' sign as per Lenz's law.

Mathematically $e_1(t) = -N_1 \dfrac{d\phi_1(t)}{dt}$ ——— (1)

Again $e_1(t) = -L_1 \dfrac{dI_1(t)}{dt}$ ——— (2)

Compairing equ^n (1) and (2)

$$L_1 = N_1 \frac{d\phi_1(t)}{dI_1(t)} = N_1 \frac{d\phi_1}{dI_1} \quad \text{——— (3)}$$

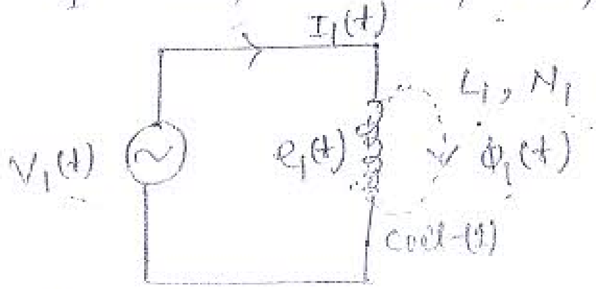
where $L_1 \rightarrow$ coefficient of self Inductance.

$I_1 \rightarrow$ Excitation current flowing through coil-(1).

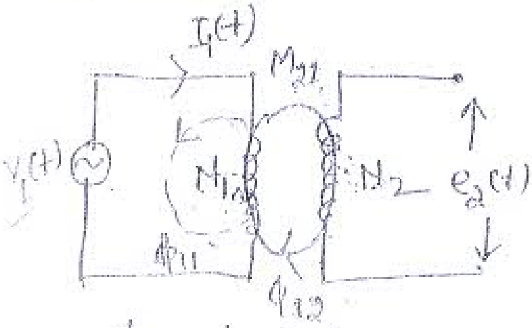
$N_1 \rightarrow$ No. of turns of a coil.

$\phi_1 \rightarrow$ Magnetic flux linking to coil-(1)

$e_1 \rightarrow$ self induced emf in coil-1.



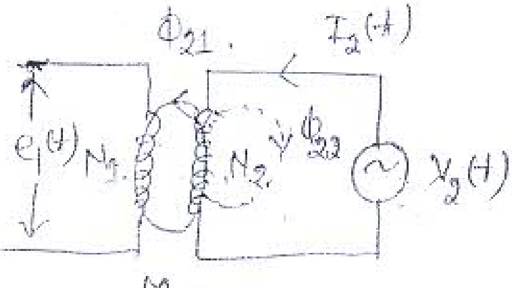
<u>Mutual Inductance</u> :→ Mutual inductance of a coil w.r.t. another coil is defind as the property of the coil which opposes any change in flux linkages in the 2nd coil due to the change in excitation in the 1st coil by inducing an emf across the 2nd coil.



$\phi_1 = \phi_{11} + \phi_{12}$.

(coil-1 is excited)

fig-(a)



$\phi_2 = \phi_{22} + \phi_{21}$.

(coil-2 is excited)

fig-(b)

[circuit for Mutual Inductance]

Case-1. When coil-(1) is excited, mutually induced emf appears in coil-(2)

$$e_2(t) = -N_2 \frac{d\phi_{12}(t)}{dt} \quad \text{———} \; (1)$$

$$+ \; e_2(t) = -M_{21} \frac{dI_1(t)}{dt} \quad \text{———} \; (2)$$

Comparing (1) and (2)

$$-\frac{N_2 \, d\phi_{12}(t)}{dt} = -M_{21} \frac{dI_1(t)}{dt}.$$

$$M_{21} = N_2 \frac{d\phi_{12}(t)}{dI_1(t)} = N_2 \frac{d\phi_{12}}{dI_1} \quad \text{———} \; (3)$$

where,
I_1 = Excitation current in coil-1

ϕ_1 = Total magnetic flux in coil-1.

ϕ_{11} = Magnetic flux that doesn't link coil-2

ϕ_{12} = common magnetic flux linking both coil(1) and coil (2)

$$\boxed{\phi_1 = \phi_{11} + \phi_{12}}$$

e_2 = Mutual induced emf in coil (2)

Case-2 Coil-(2) is excited and mutually induced emf appears in coil-(1)

$$e_1(t) = -N_1 \frac{d\phi_{21}(t)}{dt}.$$

Also, $e_1(t) = -M_{12} \frac{dI_2(t)}{dt}.$

$$M_{12} = N_1 \frac{d\phi_{21}(t)}{dI_2(t)} = N_1 \frac{d\phi_{21}}{dI_2} \quad \text{———} \; (4)$$

where. I_2 = Excitation current in coil (2)

ϕ_2 = Total magnetic flux in coil (2).

$$\boxed{\phi_2 = \phi_{22} + \phi_{21}.}$$

Note. For a particular arrangement, mutual inductance remain same, so $\boxed{M_{12} = M_{21} = M}$

Coefficient of coupling :→

→ It is defind as the fraction of total flux that links the coils i.e. K, the coefficient of coupling $= \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$

where ϕ_1 and ϕ_2 → Total flux in the corresponding coils.

ϕ_{12} and ϕ_{21} → Fluxes linked with the coils.

$\because \phi_{12} < \phi_1$ and $\phi_{21} < \phi_2$.

Hence maximum value of 'K' is unity.

Multiplying equn (3) and (4)

$$M^2 = N_1 N_2 \frac{\phi_{12}\phi_{21}}{i_1 i_2} = \frac{N_1 N_2 \, K\phi_1 \cdot K\phi_2}{i_1 \cdot i_2}$$

$$\boxed{K = \frac{\phi_{12}}{\phi_1}}$$

$$= \frac{N_1 N_2 K^2 \phi_1 \phi_2}{I_1 \cdot I_2}$$

$$= K^2 \frac{N_1 \phi_1}{I_1} \cdot \frac{N_2 \phi_2}{I_2} = K^2 \cdot L_1 \cdot L_2$$

$$\Rightarrow M^2 = K^2 L_1 L_2 \quad \left(\because L = \frac{N\phi}{i}\right)$$

$$\Rightarrow \boxed{\therefore M = K\sqrt{L_1 L_2}}$$

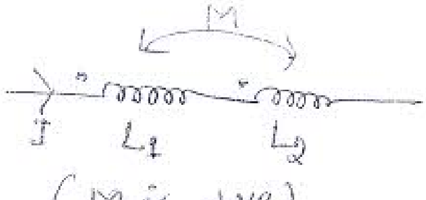
Dot convention for Representing Coupled circuits :→

To determine the relative polarity of the mutually induced voltage a convention is used. This is known as dot convention.

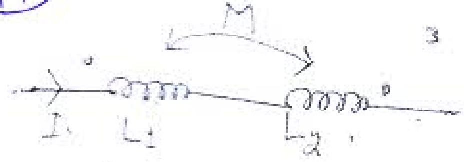
→ According to this convention,

polarity of mutual inductance of a couple coil may be treated as +ve, if the loop currents enter into the respective coils at the dotted ends of respectively coil or simultaneously coming out from the dotted ends of the respective coils and −ve if current enter the dotted end for one coil and leaves the dotted end for other coil.

Exa



(M is +ve)



(M is -ve)

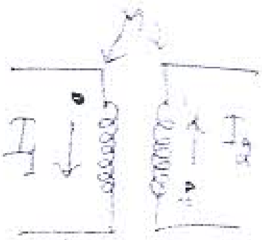
* In both cases coupled coils are connected in series

Dot Rule :→

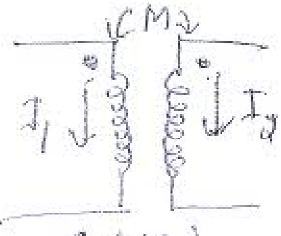
→ When both the assumed currents enter or leave a pair of dotted terminals, the signs of the M-terms and L-terms are same.

→ If one current enters a dotted terminal and the other leaves a dotted terminal, the sign of the M-terms and L-terms are opposite

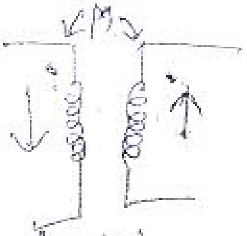
(-M)



(+M)

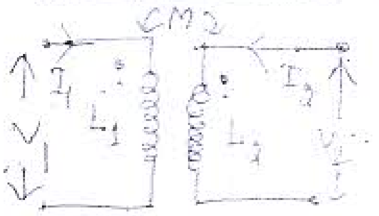


(+M)



(-M)

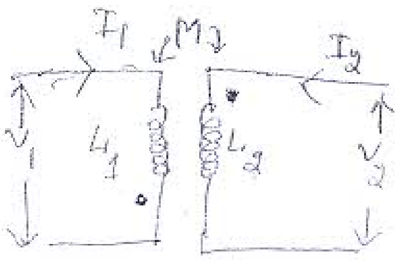
Loop equations in coupled circuits using KVL and Dot convention :→



Applying KVL

$$V_1 = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

$$V_2 = M\frac{dI_1}{dt} + L_2\frac{dI_2}{dt}$$



$$V_1 = L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$V_2 = L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}.$$