NT Module – II

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Solution of First and Second Order Networks :→

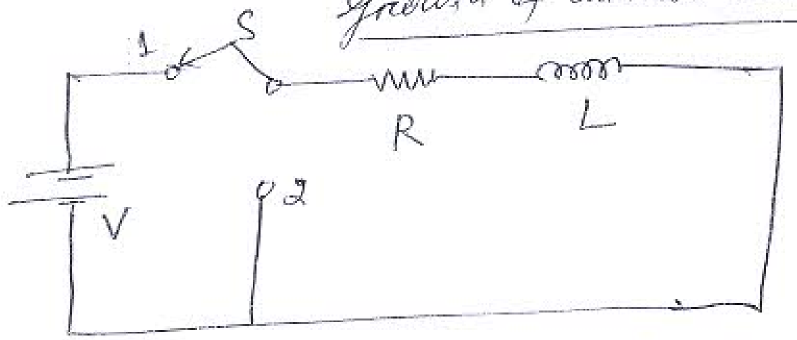
* Solution of first and second order differential equations for series and parallel R-L, R-C, R-L-C circuits. :→

1st order circuit. It contains resistance and one energy storing element i.e. one inductor or capacitor.

→ This first order circuit, during its transient state of operation, is governed by first order linear differential equation.

D-C. steady state Solutions of R-L circuit.

Growth of current in R-L circuit.



Consider some instant t seconds after the voltage is applied.

i = current flowing through the circuit at instant 't' seconds.

$\frac{di}{dt}$ = rate of growth of current at this instant.

$V_R = iR$ = voltage across R

$V_L = L\frac{di}{dt}$ = voltage across L.

Appeying KVL

$$V = Ri + L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \quad \text{——— (1)}$$

Equ^n (1) is a non homogeneous differential equ^n.
Solu^n of this equ^n is

$$i = e^{-\frac{R}{L}t} \int e^{\frac{R}{L}t} \cdot \frac{V}{L} \, dt + K e^{-\frac{R}{L}t} \quad \text{— (2)}$$

$$\Rightarrow i = i_p + i_c$$

Solution of equ^n (2) is.

$$\boxed{i = \frac{V}{R} + K e^{-\frac{R}{L}t}} \quad \text{——— (3)}$$

At $t = 0^+$ (just after switching), equ^n (3) becomes

$$0 = \frac{V}{R} + K e^{-\frac{R}{L} \times 0} = \frac{V}{R} + K$$

$$\Rightarrow K = -\frac{V}{R}.$$

putting in equ^n (3) we get.

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{(4)}$$

where $I_0 = \frac{V}{R}$ = maximum. Current or steady state current

$\tau = \frac{L}{R}$ = time constant.

Equ^n (4) is called Helmholtz equation for growth of current in R-L circuit.

If we put $t = \tau = \frac{L}{R}$ in equn (4) we get

$$i = I_0 (1 - e^{-1}) = 0.632 I_0 = 63.2\% \text{ of } I_0.$$

If we put $t = \infty$ in equn (4) we get

$$i = I_0 (1 - e^{-\infty/\tau}) = I_0$$

Thus current in R-L circuit would attain maximum value (I_0) only after infinite time.

Voltage drop across inductor is $V_L = L \frac{di}{dt}$

$$\Rightarrow V_L = L \cdot \frac{d}{dt}\left[I_0 (1 - e^{-t/\tau})\right] = L \cdot \frac{d}{dt}\left[I_0 - I_0 e^{-t/\tau}\right]$$

$$\Rightarrow V_L = L\left[\frac{d}{dt} I_0 - \frac{d}{dt} I_0 e^{-t/\tau}\right] = L\left[0 + \frac{1}{\tau} I_0 e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_L = L\left[\frac{1}{L/R} I_0 e^{-t/\tau}\right] = I_0 R e^{-t/\tau}.$$

$$\Rightarrow \boxed{V_L = V e^{-t/\tau}.} \quad \text{where } V = I_0 R$$

Voltage drop across resistor is

$$V_R = iR = I_0 (1 - e^{-t/\tau}) R = V (1 - e^{-t/\tau})$$

→ In transient period voltage across resistor exponentially raising and voltage across inductance exponentially decaying. Once the transient dies out within a short time then steady current ($I_0 = \frac{V}{R}$) remains in the circuit.

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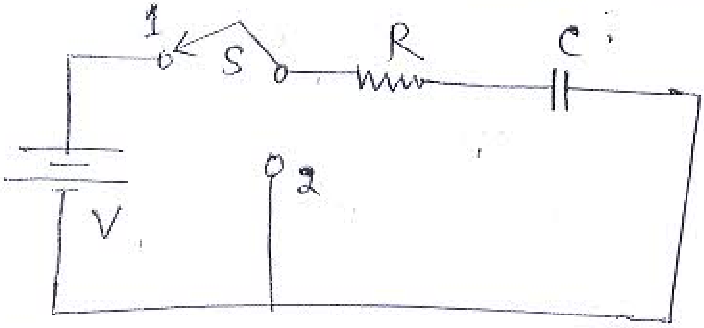
D.C. Steady state Solution of R.C circuit :→ case-I. Charging of R-C circuit.



Consider a R–C circuit connected in series with a battery of voltage V and a switch 'S'.

→ Initially capacitor is uncharged and voltage across it is zero. ($V_C = 0$)

$\Rightarrow V_R = V \rightarrow$ whole supply voltage.

Initial current in the circuit $\boxed{I_0 = \frac{V}{R}}$

→ As current flows in the capacitor, starts charging and capacitor voltage V_C increases.

→ consider some instant 't' second after the voltage is applied,

$i \rightarrow$ current flowing through the circuit at instant 't' seconds.

$V_R = Ri$ = voltage drop across R

$V_C = \frac{1}{C}\int I \cdot dt$ = Voltage across 'C'.

Applying KVL, $\boxed{Ri + \frac{1}{C}\int i \cdot dt = V}$

Differentiating both sides w.r.t time 't', we get

$R\frac{di}{dt} + \frac{i}{C} = 0 \Rightarrow \boxed{\frac{di}{dt} + \frac{1}{RC}i = 0}$ —— (1)

Equation (1) is a homogeneous equation. solution of this equ^n (1)

$$i = Ke^{-\frac{t}{RC}} \quad \text{--- (2)}$$

where K is a constant, whose value. can be calculated from the initial. condition.

$t = 0^+$

Current $i(0^+) = \frac{V}{R}$.

equ^n (2) becomes.

$$\frac{V}{R} = Ke^{-\frac{0}{RC}}$$

$$\Rightarrow \boxed{K = \frac{V}{R}}$$

putting this value. of 'K' in equ^n (2)

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$\Rightarrow \boxed{i = I_0 e^{-\frac{t}{\tau}}} \quad \text{--- (3)}$$

where $I_0 = \frac{V}{R}$ = maximum current. in the circuit.

$\tau = RC$ = Time constant or. Capacitive time constant.

Voltage across the resistor.

$$V_R = iR = I_0 R e^{-t/\tau} = Ve^{-t/\tau} \quad \text{--- (4)}$$

Voltage across the capacitor :→

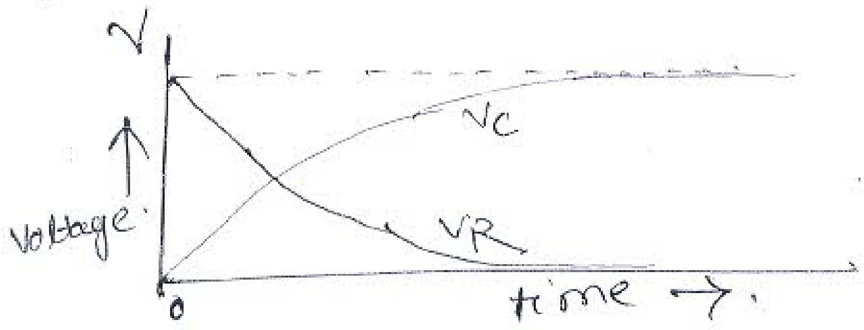
$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_0^t I_0 e^{-t/\tau}$$

$$\Rightarrow \boxed{V_C = V\left(1 - e^{-t/\tau}\right)} \quad \text{---- (5)}$$

→ The charge stored in capacitor during charging is $q = CV_C = CV(1-e^{-t/\tau}) = Q(1-e^{-t/\tau})$

→ If we put $t = \tau = RC$, in eqn (5) we get.

$$V_C = V(1-e^{-1}) = 0.632V = 63.2\%\ \text{of}\ V$$



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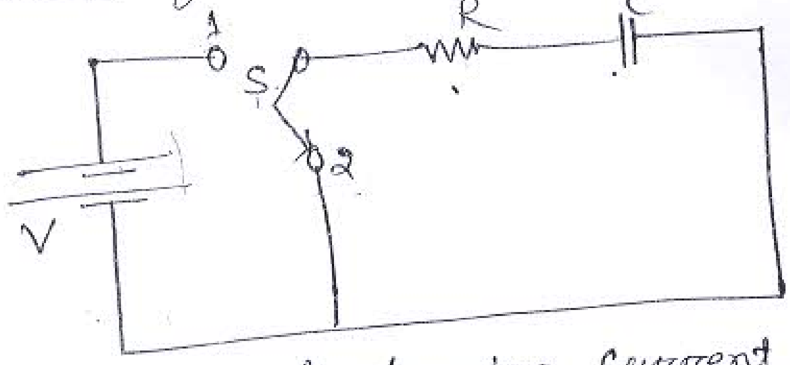
Discharging of RC circuit :→



Let i = discharging current at any instant.

Applying KVL

$$Ri + \frac{1}{C}\int i \cdot dt = 0$$

Differentiating both sides w.r.t time (t), we get

$$R\frac{di}{dt} + \frac{i}{C} = 0 \quad \text{— (6)}$$

homogenous diffential equⁿ

Equⁿ (6) is a homogenous diffential equⁿ.
Soluⁿ of this equⁿ.

$$i = Ke^{-\frac{t}{RC}} \quad \text{— (7)}$$

where 'K' is a const., whose value can be calculated from initial condition.

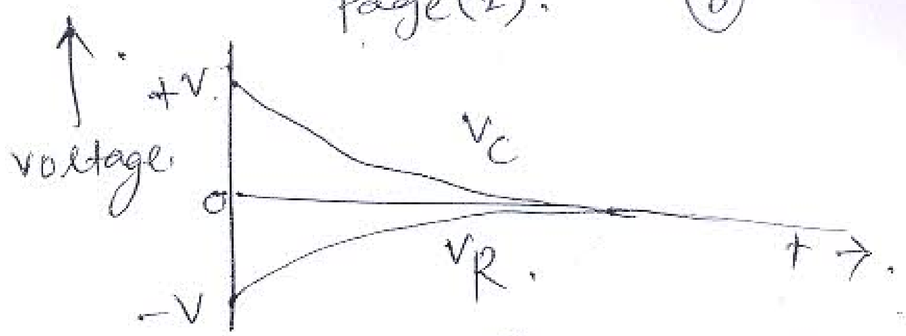
at $t=0$, $i = -\frac{V}{R}$, equⁿ (7) becomes.

$$-\frac{V}{R} = Ke^{\frac{0}{RC}}$$

$$\Rightarrow \boxed{K = -\frac{V}{R}}$$ putting this value in equⁿ (7)

$$i = -\frac{V}{R}e^{-t/RC} = -I_0 e^{-t/\tau}$$

$$\boxed{i = -I_0 e^{-t/\tau}} \quad \text{— (8)}$$



$$V_R = Ri = -I_0 R e^{-t/\tau} = -V e^{-t/\tau}$$

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int -I_0 e^{-t/\tau}$$

$$\Rightarrow \boxed{V_C = I_0 R e^{-t/\tau} = V e^{-t/\tau}}$$

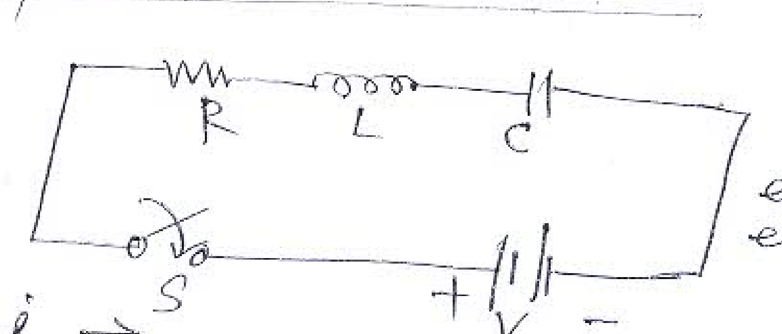
The charge in capacitor during discharging is.

$$q = CV_c = C V e^{-t/\tau} = Q e^{-t/\tau}.$$

where $Q = CV$ = maximum charge in capacitor.

Transient Response of second order

Solu^n of second order circuit.



Two types of energies

electromagnet

electrostatic

Applying KVL.

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$

$$\Rightarrow R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{d^2 i}{dt^2} + \frac{R}{L}\cdot\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \text{--- (1)}$$

homogeneous.

Equ^n (1) is a second order linear homogeneous differential equ^n.

Its characteristics equ^n is.

$$\boxed{p^2 + \frac{R}{L}p + \frac{1}{LC} = 0} \quad \text{— (2)} \quad \left[\because p = \frac{d}{dt}\right]$$

The roots of this char. equ^n

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}.$$

$$= \alpha \pm \beta.$$

$$P_1 = \alpha + \beta \quad , \quad P_2 = \alpha - \beta.$$

where $\alpha = \frac{-R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$.

The solu^n of the differential equ^n is.

$$\boxed{i = K_1 e^{P_1 t} + K_2 e^{P_2 t}} \quad \text{— (3)}$$

where K_1 and K_2 are constants whose values are calculated from boundary conditions.

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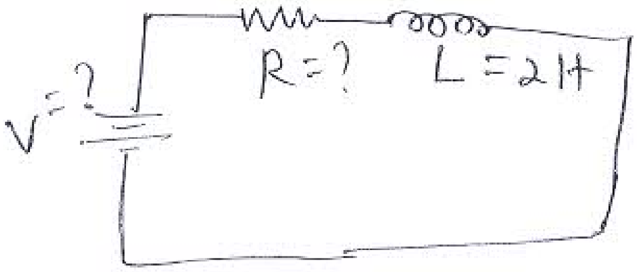
Solved Examples :→

Exa-1 A constant Voltage is applied to R-L Series circuit at $t=0$, by closing a switch. The voltage across 'L' is 25 V at $t=0$ and drops to 5V at $t = 0.025$ sec. $L = 2H$. find.

(a) the applied voltage.
(b) the value of R

Soln



(a) At $t=0$, the entire voltage is dropped across inductor and no voltage is dropeed across the resistor. so applied Voltage $V = 25$ volt.

(b) At any instant voltage across inductor is 5V ($V_L = 5$ volt)

$$V_L = V e^{-t/\tau}$$

$$\Rightarrow 5 = 25 e^{\frac{-0.025}{\tau}}$$

$$\Rightarrow \tau = 0.01553$$

$$\Rightarrow \frac{L}{R} = 0.01553$$

$$\Rightarrow R = \frac{L}{0.01553} = \frac{2}{0.01553} = 128.78\Omega$$

$$\Rightarrow \boxed{R = 128.78\Omega}$$

Exa 2 The winding of an electromagnet has an inductance of 3H and resistance of 15 Ω when it is connected to 120 V dc supply. Calculate.

(a) The steady state value of current flowing in the winding.

(b) The time constant of the circuit.

(c) The value of induced emf after 0.1 sec.

(d) The time for the current to rise to 85 % of its final value.

(e) The value of current after 0.3 sec.

(a) Given. $R = 15\Omega$, $L = 3H$ and $V = 120V$

steady state current $I_0 = \frac{V}{R} = \frac{120}{15} = 8A$

(b) $\tau = \frac{L}{R} = \frac{3}{15} = 0.2$ sec.

(c) The value of induced emf after 0.1 sec is

$$V_L = Ve^{-t/\tau} = 120\left(e^{\frac{-0.1}{0.2}}\right) = 72.783 \text{ volt.}$$

(d) Current at any instant during rise is,

$$i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \frac{85}{100} I_0 = I_0\left(1 - e^{\frac{-t}{0.2}}\right)$$

$$\Rightarrow \boxed{t = 0.379 \text{ sec}}$$

(e) $$i = I_0\left(1 - e^{-\frac{t}{\tau}}\right) = 8\left(1 - e^{\frac{-0.3}{0.2}}\right)$$

$$\Rightarrow \boxed{i = 6.21 A}$$