

Registration No :

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Total Number of Pages : 02

B.Tech  
RMA3A001

3<sup>rd</sup> Semester Regular / Back Examination: 2021-22

MATHEMATICS - III

BRANCH(S): AEIE, AERO, AG, AUTO, BIOMED,  
BIOTECH, C&EE, CHEM, CIVIL, CSE, CSEAIME, CST, ECE, EEE, EIE, ELECTRICAL, ELECTRICAL  
& C.E, ELECTRONICS & C.E, ENV, ETC, IT, MANUTECH, MECH, METTA, MINERAL, MINING, MME,  
MMEAM, PE, PLASTIC, PT

Time : 3 Hour

Max Marks : 100

Q.Code : OF575

Answer Question No.1 (Part-1) which is compulsory, any eight from Part-II and any two from Part-III.

The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions :

(2×10)

- Rate of convergence of the Newton-Raphson method is generally \_\_\_\_\_
- Using Newton's Forward formula, find  $\sin(0.1604)$  from the following table.

x	0.160	0.161	0.162
f(x)	0.1593182066	0.1603053541	0.1612923412
- Rewrite  $dy/dx + 2y = 1.3e^{-x}$   $y(0) = 5$   
In  $dy/dx = f(x,y)$   $y(0) = y_0$  form
- Given that  $x = 2y + 4$  and  $y = kx + 6$  are the lines of regression of  $x$  on  $y$  and  $y$  on  $x$  respectively. Find the value of  $k$  if  $r$  is 0.5.
- A coin is tossed three times.  
What is the probability of three heads?
- What is the meaning of probability in statistics?
- What will be the variance of the Bernoulli trials, if the probability of success of the Bernoulli trial is 0.3.
- Explain the primary philosophical difference between the parameters of the probability distribution function (PDF) and the cumulative distribution function (CDF).
- The mean of hypergeometric distribution is \_\_\_\_\_
- A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)

(6×8)

- Using Lagrange's interpolation formula find  $y(10)$  from the following table:

x	5	6	9	11
y	12	13	14	16

Find the cube root of 12 using the Newton Raphson method assuming  $x_0 = 2.5$ .

- Baby boys have a mean weight of 6.4 kg, with a standard deviation of 0.7. Baby girls have a mean weight of 5.9 kg, with a standard deviation of 0.7. The weights of 3-month old babies are normally distributed. What is the probability that a 3-month old baby boy weighs more than 7.3 kg?
- For an integral  $\int_a^b f(x) dx$  derive the one-point Gauss quadrature rule.
- valuate  $\int_0^2 x^2 dx$  using the Trapezoidal Rule, with  $n = 2$ .

- e) Find the value of  $k_1$  by Runge-Kutta method of fourth order if  $dy/dx = 2x + 3y^2$  and  $y(0.1) = 1.1165$ ,  $h = 0.1$
- f) A die is rolled twice and two numbers are obtained, let  $X$  be the outcome of first roll and  $Y$  be the outcome of the second roll. Given that  $X+Y=5$ , what is the probability of  $X=4$  or  $Y=4$ ?
- g) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?
- h) The probability of simultaneous occurrence of at least one of two events  $A$  and  $B$  is  $p$ . If the probability that exactly one of  $A, B$  occurs is  $q$ , then prove that  $P(A') + P(B') = 2 - 2p + q$ .
- i) 9 Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If  $X$  denotes the number of red ball drawn, find the probability distribution of  $X$ .
- j) A binomial probability experiment is conducted with the given parameters. Compute the probability of  $x$  successes in the  $n$  independent trials of the experiment  $n=10$ ,  $p=0.75$ ,  $x=8$
- k) Let's return to the example in which  $X_1, X_2, \dots, X_n$  are normal random variables with mean  $\mu$  and variance  $\sigma^2$ . What are the method of moments estimators of the mean  $\mu$  and variance  $\sigma^2$ ?
- l) Suppose that  $X$  is a discrete random variable with the following probability mass function: where  $0 \leq \theta \leq 1$  is a parameter. The following 10 independent observations
- |        |             |            |                 |                |
|--------|-------------|------------|-----------------|----------------|
| $X$    | 0           | 1          | 2               | 3              |
| $P(X)$ | $2\theta/3$ | $\theta/3$ | $2(1-\theta)/3$ | $(1-\theta)/3$ |
- were taken from such a distribution: (3,0,2,1,3,2,1,0,2,1). What is the maximum likelihood estimate of  $\theta$ .

### Part-III

#### Only Long Answer Type Questions (Answer Any Two out of Four)

- Q3 a) Find the cube root of 12 using the Newton Raphson method assuming  $x_0 = 2.5$ . (8)  
 b) Solve Equations  $2x+5y=16, 3x+y=11$  using Gauss Seidel method (8)
- Q4 a) Find an approximate value of  $\int_5^8 6x^3 dx$  using Euler's method of solving an ordinary differential equation. Use a step size of  $h = 1.5$ . (8)  
 b) A pair of dice is thrown and let  $X$  be the random variable which represents the sum of the numbers that appear on the two dice. Find the mean of  $X$ . (8)
- Q5 a) Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of  $n$ . Round your answer to four decimal places and compare the results with the exact value of the definite integral.  $\int_0^2 x\sqrt{x^2+1} dx$   $n=4$  (8)  
 b) Let  $X$  and  $Y$  be two independent Uniform (0,1) random variables. Let also  $Z = \max(X, Y)$  and  $W = \min(X, Y)$ . Find  $\text{Cov}(Z, W)$ . (8)
- Q6 a) Calculate the regression coefficient and obtain the lines of regression for the following data (8)
- |     |   |   |    |    |    |    |    |
|-----|---|---|----|----|----|----|----|
| $X$ | 1 | 2 | 3  | 4  | 5  | 6  | 7  |
| $y$ | 9 | 8 | 10 | 12 | 11 | 13 | 14 |
- b)  $X$  is a normally distributed variable with mean  $\mu = 30$  and standard deviation  $\sigma = 4$ . (8)  
 Find :  
 a)  $P(x < 40)$   
 b)  $P(x > 21)$   
 c)  $P(30 < x < 35)$