

SRINIX COLLEGE OF ENGINEERING, BALASORE



**DEPARTMENT OF
COMPUTER SCIENCE & ENGINEERING**

**ASSIGNMENT ON
DISCRETE MATHEMATICS**

DISCRETE MATHEMATICS

ASSIGNMENT -I

1. a)	Explain the connectives and their truth tables
b)	Define converse, inverse contra positive with an example
2. a)	Construct the truth table for the following formula $\neg(\neg P \vee \neg Q)$
b)	Construct the truth table to Show that $\neg P \wedge (Q \wedge P)$ is a contradiction.
3. a)	Define NAND, NOR & XOR and give their truth tables.
b)	Show that $((P \rightarrow Q) \rightarrow Q) \Rightarrow P \vee Q$ without constructing truth table.
4. a)	Show that $S \vee R$ is a tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$
b)	Show that $(P \rightarrow Q) \wedge ((Q \rightarrow R) \Rightarrow (P \rightarrow Q))$
5. a)	What is principle disjunctive normal form? Obtain the PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
b)	What is principle conjunctive normal form? Obtain the PCNF of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$
6. a)	Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$
b)	Show that $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R(x)) \Rightarrow (\forall x)(P(x) \rightarrow R(x))$
7. a)	Define Quantifiers and types of Quantifiers with examples.
b)	Show that $(\exists x) M(x)$ follows logically from the premises $(\forall x)(H(x) \rightarrow M(x))$ and $(\exists x)H(x)$
8. a)	Use indirect method of proof to prove $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$
b)	Define Maxterms & Minterms of P & Q and give their truth tables
9. a)	Show that S is a valid conclusion from the premises $p \rightarrow q, p \rightarrow r, \neg(q \wedge r)$ and $(s \vee p)$
b)	Obtain PCNF of $A = (p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$ by constructing PDNF
10. a)	Show that $P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R, P$ are inconsistent.
b)	Explain the Universal Quantifier and Existential quantifier with examples

ASSIGNMENT -II

1. a)	Define Relation? List out the properties of Binary operations.
b)	Let $X = \{1,2,3,4\}$ and $R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$. Then prove that R is an equivalence relation.
2. a)	Define an equivalence relation? If R be a relation in the set of integers Z defined by $R = \{(x, y) : x \in Z, y \in Z, (x - y) \text{ is divisible by } 6\}$ Then prove that R is an equivalence relation.
b)	Define a binary relation with an example. Let R be the relation from the set $A = \{1, 3, 4\}$ on itself and defined by $R = \{(1, 1), (1, 3), (3, 3), (4, 4)\}$ then find the matrix of R draw the graph of R.
3.	Define Equivalence class. Determine A/R when $A = \{1,2,3,4\}$ and let $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ be an equivalence relation on R?
4.	Let A be a given finite set and P(A) its power set. let \subseteq be the inclusion relation on the elements of P(A). Draw the Hasse diagram of $(P(A), \subseteq)$ for i) $A = \{a\}$ ii) $A = \{a, b\}$ iii) $A = \{a, b, c\}$ iv) $A = \{a, b, c, d\}$
5. a)	Define a function and write the types of functions
b)	Define primitive recursive function. Show that the function $f(x, y) = x + y$ is primitive recursive.
6.	If $f : R \rightarrow R$ and $g : R \rightarrow R$ defined by $f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2 + 1}$, $h(x) = x^4$, find the following composition functions: a) $(f \circ g \circ h)(x)$ b) $(h \circ g \circ f)(x)$ c) $(g \circ g)(x)$ d) $(g \circ h)(x)$
7. a)	Let $f : A \rightarrow B$, $g : B \rightarrow C$, $h : C \rightarrow D$ then prove that $h \circ (g \circ f) = (h \circ g) \circ f$
b)	If $f : R \rightarrow R$ such that $f(x, y) = 2x + 1$ and $g : R \rightarrow R$ such that $g(x) = \frac{x}{3}$ then verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
8. a)	Show that the set of all positive rational numbers forms an abelian group under the composition defined by $a * b = \frac{(ab)}{2}$.
b)	Show that $S = \{1, 2, 3, 4, 5\}$ is not a group under addition & multiplication modulo 6.
9. a)	Prove that the set of all integers Z is an abelian group with the operation '*' defined as $a * b = a + b + 1, \forall a, b \in Z$.
b)	The necessary and sufficient condition for a non-empty subset H of a group $(G, *)$ to be a subgroup is $a \in H, b \in H \Rightarrow a * b^{-1} \in H$.
10. a)	Define abelian group, homomorphism and isomorphism.
b)	If a group $(G, .)$, $f : G \rightarrow G$ is given by $f(a) = a^2 \forall a \in G$ is homomorphism prove that G is abelian group.

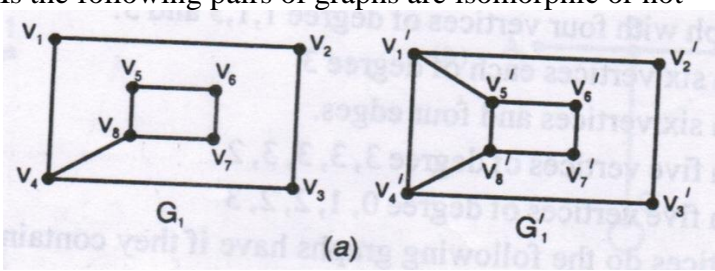
ASSIGNMENT -III

1.a)	How many different license plates are there that involve 1,2or 3 letters followed by 4 digits?
b)	How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no repetitions are allowed?
2. a)	In how many ways can the letters of the word COMPUTER be arranged? How many of them begin with C and end with R? How many of them do not begin with C but end with R?
b)	Out of 9 girls and 15 boys, how many different committees can be formed each consisting of 6 boys and 4 girls?
3. a)	How many permutations can be formed out of the letters of word “SUNDAY”? How many of these (i) Begin with S? (ii) End with Y? (iii) Begin with S & end with Y? (iv) S & Y always together?
b)	The question paper of mathematics contains two questions divided into two groups of 5 questions each. In how many ways can an examine answer six questions taking atleast two questions from each group.
4. a)	Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included?
b)	Enumerate the number of non negative integral solutions to the inequality $x_1 + x_2 + x_3 + x_4 + x_5 \leq 19.$
5. a)	How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where each (i) $x_i \geq 2$ (ii) $x_i > 2$
b)	(i) Define permutation, (ii) Define combination, (iii) State Binomial theorem
6. a)	What is the co-efficient of (i) $x^3 y^7$ in $(x + y)^{10}$ (ii) $x^2 y^4$ in $(x - 2y)^6$
b)	Find the coefficient of (i) $x^3 y^2 z^2$ in $(2x - y + z)^7$ (ii) $x^6 y^3$ in $(x - 3y)^9$
7. a)	Find the number of arrangements of the letters in the word ACCOUNTANT
b)	Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7. Also indicate how many are divisible by 3 or 7 but not by 5 and divisible by 3 or 5.
8. a)	A Survey among 100 students shows that of the three ice cream flavors vanilla, chocolate, straw berry. 50 students like vanilla, 43 like chocolate, 28 like straw berry, 13 like vanilla and chocolate, 11 like chocolate and straw berry, 12 like straw berry and vanilla and 5 like all of them. Find the following. 1. Chocolate but not straw berry 2. Chocolate and straw berry but not vanilla 3. Vanilla or chocolate but not straw berry
b)	Out of 80 students in a class, 60 play foot ball, 53 play hockey and 35 both the games. How many students (i) do not play of these games? (ii) Play only hockey but not foot ball
9. a)	Find how many integers between 1 and 60 that are divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5 not by 2, not by 3.
b)	Show that if 8 people are in a room, at least two of them have birthdays that occur on the same day of the week.
10.a)	Applying pigeon hole principle show that if any 14 integers are selected from the set $S = \{1, 2, 3, \dots, 25\}$ there are at least two whose sum is 26. Also write a statement that generalizes this result
b)	Find the minimum number of students in a class to be sure that 4 out of them are born on the same month

ASSIGNMENT –IV

1. a)	Find the generating function for the sequence 1,1,1,3,1,1,.....
b)	Determine the sequence generated by (i) $f(x) = 2e^x + 3x^2$ (ii) $7e^{8x} - 4e^{3x}$.
2. a)	Find the sequence generated by the following generating functions (i) $(2x - 3)^3$ (ii) $\frac{x^4}{1-x}$
b)	Find the coefficient of x^{20} in $(x^2 + x^3 + x^4 + x^5 + x^6)^5$?
3. a)	Solve $a_n = a_{n-1} + 2a_{n-2}, n > 2$ with initial conditions $a_0 = 0, a_1 = 1$
b)	Solve $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_1 = 1.5$ & $a_2 = 3$
4. a)	Solve $a_n = 3a_{n-1} - a_{n-2}$ with initial conditions $a_1 = -2$ & $a_2 = 4$
b)	Solve $a_n = a_{n-1} + 2a_{n-2}, n > 2$ with initial conditions $a_0 = 2, a_1 = 1$
5. a)	Solve $a_k = k(a_{k-1})^2, k \geq 1, a_0 = 1$
b)	Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with condition the initial $a_0 = 1, a_1 = -1$
6. a)	Solve the R.R $a_{n+2} - 2a_{n+1} + a_n = 2^n$ with initial condition $a_0 = 2, a_1 = 1$
b)	Solve the recurrence relation $a_n = a_{n-1} + \frac{n(n+1)}{2}$
7. a)	Solve the following $y_{n+2} - y_{n+1} - 2y_n = n^2$
b)	Solve $a_n - 5a_{n-1} + 6a_{n-2} = 1$
8. a)	Solve $a_n - 7a_{n-1} + 10a_{n-2} = 4^n$
b)	Solve $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)^2$ given $a_0 = 0, a_1 = 1$
9. a)	Solve the recurrence relation using generating functions $a_n - 9a_{n-1} + 20a_{n-2} = 0$ for $n \geq 2$ and $a_0 = -3, a_1 = -10$
b)	Solve the recurrence relation $a_r = a_{r-1} + a_{r-2}$ Using generating function.
10.a)	Using generating function solve $a_n = 3a_{n+1} + 2, a_0 = 1$
b)	Solve $a_n - 5a_{n-1} + 6a_{n-2} = 2^n, n > 2$ with condition the initial $a_0 = 1, a_1 = 1$. Using generating functions.

ASSIGNMENT -V

1. a)	Determine the number of edges in (i) Complete graph K_n (ii) Complete bipartite graph $K_{m,n}$ (iii) Cycle graph C_n (iv) Path graph P_n (v) Null graph N_n
b)	Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$
2. a)	Define isomorphism. Explain Isomorphism of graphs with a suitable example.
b)	Explain graph coloring and chromatic number give an example.
3. a)	Explain about complete graph and planar graph with an example
b)	Define the following graph with one suitable example for each graphs (i) complement graph (ii) sub graph (iii) induced sub graph (iv) spanning sub graph
4. a)	Explain In degree and out degree of graph. Also explain about the adjacency matrix representation of graphs. Illustrate with an example.
b)	Give an example of a graph that has neither an Eulerian circuit nor a Hamiltonian circuit
5. a)	Define Spanning tree and explain the algorithm for Depth First Search (DFS) traversal of a graph with suitable example.
b)	A graph G has 21 edges, 3 vertices of degree 4 and the other vertices are of degree 3. Find the number of vertices in G ?
6. a)	Suppose a graph has vertices of degree 0, 2, 2, 3 and 9. How many edges does the graph have?
b)	Give an example of a graph which is Hamiltonian but not Eulerian and vice versa.
7. a)	Let G be a 4 – Regular connected planar graph having 16 edges. Find the number of regions of G .
b)	Draw the graph represented by given Adjacency matrix <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>(i)</p> $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ </div> <div style="text-align: center;"> <p>(ii)</p> $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ </div> </div>
8. a)	Show that in any graph the number of odd degree vertices is even.
b)	Is the following pairs of graphs are isomorphic or not <div style="text-align: center;">  <p>(a) ?</p> </div>