Registration No : | | | | | | | | | | |

Total Number of Pages : 02

B.Tech
RMA1A001

1st Semester Regular / Back Examination: 2021-22
MATHEMATICS-I
BRANCH(S): AEIE, AERO, AG, AME, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, CSEAI, CSEAIME, CST, ECE, EEE, EIE, ELECTRICAL, ELECTRICAL & C.E, ELECTRONICS & C.E, ENV, ETC, IT, MANUTECH, MECH, METTA, MINERAL, MINING, MME, PE, PLASTIC, PT
Time : 3 Hour
Max Marks : 100
Q.Code : OF596

Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions :** **(2×10)**

a) Find the asymptotes, parallel to the axis of x of the curve $y^4 + x^2y^2 + 2x^2y + 2xy^2 - 4x^2 - y + 1 = 0$.

b) Find the radius of curvature for the catenary $s = c\tan\psi$.

c) How Beta and Gamma functions are related?

d) Find the Wronskian $W(x^4, x^4\ln x)$.

e) Define Bernoulli differential equation.

f) What is exact differential equation?

g) Write the Legendre polynomial of degree three.

h) Find the integrating factor of the differential equation $y\,dx + (x^2y - x)dy = 0$.

i) Find the inverse Laplace transformation of the function $\frac{7}{(s-1)^3}$.

j) Find the convolution $t * e^t$ by integration.

Part-II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6×8)**

a) Find all the asymptotes of $y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$.

b) Evaluate the integral $\int_0^\infty e^{-ax}\cos bx\,dx$ using Gamma function.

c) Solve the following ordinary differential equation by method of variation of parameter: $\frac{d^2y}{dx^2} + y = \cos x + \sec x$.

d) Find the maxima and minima of the function $f(x,y) = 21x - 12x^2 - 2y^2 + x^3 + xy^2$.

e) Solve the differential equation $\frac{d^2y}{dx^2} - y = 2e^{-x}$.

f) Find the value of ρ for the curve $x - s\cos\left(\frac{s}{a}\right), y - a\sin\left(\frac{s}{a}\right)$.

g) Solve the ordinary differential equation $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$.

h) Find the indicial equation of the Bessel's differential equation
$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-v^2)y=0.$

i) Solve the differential equation $\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+2y=0$ by power series method.

j) Using Laplace transform, solve the initial value problem:
$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+8y=e^{-3t}-e^{-5t}, y(0)=0=y'(0).$

k) Using Laplace transform solve the integral equation
$$y(t)=te^t-2e^t\int_0^t e^{-\tau}y(\tau)d\tau.$$

l) Using convolution, find inverse $f(t)$ of $F(s)=\frac{s}{(s^2+\pi^2)^2}$.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Show that the following improper integral is convergent: **(16)**
$\int_0^1 t^{x-1}(1-t)^{y-1}dt, x>0, y>0.$

Q4 Determine the characteristic (auxiliary) equation of the second order Euler Cauchy type ordinary differential equations. Solve the initial value problem **(16)**
$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0,\ y(1)=4, y'(1)=-2.$

Q5 Obtain Legendre polynomial of degree n, from the Legendre differential equation **(16)**
$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+m(m+1)y=0, m$ is given constant.

Q6 Solve the initial value problems **(16)**
$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=\begin{cases}1, 0<t<a\\ 0, t>a\end{cases}, y(0)=0=y'(0).$