M-5

Introduction To Quantum Mechanics →

The quantum mechanism deals with the study of the mecroscopic property of the matter.

OR

It is the branch of physics which deals with the small particle whose size must be less than 10^{-10} m.

speed →

Size ↑	For less than 3×10^8 m/s	comparable to 3×10^8 m/s
larger than 10^{-10} m	classical Mech.	Relativistic Mech.
Less than 10^{-10} m	Quantum Mech.	Quantum field Theory

classical mechanics can't explain stem of physical phenomenon are

1. quantaisation of certain physical properties.
2. Quantum & entangelment.
3. Principle of uncertainty
4. wave particle duality.

These phenomenon are completely explain by quantum mechanics

classical mechanics describe the motion of particles, rigid body, fluids etc. Again some formulation of the law of electromagnetism, thermo dynamics, geometrical & Physical optics which can explain Properly by the new corresponding Theory.

Many new Phenomenan observe during the last decade of 19 century & early years of 20 century couldn't be explain within the framework of classical physics. In an attempt some new Phenomenan like time-tested concept of classical Physics ~~were~~ there modified in favour of new revolutionary concept. This Process gave birth to the quantum Physics.

Due to the birth of quantum Physics some legends of scientist gave the various quantum Phenomenan which are more useful & more predictable in the revolutionary Physics. These scientist are

- I. Plank
- II. Einstein
- III. ~~Deo~~ de Broglie
- IV. Heizen Berg
- V. (Sordinger) Schrodinger
- VI. Bohre ~~Rutherford~~
- VII. Dirac & many more.

The quantum idea was 1st introduced by Max Plank in the year of 1900 in an attempt to explain the energy distribution in the spectrum of blackbody radiation. This idea was later successfully explain by Einestine & gave the different Phenomenon of it.

Neil Bohre use the simillar quantum concept to formulate the model of hydrogen atom & observe hydrogen spectra successfully. These devlopment leads the following conclusions that are

1. radiant energy such as electro-magnetic radiation which show the dual nature of radiation.
2. Some dynamical Properties such as energy, momentum & angular momentum at the descrete value but not the arbitary value. So hence the electromagnetic radiation further explain by Louis de Broglie ~~Dirac~~ which give the wave Particle duality which can explain the dual nature of radiation.

Some of the quantum theory which states that every system is characterised by a wave fun ψ which describe the system completely. The wave func satisfy a partial differential eqn called the Schrodinger eqn.

The wave func satisfied a partial differential eqn which can again modify by Max Born who can gave the mechanical formulation of quantum theory & this is the case when the quantum field theory advancelize the quantum mechanics. Similarly various scientist get their contribution towards the quantum mechanics which can help to our day to day life.

Photoelectric effect :—

(i) When a light is incident on the metal surface then some -ve electrons will be release out in a particular direction.

(ii) These -ve electrons are called as photoelectrons & the process is called as photoelectric effect.

Laws of Photoelectric effect :—

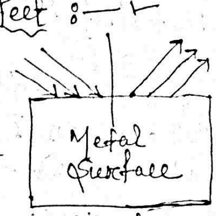
(i) Photoelectric effect is an instantaneous process.

(ii) Photoelectric current is directly proportional to the intensity of incident light & is independent of frequency.

(iii) The stopping potential & the max^n velocity of electrons depend upon the frequency of incident light but independent of intensity of incident light.

(iv) The emission of electrons stop below as certain min^n frequency known as threshold frequency.

Einsteine Photoelectric effect :—



(i) Einstein explained photoelectric effect on the basis of Plank's quantum theory.

(ii) According to this theory the radiations composed of small bondles of energy.

(iii) Einstein suggests that these bondles of energy as photon.

(iv) So, according to this theory,

$$E = h\nu$$

where, h = Plank's constant.
ν = frequency of readiation.

(v) Again he observed that the electrons are emitted from the metal surface, when they acquire some extra energy to overcome the work func (W_0) of the metal surface.

(vi) So, according to Einsteine,

$$E = W_0 + K.E$$

$$\Rightarrow h\nu = h\nu_0 + \frac{1}{2} m V_{max}^2$$

$$\Rightarrow \boxed{\frac{1}{2} m V_{max}^2 = h\nu - h\nu_0 = h(\nu - \nu_0)}$$

Where; V_{max} = max^n velocity of emitted $\bar{e}$.

m = mass of the electron.

$E = h\nu$ = energy of the incident photon.

$W = h\nu_0$ = work funᶜ.

ν_0 = threshold frequency.

This eqⁿ is known as Einstein's eqⁿ of photoelectric effect.

Work funᶜ :—

It is the minᵐ amount of energy required to pull out an $\bar{e}$ from the surface of metal i.e. $W_0 = h\nu_0$.

Compton effect :—

→ The effect of change in wavelength on scattering of x-ray photon by electrons is called compton effect.

→ The change in wavelength ($\Delta\lambda$) is called compton shift.

→ The spectrum of the scattered radiation at an angle 'θ' with the incident direction consist of two components.

(i) One of the component having wavelength same as that of the incident radiation is called thomson component.

(ii) The other component having wavelength greater than the incident wavelength is called compton component.

Compton also observed that —

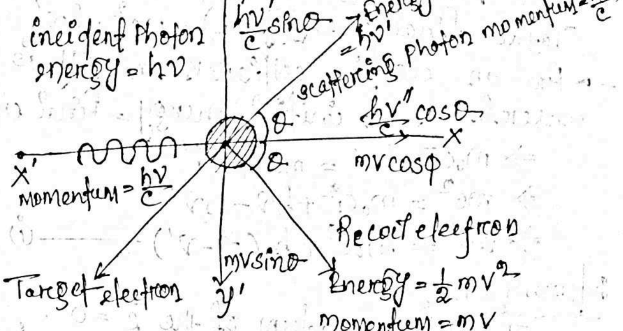
(i) The difference $\Delta\lambda$ is always +ve.

(ii) $\Delta\lambda$ is an increasing funᶜ of the angle 'θ' betⁿ the direction of incident & the direction of scattering.

(iii) $\Delta\lambda$ depends only on 'θ' & is independent of the wavelength of the incident radiation & the nature of the scattering material.

Theory :—

(i) According to compton the phenomenon of scattering is due to the elastic collision betⁿ the photons & a free electron at rest.



$$\boxed{\begin{aligned} E &= mc^2 = mc \cdot c \\ E &= pc \\ \Rightarrow p &= \frac{E}{c} = \frac{h\nu}{c} \end{aligned}}$$

(ii) In fig. a x-ray photon collide with an electron at rest. After collision both the scattered photon & the electrons move in different direction.

(iii) Let θ = angle betⁿ the incident photon dirⁿ & the scattered photon direction.

(iv) ϕ = angle betⁿ the incident photon dirⁿ & the recoil electron.

→ ν = frequency of the incident photon.

→ ν' = frequency of the scattered photon.

→ m_0 = mass of the rest $\bar{e}$.

→ m = mass of the moving $\bar{e}$.
→ v = velocity of the electron.

Energy :—

→ Initial energy of the $\bar{e} = m_0c^2$.
→ Initial energy of the photon $= h\nu$
Total initial energy (E) $= m_0c^2 + h\nu$
→ Final energy of the $\bar{e} = mc^2$
Final energy of the photon $= h\nu'$
Total final energy (E) $= mc^2 + h\nu'$
→ For an elastic collision energy is conserved. So initial energy = final energy.

$\Rightarrow m_0c^2 + h\nu = mc^2 + h\nu'$

$\Rightarrow mc^2 = m_0c^2 + h\nu - h\nu'$

$\Rightarrow mc^2 = m_0c^2 + h(\nu - \nu')$ ——— (i)

Momentum :—

→ Initial momentum of the $\bar{e} = 0$.
→ Initial momentum of the photon $= \frac{h\nu}{c}$
→ Total initial momentum $= 0 + \frac{h\nu}{c} = \frac{h\nu}{c}$.
→ Final momentum of the $\bar{e} = mv\cos\varphi$
→ Final momentum of the scattered photon $= \frac{h\nu'}{c}\cos\theta$
→ Total final momentum $= \frac{h\nu'}{c}\cos\theta + mv\cos\varphi$.

According to conservation law of momentum = Initial momentum = final momentum.

$\Rightarrow \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\varphi$

$\Rightarrow mv\cos\varphi = \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta$

$\Rightarrow mv\cos\varphi = \frac{h(\nu - \nu'\cos\theta)}{c}$

$\Rightarrow h(\nu - \nu'\cos\theta) = mvc\cos\varphi$ ——— (ii)

The perpendicular component of momentum can be written as —

$\frac{h\nu'}{c}\sin\theta + (-mv\sin\varphi) = 0$

$\Rightarrow mv\sin\varphi = \frac{h\nu'}{c}\sin\theta$

$\Rightarrow mvc\sin\varphi = h\nu'\sin\theta$ ——— (iii)

Squaring & adding eq^n (ii) & (iii) we get,

$m^2v^2c^2\cos^2\varphi + m^2v^2c^2\sin^2\varphi = h^2(\nu - \nu'\cos\theta)^2 + h^2\nu'^2\sin^2\theta$

$\Rightarrow m^2v^2c^2 = h^2[\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta] + h^2\nu'^2\sin^2\theta$

$= h^2[\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta + \nu'^2\sin^2\theta]$

$\Rightarrow m^2v^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta)$ ——— (iv)

Squaring eq^n (i) both sides we get

$(mc^2)^2 = [m_0c^2 + h(\nu - \nu')]^2$

$\Rightarrow m^2c^4 = m_0c^4 + h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2m_0c^2(h\nu - h\nu')$ ——— (v)

Subtracting eq^n (iv) from eq^n (v) we get

$m^2c^4 - m^2v^2c^2 = m_0^2c^4 + h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2m_0c^2h(\nu - \nu') - h^2\nu^2 - h^2\nu'^2 + h^2 2\nu\nu'\cos\theta$

$\Rightarrow m^2c^4 - m^2v^2c^2 = m_0^2c^4 + h^2\nu^2 + h^2\nu'^2 - 2\nu\nu'h^2 + 2m_0c^2h(\nu - \nu') - h^2\nu^2 - h^2\nu'^2 + h^2 2\nu\nu'\cos\theta$

$\Rightarrow m^2c^2(c^2 - v^2) = m_0^2c^4 + h^2(\cancel{\nu^2} + \cancel{\nu'^2} - 2\nu\nu' - \cancel{\nu^2} - \cancel{\nu'^2} + 2\nu\nu'\cos\theta) + 2m_0c^2h(\nu - \nu')$

$$= m_0^2c^4 + h^2(2\nu\nu'\cos\theta - 2\nu\nu') + 2m_0c^2h(\nu-\nu')$$

$$= m_0^2c^4 + h^2[2\nu\nu'(\cos\theta - 1)] + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow m^2c^2(c^2-v^2) = m_0^2c^4 + h^2[2\nu\nu'(\cos\theta-1)] + 2m_0c^2h(\nu-\nu')$$

Putting the value of $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ in the above eq^n we get ——

$$\Rightarrow \frac{m_0^2}{1-\frac{v^2}{c^2}} \cdot c^2(c^2-v^2) = m_0^2c^4 + h^2[2\nu\nu'(\cos\theta-1)] + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow \frac{m_0^2c^2}{(c^2-v^2)} \cdot c^2(c^2-v^2) = m_0^2c^4 + h^2[2\nu\nu'(\cos\theta-1)] + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow m_0^2c^4 = m_0^2c^4 + h^2[2\nu\nu'(\cos\theta-1)] + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow h^2[2\nu\nu'(\cos\theta-1)] + 2m_0c^2h(\nu-\nu') = 0$$

$$\Rightarrow h^2[2\nu\nu'(\cos\theta-1)] = -2m_0c^2h(\nu-\nu')$$

$$\Rightarrow h[\nu\nu'(\cos\theta-1)] = -m_0c^2(\nu-\nu')$$

$$\Rightarrow h\nu\nu'(1-\cos\theta) = m_0c^2(\nu-\nu')$$

$$\Rightarrow h(1-\cos\theta) = \frac{m_0c^2(\nu-\nu')}{\nu\nu'}$$

$$\Rightarrow \frac{h}{m_0c^2}(1-\cos\theta) = \frac{(\nu-\nu')}{\nu\nu'}$$

$$\Rightarrow \frac{h}{m_0c^2}(1-\cos\theta) = \frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'}$$

$$\Rightarrow \boxed{\frac{h}{m_0c^2}(1-\cos\theta) = \frac{1}{\nu'} - \frac{1}{\nu}}$$

$$\Rightarrow \frac{h}{m_0c}(1-\cos\theta) = \frac{c}{\nu'} - \frac{c}{\nu}$$

Using the formula $c = \nu\lambda$ we get $\left(\because c = \nu\lambda \Rightarrow \lambda = \frac{c}{\nu}\right)$

$$\Rightarrow \frac{h}{m_0c}(1-\cos\theta) = \lambda' - \lambda = \Delta\lambda$$

$$\Rightarrow \boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)} \quad \text{——— (vi)}$$

This is the expression for compton shift & from the above eq^n it is clear that, there is increase in wavelength $\Delta\lambda$ is independent of the wavelength of incident radiations.

So, again $1-\cos\theta = 2\sin^2\frac{\theta}{2}$

$$\Rightarrow \boxed{\Delta\lambda = \frac{h}{m_0c} \cdot 2\sin^2\frac{\theta}{2}}$$

<u>Case-I</u> :— (When $\theta = 0°$)

$$\Delta\lambda = \frac{h}{m_0c} \cdot 2 \times 0 = 0$$

No, scattering effect.

<u>Case-II</u> :— (When $\theta = 90°$)

$$\Delta\lambda = \frac{h}{m_0c} \times 2 \times \frac{1}{2} = \frac{h}{m_0c}$$

$$\Rightarrow \boxed{\lambda_c = \frac{h}{m_0c}}$$ (It is known as compton length)

<u>Case-III</u> :— (When $\theta = 180°$)

$$\Delta\lambda = \frac{h}{m_0c} \times 2 = \frac{2h}{m_0c}$$

$\Delta\lambda = 2 \times$ compton wavelength $\left(\because \frac{h}{m_0c} = \text{compton wavelength}\right)$

It gives the max^m wavelength of the scattered Photon, and also it can be written as

$\Delta\lambda = 2 \times$ compton wavelength.

Also it is the special case of head on elastic collision.

Dual nature of radiation :-

→ light exhibits the reflection, refraction, dispersion, diffraction, Transmission, photo electric effect & compton effect.

→ Among the all Phenomenon reflection, refraction, dispersion, diffraction & transmission. Show the wave nature of the light Particle & the remaining Phenomenon photoelectric effect & compton effect which show the Particle nature of the light Particle.

→ So, the wavelength of the light Particle can be written as,

$$\lambda = \frac{h}{P}$$

where, h = Plank's constant.
P = momentum.

→ The expression for energy,

$$E = h\nu$$

Where, ν = frequency of the light Part.

→ So, from the above as the light show the wave nature along with Particle nature but not simultaneously. So this is called the dual nature of radiation.

De-Broglie Hypothesis :-

→ The electromagnetic ~~radation~~ radiation show the dual nature.

→ It exhibits the particle nature along with the wave nature but not simultaneously & acts two different experiment.

→ If 'ν' is the frequency of the light Particle then we can write energy of the light Particle $E = h\nu$.

→ In electromagnetic radiation the Particle & the wave can stay in the form of energy Packet & the energy of the each Packet by Einstein is $E = h\nu$.

→ Then in electromagnetic radiation the frequency & the wavelength show the wave character & the momentum & energy show the Particle characteristics.

→ So, $E = \hbar\omega = \frac{h}{2\pi} \times 2\pi\nu = h\nu$

$$P = \hbar K = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \frac{h}{\lambda}$$

Here, h = Plank's constant.

→ From the above it is show that the energy & the momentum can relate to each other by a Plank's constant 'h'.

→ So, Lewis De-Broglie Postulates that, if light Particle exhibits the wave nature then the Particle such as electron also show the Particle nature.

→ Hence, light wave exhibit a Particle nature & the Particle is also wave characteristics.

→ So, according to De-Broglie if λ is the wavelength of any Particle in the wave then, the De-Broglie wavelength —

$$\boxed{\lambda = \frac{h}{P}}$$

where, h = Plank's constant
P = momentum.

→ It also can be derived from the relativistic

we have, $E = h\nu$ —— (i)

Also in case of relastivestic

$$E = \sqrt{m_0c^2 + P^2c^2}$$

As the rest mass of Photon
$m_0 = 0$

So $E = pc$ ——— (ii)

comparing eq^n (i) & (ii)

$h\nu = pc$

$\Rightarrow p = \frac{h\nu}{c}$

Again we have, $c = \nu\lambda$.

So, $p = \frac{h\nu}{\nu\lambda} = \frac{h}{\lambda}$

$\Rightarrow \boxed{\lambda = \frac{h}{p}}$

Different forms of De-Broglie Wave length :-

(i) The De-Broglie wavelength (In general case)

$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$

where, m = mass of electron.
v = velocity

(ii) In case of non-relativistic case;

$E_K = p^2/2m$

$\Rightarrow p^2 = 2m E_K$

$\Rightarrow p = \sqrt{2mE_K}$

So; $\boxed{\lambda = \frac{h}{\sqrt{2mE_K}}}$ $\left(\because \lambda = \frac{h}{p}\right)$

where E_K = Kinetic energy.

(iii) When a charge particle having a charge 'q' & Potential difference 'v' then the energy of the charge particle,

$E_K = qV$

So, the De-Broglie wavelength

$\Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2mE_K}} = \frac{h}{\sqrt{2mqV}}}$

(In electrostatic field)

(iv) Consider a matter in thermal radiation,
So, the De-Broglie wavelength

$\lambda = \frac{h}{\sqrt{2mE_K}} = \frac{h}{\sqrt{2m \times \frac{3}{2}KT}}$

$\Rightarrow \boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$

(v) In case of relativistic effect,

$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

So, the De-Broglie wavelength,

$\lambda = \frac{h}{p} = \frac{h}{mv}$

$\Rightarrow \lambda = \frac{h}{m_0 v/\sqrt{1 - v^2/c^2}}$

$\Rightarrow \boxed{\lambda = \frac{h\left(\sqrt{1 - v^2/c^2}\right)}{m_0 v}}$

Schrodinger time dependent Eq^n :-

Wave fun^c for a particle having energy 'E' & momentum 'p' travelling x-direction is given by —

$\psi(x,t) = A e^{-\frac{i}{\hbar}(Et - px)}$ ——— (i)

Differentiating twice w.r.t x

$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} A e^{-\frac{i}{\hbar}(Et - px)} = \left(\frac{ip}{\hbar}\right) A e^{-\frac{i}{\hbar}(Et - px)}$

Again

$\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)^2 A e^{-\frac{i}{\hbar}(Et - px)}$

$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \left(-\frac{p^2}{\hbar^2}\right) A e^{-\frac{i}{\hbar}(Et - px)}$

$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi$

$$\Rightarrow p^2\psi = -\hbar^2 \frac{\partial^2\psi}{\partial x^2} \quad \text{--- (2)}$$

Differentiating eq^n (i) w.r.t time

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar} A e^{-\frac{i}{\hbar}(Et - Px)}$$

$$= -\frac{iE}{\hbar}\psi$$

$$= \frac{E}{i\hbar}\psi$$

$$\Rightarrow i\hbar \frac{\partial\psi}{\partial t} = E\psi \quad \text{--- (3)}$$

Total energy of a particle is the sum of K.E & P.E i.e $E = \frac{p^2}{2m} + V$

Where, V = Potial to which the particle is subjected.

Multiplying ψ on both sides

$$E\psi = \frac{p^2\psi}{2m} + V\psi$$

$$\Rightarrow \boxed{i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi}$$

This is schrodinger time dependant eq^n in one dimension for a particle moving in a potential V.

In 3 dimension eq^n becomes

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\psi(r,t) + V\psi(r,t)$$

$$= i\hbar \frac{\partial(r,t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,t) + V\psi(r,t) = i\hbar \frac{\partial\psi}{\partial t}(r,t)$$

Schrodinger Eq^n for a free particle :-

For a free particle P.E 'V' is independent of position. Since there is no external force. Hence for a free particle $V=0$.

Schrodinger eq^n in 1 dimension

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar \frac{\partial\psi(x,t)}{\partial t}$$

In 3-dimension

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,t) = i\hbar \frac{\partial\psi}{\partial t}(r,t)$$

Schrodinger Time Independent Eq^n :-

When P.E depends on position only & is independent times we can separate space & time variable & get the time independent schrodinger eq^n. So wave func^c for a particle of energy 'E' & momentum 'p' travelling along x-direction is

$$\psi(x,t) = Ae^{-\frac{i}{\hbar}(Et - Px)} \quad \text{--- (i)}$$

$$\Rightarrow \psi(x,t) = Ae^{iPx/\hbar} \cdot e^{-iEt/\hbar}$$

Again $\psi(x) = Ae^{iPx/\hbar}$ & $T(t) = e^{-iEt/\hbar}$

Now $\psi(x,t) = \psi(x) \cdot T(t)$ --- (ii)

Where $\psi(x)$ varies with time only & $T(t)$ varies with time only.

Generally Times depends schrodinger eq^n is —

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar \frac{\partial\psi(x,t)}{\partial t} \quad \text{--- (iii)}$$

Substituting eq^n(ii) in eq^n(iii) we get

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\psi(x)T(t)\right]+V\psi(x)T(t) = i\hbar\frac{\partial}{\partial t}\psi(x)T(t)$$

$$\Rightarrow -\frac{\hbar^2}{2m}T(t)\frac{\partial^2\psi(x)}{\partial x^2}+V\psi(x)T(t) = i\hbar\,\psi(x)\frac{\partial T(t)}{\partial t} \quad \text{—(iv)}$$

Now $\frac{\partial T(t)}{\partial t} = \frac{\partial}{\partial t}\left[e^{-iEt/\hbar}\right] = -\frac{iE}{\hbar}\left[e^{-iEt/\hbar}\right]$

$$= -\frac{iE}{\hbar}T(t) \quad \text{—(v)}$$

Substituting eq^n(v) in eq^n(iv) we get

$$-\frac{\hbar^2}{2m}T(t)\frac{\partial^2\psi}{\partial x^2}+V\psi(x)T(t) = i\hbar\psi(x)\left[-\frac{iE}{\hbar}T(t)\right]$$

Dividing through out by $T(t)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V\psi(x) = i\hbar\,\psi(x)\left[-\frac{iE}{\hbar}\right]$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V\psi(x) = E\psi$$

$$\Rightarrow \boxed{\frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}(E-V)\psi(x) = 0}$$

This is Schrodinger time independent eq^n in 1-dimension.

In 3-dimension ⟶

$$\nabla^2\psi(x)+\frac{2m}{\hbar^2}(E-V)\psi = 0$$

Sol^n of Schrodinger Wave Eq^n :—

We know wave fun^c can be written as

$$\psi(x,t) = \psi(x)\,T(t)$$

In this fun^c $\psi(x)$ depends on potential $V(x)$ & can be obtained by solving the Schrodinger time independent eq^n.

The fun^c $T(t)$ is same for all potential & can be determined as follows —

$$T(t) = e^{-iEt/\hbar}$$

$$\frac{\partial T(t)}{\partial t} = -\frac{iE}{\hbar}T(t)$$

Since t is a fun^c of time only we can replace the partial derivative by a complete derivative & write the above eq^n as—

$$\frac{\partial T}{\partial t} = -\frac{iE}{\hbar}T$$

$$\Rightarrow \frac{dT}{T} = -\frac{iE}{\hbar}dt$$

Integrating both sides

$$\ln T = -\frac{iEt}{\hbar}$$

Hence $T(t) = e^{-iEt/\hbar}$

As general wave fun^c is

$$\psi(x,t) = \psi(x)T(t)$$

We can write

$$\boxed{\psi(x,t) = \psi(x)\,e^{-iEt/\hbar}}$$

This is the sol^n of Schrodinger wave eq^n & we represents the quantum wave eq^n.

Sol^n of Schrodinger eq^n

$$\psi(x,t) = \psi(x)e^{-iEt/\hbar}$$

According to maxborn probability density i.e probability per unit length of finding the particle in the quantum state $\psi(x,t)$ is given by —

$$P(x,t) = \psi^*(x,t)\,\psi(x,t)$$

$$= \psi^*(x)e^{iEt/\hbar}\cdot\psi(x)e^{-iEt/\hbar}$$

$$\Rightarrow \boxed{P(x,t) = \psi^*(x)\psi(x)}$$

From the above expression we draw the infinite probability density is independent of time i.e while these are called Stationary State. So sol^n of a schrodinger eq^n represent as Stationary State.

Wave func^c & its Physical Significance →

Wave fun^c is a complex mathematical entity whose variation w.r.t space & time will give raise to matter wave & it also satisfy schrodinger eq^n.

Wave fun^c ψ has no direct Physical Significance how ever a Statical interpretation of Physical Significance of the wave fun^c was given by Max Born.

According to Max Born →

(i) Wave fun^c are mathematical represantation of Particles which contains all the information require for the probabilistic description of the particle & is generally a complex no.

(ii) Wave fun^c ψ is called the probability amplitude so that the product of ψ with its complex conjugate gives the probability of Particle

$$\psi^*\psi = |\psi|^2 = \text{Probability density.}$$

(iii) So probability density is a real & +ve no. & hence is an observable quantity.

(iv) Greater the value of $|\psi|^2$ higher is the Probability of finding the Particle at a given position & time.

(v) The Probability that a particle may be found in the region bet^n x & $x+dx$ at a time 't' is given by —

$$P(x,t)\,dx = \psi^*(x,t)\,\psi(x,t)\,dx$$
$$= |\psi(x)|^2\,dx$$

Properties of Wave fun^c :—

The Probability interpretation of wave fun^c requires that $\psi(x)$ must fullfill following general condition –

(i) Wave fun^c ψ & its order derivative $\frac{\partial\psi}{\partial x}$ must be finite everywhere. This will assure that $|\psi|^2$ which is equal to finding the particle at position 'x' is finite.

(ii) Wave fun^c ψ ~~must be continuous or~~ & its 1st order derivative $\frac{\partial\psi}{\partial x}$ must have a single value or unique value at every point.

(iii) Wave fun^c ψ must be continuous everywhere.

(iv) Wave fun^c ψ must be vanished at infinite $\psi(x) \to 0$ when $x \to \pm\infty$

In other word $\int_{-\infty}^{\infty} \psi^*\psi\,dx$ should be infinite

Normalisation of a wave fun^c :—

For a dimensional system probability of finding the particle in a particular volume element $d\tau$.

$$dP = \psi\psi^*\,d\tau$$
$$dP = |\psi(r,t)|^2\,d\tau$$

Hence the total Probability in all this Space is given by —

$$P = \int |\psi(r,t)|^2 d\tau$$

Normalisation means that the integral of the square of a func should be equal to one i.e probability of finding the particle over a space must be equal to one.
Hence condn of normalisation of a wave func in one dimension can be written as —

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

Any wave func which satisfy the above eqn is s.t.b normalisation.

In order to normalise a given wave func $\psi(x,t)$ we multiply with a normalisation constant A because if ψ is a soln Schrodinger wave eqn then $A\psi$ is also a soln.

Normalisation condn becomes

$$\int_{-\infty}^{\infty} (A\psi)^* (A\psi)\, dx = A^2 \int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$$\Rightarrow A^2 = \frac{1}{\int_{-\infty}^{\infty} \psi^* \psi \, dx}$$

Probability current density →

The Probabilistic interpretation of a wave func ψ requires that the probability must be a conserve quantity i.e if the Probability of finding the particle in same bounded region of space decreases in time then the Probability of finding its outside the region must increase by same amount.

We will write the eqn of continue detecting conservation law of Probability which will called as Probability current density.

If P(r) is the Probability of finding the particle at a particular point in space then the Probability finding the particle in a region space of the volume 'V' bounded the surface 'S' is given by —

$$P = \int_V P(r)\, d\tau = \int_V \psi^* \psi \, d\tau \quad \text{——(i)}$$

Where $d\tau = dx\,dy\,dz$ = volume element

In order to find the flow of Probability we must know the change in Probability with time with in the region.

So differentiating eqn (i) w.r.t 't'

$$\frac{dP}{dt} = \frac{d}{dt}\int_V (\psi^* \psi)\, d\tau = \int_V \left(\psi \frac{\partial \psi^*}{\partial t} + \frac{\partial \psi}{\partial t}\psi^*\right) d\tau \text{ —(ii)}$$

Time dependent Schordinger eqn

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t} \text{ —— (iii)}$$

complex conjugate of its eqn is

$$-\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^* = i\hbar\frac{\partial \psi^*}{\partial t} \text{ —(iv)}$$

Multiplying eq^n (iii) by ψ^* & eq^n (iv) by ψ then substract eq^n (iiia) & eq^n (iva) we get

$$-\frac{\hbar^2}{2m}\psi^*\nabla^2\psi + V\psi\psi^* = i\hbar\psi^*\frac{\partial\psi}{\partial t} \quad \text{--- (iii)a}$$

$$-\frac{\hbar^2}{2m}\psi\nabla^2\psi^* + V\psi\psi^* = -i\hbar\psi\frac{\partial\psi^*}{\partial t} \quad \text{--- (iv)a}$$

$$-\frac{\hbar^2}{2m}\psi\nabla^2\psi^* + V\psi\psi^* - \frac{\hbar^2}{2m}\psi^*\nabla^2\psi - V\psi\psi^*$$
$$= -i\hbar\psi\frac{\partial\psi^*}{\partial t} - i\hbar\psi^*\frac{\partial\psi}{\partial t}$$

$$\Rightarrow -\frac{\hbar^2}{2m}\psi\nabla^2\psi^* + \frac{\hbar^2\psi^*}{2m}\nabla^2\psi = -i\hbar\psi\frac{\partial\psi^*}{\partial t} - i\hbar\psi^*\frac{\partial\psi}{\partial t}$$

$$\Rightarrow \frac{\hbar^2}{2m}(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*) = i\hbar\left(\psi\frac{\partial\psi^*}{\partial t} + \psi^*\frac{\partial\psi}{\partial t}\right)$$

$$\Rightarrow \left(\psi\frac{\partial\psi^*}{\partial t} + \psi^*\frac{\partial\psi}{\partial t}\right) = -\frac{\hbar}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right) \quad \text{--- (v)}$$

Substituting eq^n (v) in eq^n (ii) we get

$$\frac{dP}{dt} = -\frac{\hbar}{2m}\int(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*)\,d\tau$$

This integral can be transform into surface integral over the surface 's' & write

$$\frac{dP}{dt} = -\frac{\hbar}{2im}\int_S(\psi^*\nabla\psi - \psi\nabla\psi^*)\,ds \quad \text{--- (vi)}$$

Let's the vector J called Probability current density as —

$$J = \frac{\hbar}{2im}(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

So eq^n (vi) reduces to

$$\frac{dP}{dt} = -\int_S J\,ds$$

Using Gauss divergence theorem

$$\frac{dP}{dt} = -\nabla\cdot J$$

$$\Rightarrow \boxed{\frac{dP}{dt} + \nabla\cdot J = 0}$$

This eq^n represents probability density & shows that the rate of change of probability of a particle inside a surface is equal to rate of decrease of probability current to the surface 's'. Hence this represents conservation of probability finding or not finding in a specified volume.

Operators & Eigen Values:

An operator can be defined as an instruction or mathematical root which can be used to change by a given fun^c into another fun^c.

if $\hat{A}$ is an operator, ψ = wave fun^c

then $\hat{A}\psi = a_n\psi$.

where a_n = Eigen value.

ψ = Eigen fun^c.

Observables & Operators:

Observables are physical qty such as energy & momentum which gain

Expectation Value :-

In quantum mechanics every physical parameters of observable has an inherent uncertainty several measurement of a parameter of a system under similar conditions doesn't if the same value always ever as values of all the measured quantity is called expectation value.

If a physical parameter x is measured several time such that if value x_1 appear x_1 time x_2 appearance x_2 time & x_n appears x_n time then expectation value of observable x is —

$$\langle x \rangle = \frac{\sum_{i=1}^{n} n_i x_i}{\sum_{i=1}^{n} n_i} = \frac{\sum_{i=1}^{n} n_i x_i}{N}$$

Here $\frac{n_i}{N} = P_i$ = Probability of getting the value x_i.

Therefore $\langle x \rangle = \sum_{i=1}^{n} x_i \frac{n_i}{N} = \sum_{i=1}^{n} x_i P_i$

If the variable x is continuous Σ changes to integration —

$$\langle x \rangle = \int x P dx$$

According to Max Born probability density P associates with a particle described by the wave func $\psi(x,t)$ is given by —

$$P(x,t) = \psi^*(x,t)\,\psi(x,t)$$

Therefore $\langle x \rangle = \int_{-\infty}^{\infty} x \psi^*(x,t)\,\psi(x,t)\,dx$

or $\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi(x,t)\,dx$

So in general expectation value of any observable small 'a' is given by

$$\langle a \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\,\hat{A}\psi(x,t)\,dx$$

Where $\hat{A}$ quantum mechanical operator corresponding to the observable 'a'

$\langle a \rangle = \langle \psi | \hat{A} | \psi \rangle$ (Dirac notation)

$\langle \psi |$ → Bra vector

$| \psi \rangle$ → Ket vector

So expectation value x component of momentum is written as —

$$\langle P_x \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\,P_x \psi(x,t)\,dx$$

Since $P_x = \left(-i\hbar \frac{\partial}{\partial x}\right)$

$$\langle P_x \rangle = -i\hbar \int_{-\infty}^{\infty} \psi^*(x,t)\,\frac{\partial}{\partial x}\psi(x,t)\,dx$$

In 3-dimension

$$\langle P \rangle = -i\hbar \int_{-\infty}^{\infty} \psi^*(r,t)\,\nabla\psi(r,t)\,dV$$

Similarly expectation value of energy is

$$\langle E \rangle = i\hbar \int_{-\infty}^{\infty} \psi^*(x,t)\,\frac{\partial}{\partial t}\psi(x,t)\,dx$$