Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech**

15BS1104

2nd Semester Back Examination 2018-19

MATHEMATICS - II

BRANCH : AEIE, AERO, AUTO, BIOTECH, CHEM, CIVIL, CSE, ECE, EEE, EIE, ELECTRICAL, ETC, FAT, IEE, IT, MECH, MINERAL, MINING, MME, PE, PLASTIC, TEXTILE

Max Marks : 100

Time : 3 Hours

Q.CODE : F130

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.

The figures in the right hand margin indicate marks.

Part- I

Q1 **Only Short Answer Type Questions (Answer All-10)** **(2 x 10)**

a) Write Laplace Transform of Unit step function.

b) $L(f * g) =$ _____ .

c) Write the relation between beta and gamma function.

d) If $f(x) = x^2, -\pi < x < \pi$, then the value of Fourier coefficient $b_n =$ _____.

e) Determine Inverse Laplace transform of $\frac{s}{s^2+9}$.

f) $F(x) = x + \sin x$ is even function or odd function. Justify your answer.

g) Derive gradient of $f(x, y, z) = \cos xyz + xy$.

h) Determine a normal vector n of $z^2 = 4(x^2 + y^2)$ at the point P: $(1, 0, 2)$.

i) State Stokes's theorem.

j) State whether the vector $\vec{v} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is irrotational or not.

Part- II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 x 8)**

a) Calculate Laplace transform of

i. $t \sin at$

ii. $e^{2t}\cos 4t$

b) Solve the integral equation $y(t) = t + \int_0^t y(u)\sin(t-u)du$.

c) Calculate Inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$.

d) Formulate $Y(s)$ for the given initial value problem $y'' + 2y' + 5y = 1, y(0) = 0, y'(0) = 1$.

e) Calculate Fourier series of $f(x) = x^2, -\pi < x < \pi, p = 2\pi$.

f) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

g) Determine length of the curve $\vec{r}(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath} + 3t\,\hat{k}$ from (2,0,0) to $(2,0,6\pi)$

h) For any scalar function f(x,y,z) and vector function $\vec{v} = [v_1, v_2, v_3]$, Prove that

i. $Curl(grad\, f) = 0$

ii. $Div(curl\, \vec{v}) = 0$

i) Calculate unit normal vector of the surface $r(u, v) = [u\cos v, u \sin v, u^3]$.

j) Explain, the given differential $3x^2dx + 2yz\,dy + y^2\,dz$ is exact or not.

k) Evaluate the integral :

$\int_C y^3dx + x^3dy)$, C: Straight line segment from (0,0) to(1,1).

l) Develop Fourier Cosine transformation of $f(x) = e^{-kx}, k > 0, x > 0$.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Evaluate solution of initial value problem $y'' - 4y' + 3y = 6t - 8, y(0) = 0, y'(0) = 0$ by using Laplace transformation. **(16)**

Q4 **a)** Evaluate Fourier integral representation of $f(x) = \begin{cases} 0, & if\ x < 0 \\ \frac{1}{2}, & if\ x = 0 \\ e^{-x}, & if\ x > 0 \end{cases}$. **(10)**

b) Evaluate Fourier sine transform of $f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$. **(6)**

Q5 Prove that the integral : **(16)**

$$\int_C [\, 2xyz^2\, dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz\,]$$

Is independent of path in any domain and hence find the value of I from A: (0,0,1) to $(1, \frac{\pi}{4}, 2)$.

Q6 Verify Green's theorem for the given integral. **(16)**

$\int_C [(xy + y^2)dx + x^2dy]$, Where C is bounded by the curve $y = x$ and $y = x^2$.