# COURSE OUTLINES OF MODULE-II

1. **Turning Moment Diagram with different types of Engine.**
2. **Fluctuation of Energy and Speed.**
3. **Flywheel.**
4. **Gear and its Classification.**
5. **Gear Terminology.**
6. **Law of Gearing.**
7. **Velocity of Sliding.**
8. **Forms of Teeth.**
9. **LPC/LAC/CR**
10. **Interference and Undercutting.**
11. **Difference between Cycloidal and Involute Teeth.**

# Turning Moment Diagrams

- Turning moment diagram is a diagram which shows the variation of turning moment (torque) on a crank for various position of the crank.

## * Turning moment diagram for a single cylinder double acting steam engine

### D' Alembert Principle

According to D' Alembert Principle "The system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of body".


$F_i = ma$

$= ma$

$\Sigma F = 0$

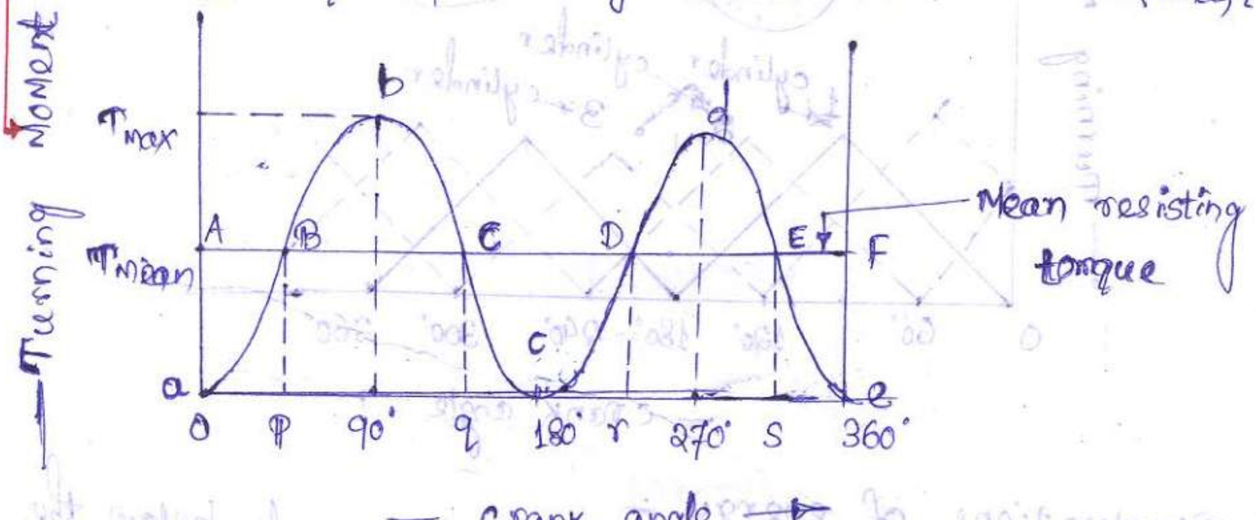
**Inertia force** :- The inertia force is an imaginary force, which when acts upon a rigid body brings it in an equilibrium position.

Inertia force = - Accelerating force = $-ma$

$m$ = Mass of the body

$a$ = linear acceleration of the centre of gravity ~~of the~~ body.

**Crank effort**:- crank effort is the net effort (force) applied at the crankpin ⊥er to the crank which gives the required turning moment on the crankshaft.




Turning moment on the crankshaft

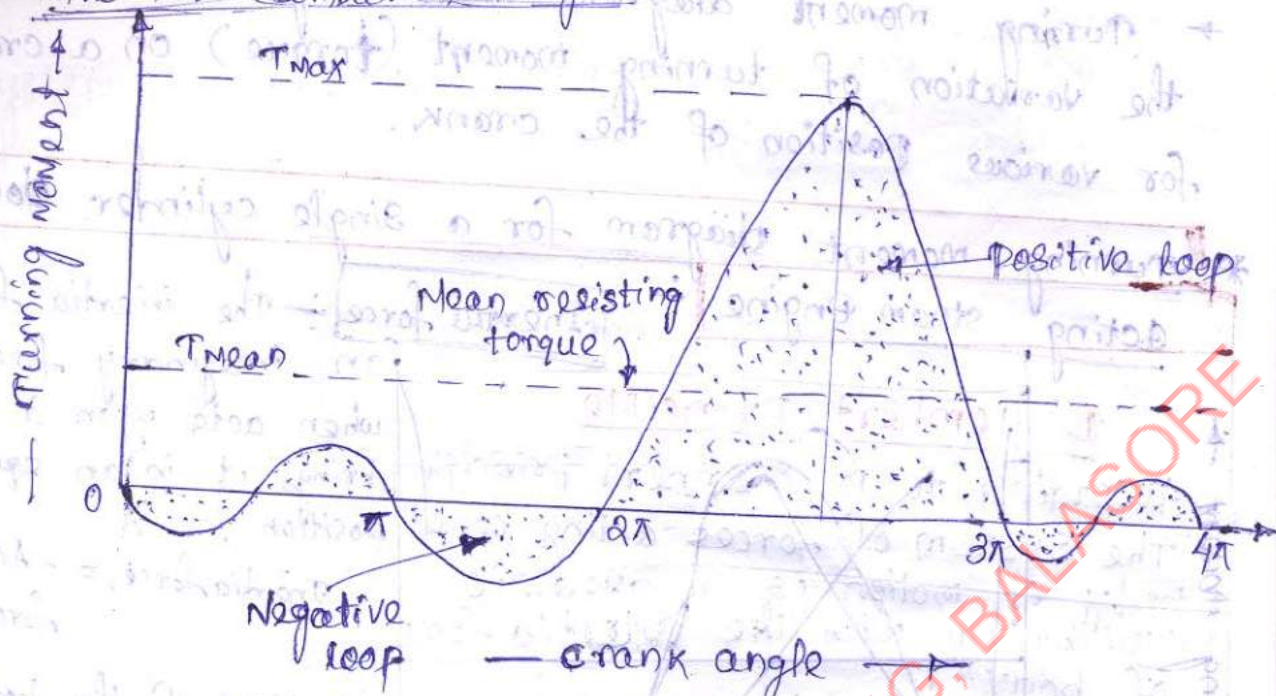
$$T = F_p \times r \left( \sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

where $F_p$ = piston effort, $r$ = Radius of crank

$n$ = Ratio of the connecting rod length and radius of crank

$\theta$ = Angle turned by the crank from inner dead centre.

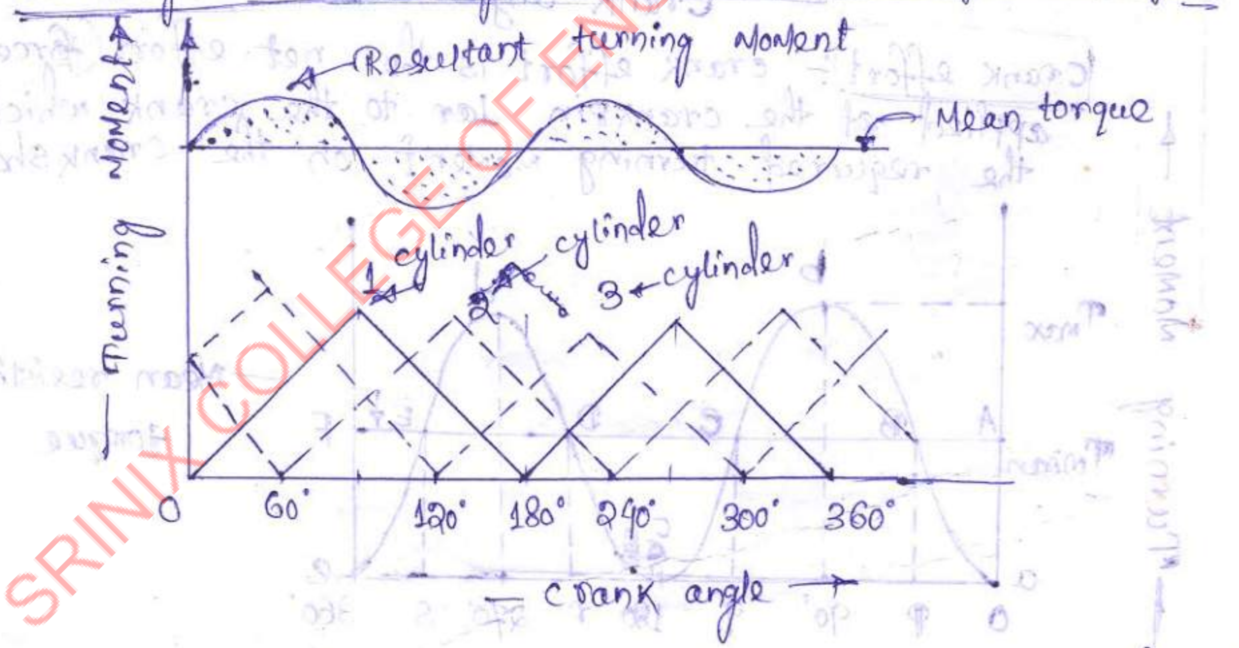
## * Turning moment Diagram for a four stroke cycle internal combustion Engine :-




0 – π ⇒ suction   2π – 3π ⇒ power

π – 2π ⇒ compression   3π – 4π ⇒ Exhaust

## * Turning moment Diagram for a multi-cylinder Engine :-



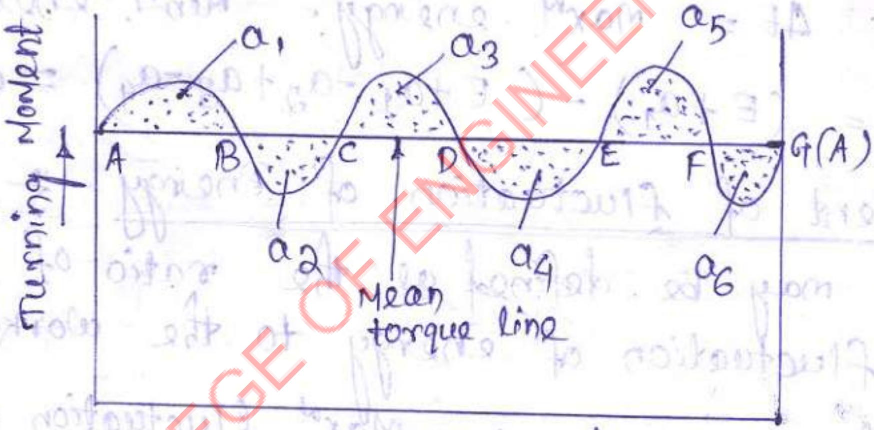

## * Fluctuations of energy :-

The variations of energy above and below the mean resisting torque line are called fluctuations of energy.

## * Max^m. fluctuations of energy :-

The difference betn the max^m. and min^m energies is known as max^m. fluctuation of energy.

* Determination of Max$^m$. fluctuation of Energy :-




A turning moment diagram for a multi-cylinder engine as shown in fig. The horizontal line AG represents the mean torque line.

Let $a_1$, $a_3$, & $a_5$ be the areas above the mean torque line and $a_2$, $a_4$ & $a_6$ be the areas below the mean torque line.

These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at A = E then from fig. we have

Energy at B = $E + a_1$

Energy at C = $E + a_1 - a_2$

Energy at D = $E + a_1 - a_2 + a_3$

Energy at E = $E + a_1 - a_2 + a_3 - a_4$

Energy at F = $E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G = $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (i.e cycle repeats after G)

Let us now suppose that the greatest of these energies is at 'B' and least at 'E'. Therefore

Max$^m$. energy in flywheel = $E + a_1$

Min$^m$. " " " = $E + a_1 - a_2 + a_3 - a_4$

∴ Max$^m$. fluctuation of energy

$\Delta E$ = Max$^m$. energy - Min$^m$. energy

$\Rightarrow \Delta E = (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

* **Coefficient of fluctuation of Energy :-**

It may be defined as the ratio of the max$^m$. fluctuation of energy to the workdone per cycle

Mathematically $(C_E) = \dfrac{\text{Max}^m\text{. fluctuation of energy}}{\text{workdone per cycle}}$

(1) Workdone/cycle = $T_{Mean} \times \theta$

$T_{Mean}$ = Mean torque

$\theta$ = Angle turned (radians) in 1 rev$^n$.

= $2\pi$, in case of steam engine & 2 stroke IC engines.

= $4\pi$, in case 4 stroke IC engines

(2) WD/cycle = $\dfrac{P \times 60}{n}$

n = No. of working strokes / min.

= N, in case of steam engine & 2 stroke IC engines.

= $\dfrac{N}{2}$, in case of 4 stroke IC engines.

## * Coefficient of fluctuation of speed :-

The difference betn the maxm. and minm. speeds during a cycle is called the maxm. fluctuation of speed.

The ratio of the maxm. fluctuation of speed to the mean speed is called coefficient of fluctuation of speed :-

Let $N_1$ & $N_2$ = Maxm. and minm. speeds in r.p.m during the cycle.

$$N = \text{Mean speed} = \frac{N_1 + N_2}{2}$$

∴ coefficient of fluctuation of speed

$$C_S = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

In terms of angular speed

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

In terms of linear speed

$$C_S = \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

The reciprocal of the coefficient of fluctuation of speed is known as coefficient of steadiness.

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

* Flywheel :-

→ A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement & releases it during the period when the requirement of energy is more than the supply.

→ A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

# Flywheel

Let $N_1$ = max$^m$ speed of flywheel in rpm during cycle

$N_2$ = min$^m$ speed during the cycle.

$\omega_1$ & $\omega_2$ = corresponding max$^m$ & min$^m$ angular speeds

$I$ = moment of inertia of the flywheel $= mK^2$

$m$ = mass of flywheel.

$K$ = Radius of gyration of the flywheel.

$\Delta E$ = max$^m$ fluctuation of energy in N-m or joules.

$C_E$ = coefficient of fluctuation of Energy

$C_S$ = coefficient of fluctuation of speed.

$N$ = mean speed of flywheel during cycle $= \frac{N_1+N_2}{2}$

$\omega$ = mean angular speed of flywheel $= \frac{\omega_1+\omega_2}{2}$

$E$ = Kinetic energy of flywheel at mean speed

We know that the KE of the flywheel corresponding to mean angular velocity is given by

$$E = \frac{1}{2} \times I \times \omega^2 = \frac{1}{2} \times mK^2 \times \omega^2$$

$$\Delta E = \text{max}^m\ KE - \text{min}^m\ KE$$

$$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$$

$$= \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} I(\omega_1+\omega_2)(\omega_1-\omega_2)$$

$$= I\left(\frac{\omega_1+\omega_2}{2}\right)(\omega_1-\omega_2)$$

$$= I\omega^2\left(\frac{\omega_1-\omega_2}{\omega}\right)$$

$$\boxed{\Delta E = I\omega^2 C_S} \quad -(1)$$

$$\Delta E = \frac{1}{2} I\omega^2 \times 2 \times C_S$$

$$\boxed{\Delta E = E \times 2 \times C_S} \quad -(2)$$

$$\boxed{\Delta E = I\omega^2 C_S = mK^2\omega^2 C_S} \quad -(3)$$

If the thickness of the rim of the flywheel is very small as compared to the diameter of flywheel, then radius of gyration = mean radius of the flywheel

i.e $K = R$

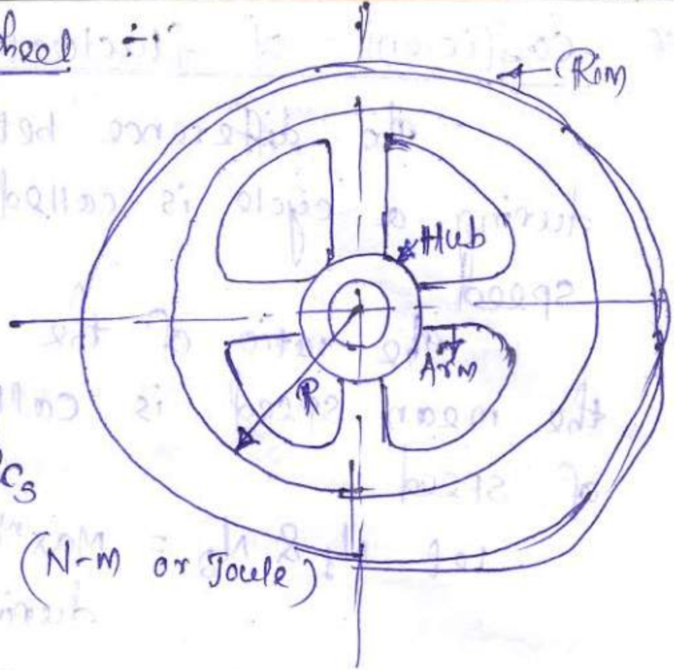
$$\Delta E = mK^2\omega^2 C_s$$

$$\boxed{\Delta E = mR^2\omega^2 C_s} \quad (4) \qquad v = \omega R$$

$$\boxed{\Delta E = mv^2 C_s} \quad (5)$$

diagram for a multi cylinder

## * Energy stored in a Flywheel :-




Mean KE of the Flywheel

$$(E) = \frac{1}{2} \times mK^2\omega^2$$

Max^m. Fluctuation of energy

$$(\Delta E) = I\omega^2 C_s = mK^2\omega^2 C_s$$

$$= 2EC_s \text{ (N-m or Joule)}$$

$$(\Delta E) = mR^2\omega^2 C_s = mv^2 C_s$$

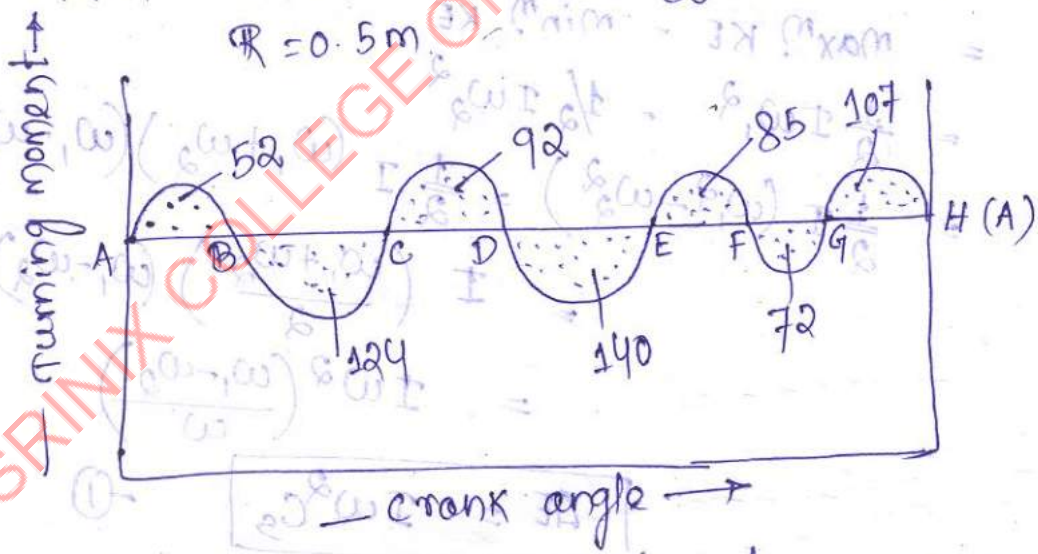
v = Mean linear velocity (at Mean radius) in m/s $= \omega \cdot R$

$$\boxed{C_s = \frac{N_1 - N_2}{N}}$$

* The turning moment diagram for a multi-cylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas betn the o/p torque curve & the mean resistance line, taken in order from one end are as follows :- +52, -124, +92, -140, +85, -72 and +107 mm², when the engine is running at a speed of 600 rpm. If the total fluctuation of speed is not to exceed ±1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.

<u>Ans</u>:- Given $N = 600$ rpm, $\omega = \frac{2\pi \times 600}{60} = 62.84$ rad/s

$R = 0.5$ m




Since the total fluctuation of speed is not to exceed ±1.5% of the mean speed, therefore,

$\omega_1 - \omega_2 = 3\%\, \omega = 0.03\omega$

$\Rightarrow \frac{\omega_1 - \omega_2}{\omega} = 0.03$

$\Rightarrow C_s = 0.03$

Since the turning moment scale is 1 mm = 600 N-m and crank angle scale is 1 mm = $3^\circ = 3 \times \frac{\pi}{180} = \frac{\pi}{60}$ rad

1 $mm^2$ on TM diagram = $600 \times \frac{\pi}{60} = 31.42$ N-m

Let the total energy at A = E

Energy at point 'B' = E + 52 ——— Max$^m$ Energy

" " " 'C' = E + 52 − 124 = E − 72

" " " 'D' = E − 72 + 92 = E + 20

" " " 'E' = E + 20 − 140 = E − 120 — Min$^m$ Energy

" " " 'F' = E − 120 + 85 = E − 35

" " " 'G' = E − 35 − 72 = E − 107

" " " 'H' = E − 107 + 107 = E = Energy at 'A'

$\Delta E$ = max$^m$. energy − min$^m$. energy

= E + 52 − E + 120 = $172 \times 31.42 = 5404$ N-m

Let m = mass of flywheel in kg.

$\Delta E = mR^2\omega^2 C_s$

$\Rightarrow 5404 = m \times (0.5)^2 \times (62.84)^2 \times 0.03$

$\Rightarrow$ **m = 183 kg**

* A single cylinder, single acting, four stroke gas engine develops 20 KW at 300 rpm. The workdone by the gases during the expansion stroke is three times the workdone on the gases during compression stroke, the workdone during the suction & exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2% of the mean speed & the turning moment diagram during comp. & expansion is assumed to be triangular in shape, find the MI of the flywheel.

Ans:- Given data Power (P) = 20 KW = $20 \times 10^3$ W

speed (N) = 300 rpm. $W_E = 3W_C$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.42 \text{ rad/sec}$$

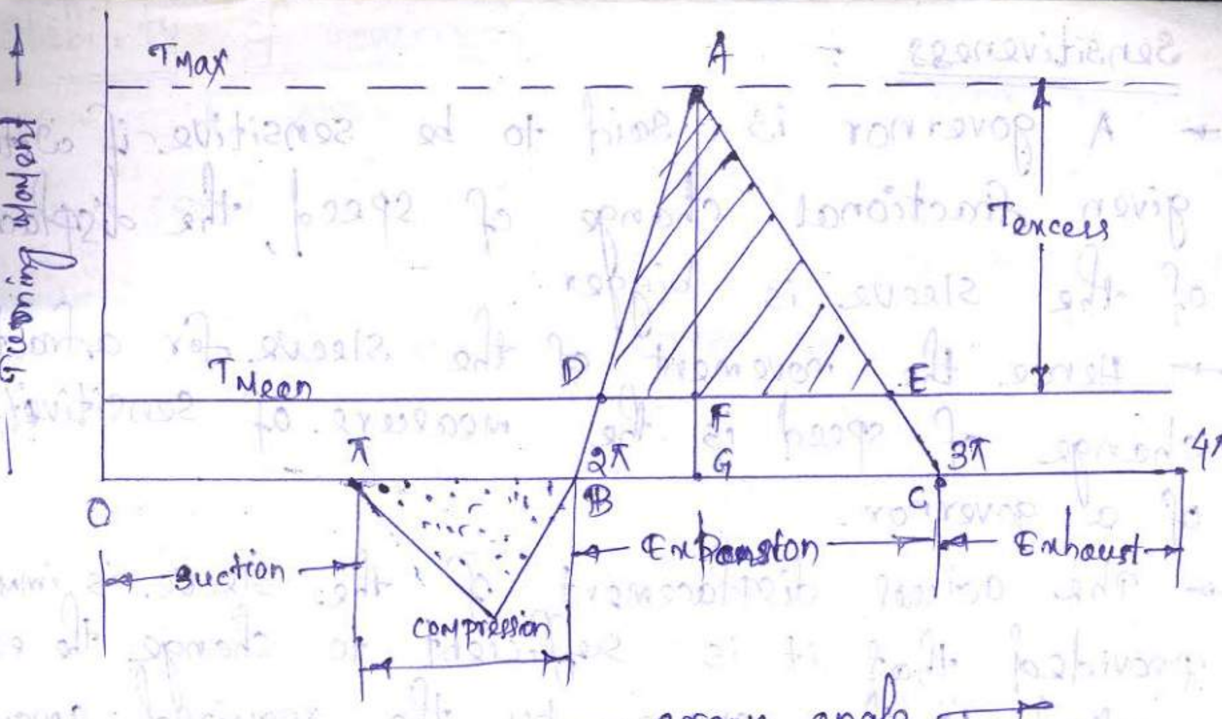
Since the total fluctuation of speed $(\omega_1 - \omega_2)$ is not to exceed ± 2% of the mean speed $(\omega)$ therefore

$$\omega_1 - \omega_2 = 4\% \omega \Rightarrow \frac{\omega_1 - \omega_2}{\omega} = 4\% = 0.04$$

coefficient of fluctuation of speed $(C_s) = \frac{\omega_1 - \omega_2}{\omega} = 0.04$

We know that for a four stroke engine, no. of working strokes/cycle $n = N/2 = \frac{300}{2} = 150$

$$WD/\text{cycle} = P \times \frac{60}{n} = 20 \times 10^3 \times \frac{60}{150} = 8000 \text{ N-m}$$




Since the WD during suction & exhaust strokes is negligible therefore Net WD/cycle (during comp. & expansion strokes)

$$= W_E - W_C = W_E - \frac{W_E}{3} = \frac{2}{3} W_E \quad (W_E = 3W_C)$$

WD/cycle = 8000 N-M

$$\Rightarrow \frac{2}{3} W_E = 8000 \Rightarrow \boxed{W_E = 12000 \text{ N-M}}$$

WD/cycle = 8000 N-m

$\Rightarrow \frac{2}{3} W_E = 8000 \Rightarrow$ $\boxed{W_E = 12000 \text{ N-m}}$

WD during expansion stroke ($W_E$) = Area of $\Delta$ ABC

$\Rightarrow 12000 = \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG$

$\Rightarrow AG = T_{max} = \frac{12000 \times 2}{\pi}$

$\Rightarrow$ $\boxed{T_{max} = 7638 \text{ N-m}}$

Mean turning Moment

$\boxed{T_{mean} = FG = \frac{\text{WD/cycle}}{\text{crange angle/cycle}}}$

$= \frac{8000}{4\pi} = 637 \text{ N-m}$

excess turning Moment $T_{excess}$ = AF = AG − FG = 7638 − 637 = 7001 N-m

from similar $\Delta$ ADE & ABC $\frac{DE}{BC} = \frac{AF}{AG}$ or $DE = \frac{AF}{AG} \times BC$

$= \frac{7001}{7638} \times \pi$

$\boxed{DE = 2.88 \text{ rad}}$

since the area above the Mean turning Moment line represents the max^m. fluctuation of energy. therefore Max^m. fluctuation of energy ($\Delta E$) = Area of ADE = $\frac{1}{2} \times DE \times AF$

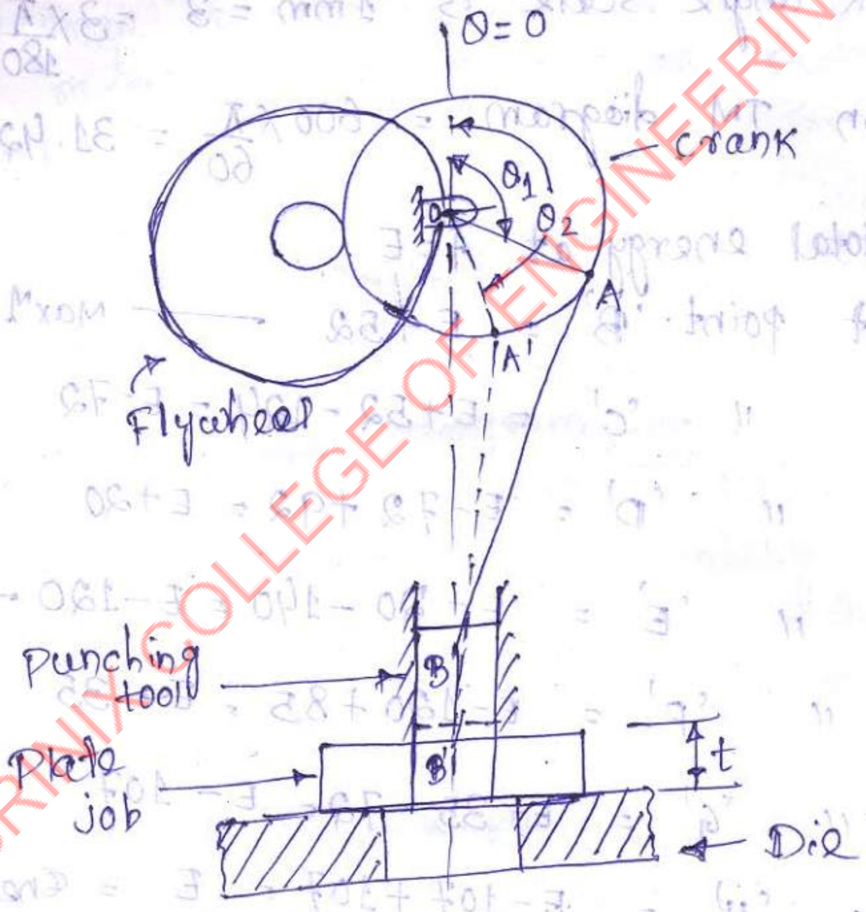
$= \frac{1}{2} \times 2.88 \times 7001 = 10081$ N-m

$\Rightarrow I \omega^2 C_s = 100081$

$\Rightarrow I \times (31.42)^2 \times 0.04 = 100081$

$\Rightarrow$ $\boxed{I = 255.2 \text{ kg-m}^2}$

## * Operation of a Flywheel in a Punching press






→ When a flywheel is attached to the crank-shaft of an engine, the load on crank-shaft is constant but the input torque varies during a cycle.

→ But when flywheel is attached to a punching press or a rivetting machine, the input torque is constant but load varies during the cycle.

→ Let $E_1$ = Energy required for one punch (for one punching operation)

$d$ = Diameter of the hole punched

$t$ = thickness of plate in which hole is to be punched.

$\tau$ = shear stress for the plate material

$F_s$ = Max$^m$ shear force required for punching

= shear stress × Area sheared

= $\tau \times \pi d \times t$

$E_2$ = Energy supplied by motor for actual punching

For one revolution (crank rotation of $2\pi$ radians) energy supplied by motor $= E_1$

For crank rotation of $(\theta_2 - \theta_1)$ radians, the energy supplied by motor will be $= \dfrac{E_1(\theta_2-\theta_1)}{2\pi}$

$\therefore$ Energy supplied by motor during actual punching

$$E_2 = \frac{E_1(\theta_2-\theta_1)}{2\pi}$$

The balance energy required for punching is equal to $= (E_1 - E_2) = E_1 - \dfrac{E_1(\theta_2-\theta_1)}{2\pi}$

$$= E_1\left(1 - \frac{\theta_2-\theta_1}{2\pi}\right)$$

This energy is supplied by the flywheel. Hence the kinetic energy of the flywheel decreases. Hence maxm. fluctuation of energy of flywheel

$$\Delta E = E_1 - E_2 = E_1\left[1 - \frac{\theta_2-\theta_1}{2\pi}\right]$$

But $\Delta E = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$$\boxed{\frac{\theta_2-\theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}}$$

where $t$ = Thickness of plate
$s$ = stroke of the punch
$= 2 \times r$ where $r$ = crank radius

The values of $\theta_1$ & $\theta_2$ are determined if the lengths of connecting rod, crank radius and the relative position of the job with respect to the crank-shaft axis are known. In absence of these data.

* A punching press is required to punch 30 mm diameter holes in a plate of 20mm thickness at the plate of 20 holes per minute. It requires 6 N/m of energy per $mm^2$ of sheared area. If punching takes place in $\frac{1}{10}$ of a second & the rpm of the flywheel varies from 160 to 140, determine the mass of the flywheel having radius of gyration of 1m.

Ans:- Given data

Diameter of holes (d) = 30mm, punching time = $\frac{1}{10}$ s = 0.1s

plate thickness (t) = 20 mm, radius of gyration (k) = 1m

No. of holes = 20 holes/min.

Energy required (E) = 6 $Nm/mm^2$ of sheared area

variation of rpm of flywheel = 160 to 140

$N_1$ = 160 rpm, $N_2$ = 140 rpm.

Let m = Mass of flywheel

We know that sheared area per hole (A) = $\pi d t$

$= \pi \times 30 \times 20 = 600\pi \ mm^2$

Energy required to punch a hole

($E_1$) = Energy required per $mm^2$ sheared area × sheared area

= E × A = 6 × 600π = 11309.73 Nm.

No. of holes punched per min = 20

(or) 20 holes are punched in 60 sec.

∴ Time required to punch a hole = $\frac{60}{20}$ = 3 sec.

∴ Energy required for punching work per second

$= \frac{\text{Energy required to punch a hole}}{\text{Time required to punch a hole}}$

$= \frac{11309.73}{3} = 3769.91 \ Nm/s.$

Since the punching takes place $\frac{1}{10}$ of a second, therefore energy supplied by motor in $\frac{1}{10}$ second.

$E_2$ = (Energy required for punching per second) x Time

$= 3769.91 \times \frac{1}{10} = 376.991$ Nm

∴ Energy to be supplied by the flywheel during punching a hole (max^m fluctuation of energy of flywheel)

$$\Delta E = E_1 - E_2 = 11309.73 - 376.991$$

$$\boxed{\Delta E = 10932.739 \text{ Nm}}$$

But $\Delta E = \frac{1}{2} \times I \times \omega_1^2 - \frac{1}{2} \times I \times \omega_2^2$

$$= \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I \left[ \left( \frac{2\pi \times N_1}{60} \right)^2 - \left( \frac{2\pi N_2}{60} \right)^2 \right]$$

$$= \frac{1}{2} I \times 32.898$$

$$\Rightarrow 10932.739 = \frac{1}{2} \times m K^2 \times 32.898 = 32.898\, m$$

$$\Rightarrow \boxed{m = 332.32 \text{ kg}}$$

# Toothed Gearing

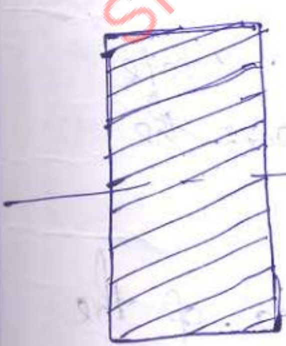
## Gear

→ Gears are simply wheel, If we provide projection overs this wheel, then it becomes gears

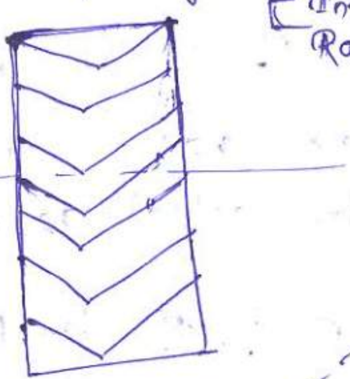
→ Gears are components used in the assembly of machines for transmission of power and motion.

## classification of gear

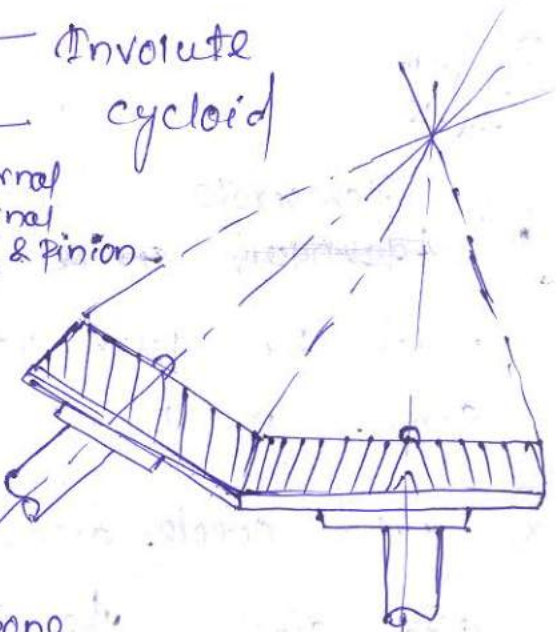
Gear
- Position of axes of shafts
  - Parallel axes – Spur & Helical
  - Non-Parallel Intersecting axes – Bevel
  - Non-Intersecting Non-parallel axes – worm gear, spiral gear, hypoid gear
- Relative motion
  - Low (Less than 3 m/s)
  - Medium (betn 3 & 15 m/s)
  - High (more than 15 m/s)
- forms of teeth
  - Involute
  - cycloid
- Type of gearing
  - External
  - Internal
  - Rack & Pinion



Single helical gear

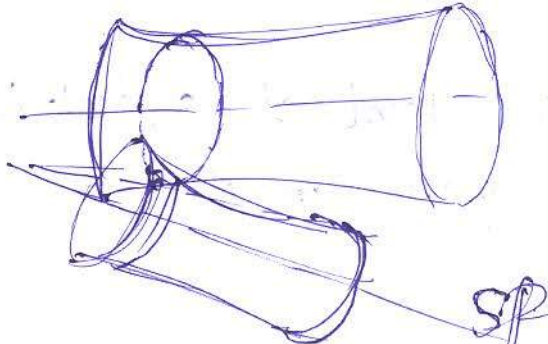


Double (Herringbone helical gear gear)



Bevel gear

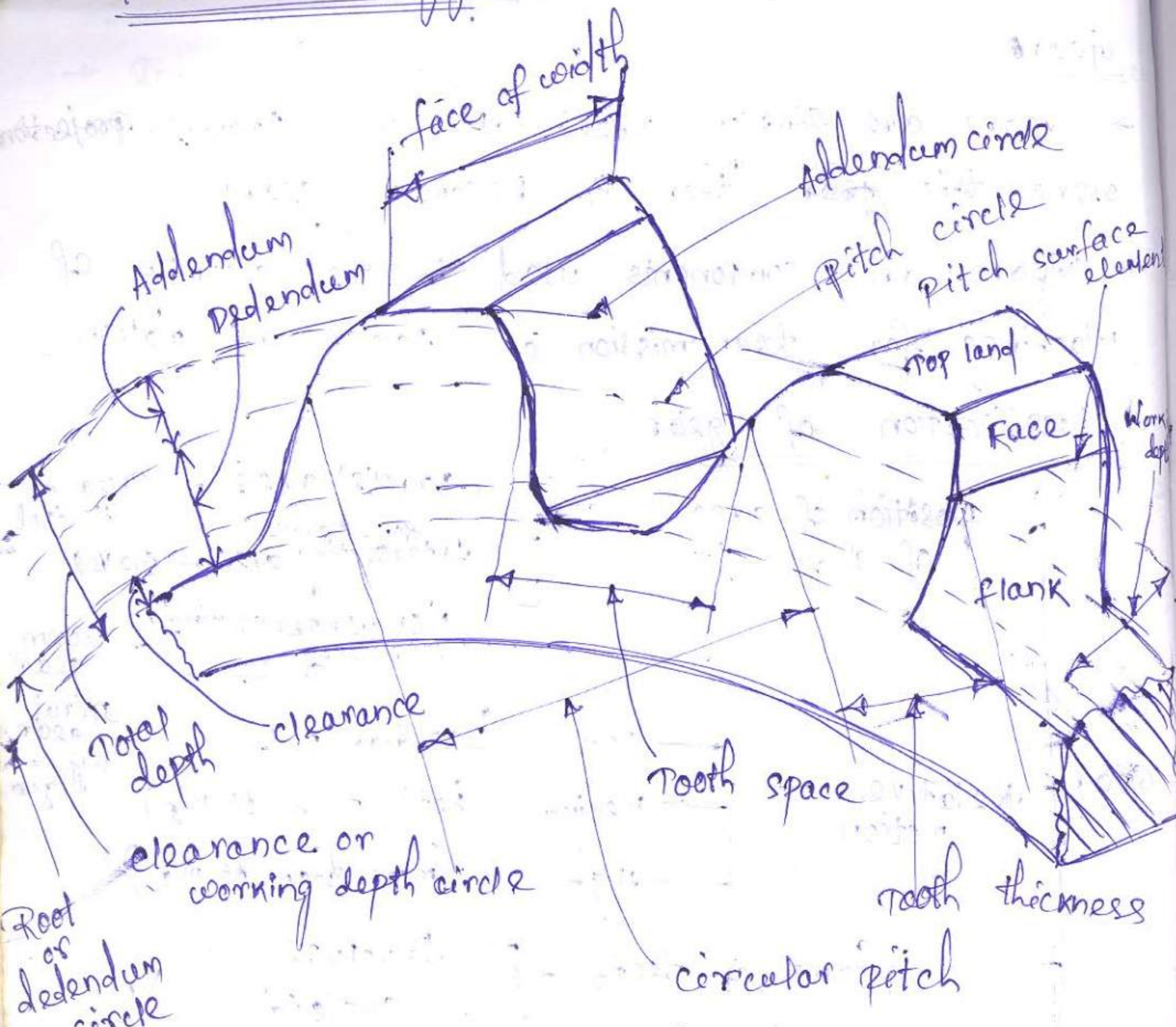
gear



A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis)

Spiral gear

# Gear Terminology




① **Pitch circle** :- It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

② **Pitch circle diameter** :- It is the dia. of the pitch circle. The size of the gear is usually specified by the pitch circle dia. It is also known as pitch dia.

③ **Pitch point** :- It is the point of contact betn the pitch circles of two gears in mesh.

(4) **Pressure angle (or) Angle of obliquity** :-

It is the angle betn the common normal to two gear teeth at the point of contact & the common tangent at the pitch point. It is denoted by $\phi$. The standard pr. angles are $14\frac{1}{2}^\circ$ & $20^\circ$.

(5) **Addendum** :- It is the radial distance of a tooth from the pitch circle to the top of the tooth.

(6) **Dedendum** :- It is the radial distance of a tooth from the pitch circle to the bottom of tooth.

(7) **Addendum circle** :- It is the circle drawn through the top of the teeth. It is also called outside dia. of the teeth.

(8) **Dedendum circle** :- It is the circle drawn through bottom of the teeth. It is also called root circle.

(9) **Circular Pitch** :- It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It denoted by '$P_c$'

circular pitch $\boxed{(P_c) = \frac{\pi D}{T}}$

D = pitch circle dia.

T = No. of teeth on the wheel

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

(10) **Diametral Pitch** :- It is the ratio of number of teeth to the pitch circle dia. in MM

$\boxed{(P_d) = \frac{T}{D}}$

(11) Module (m) :- It is the ratio of the Pitch circle dia. in mm. to the no. of teeth.

$$m = \frac{D}{T}$$

(12) Backlash :- It is the difference betn the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors & thermal expansion.

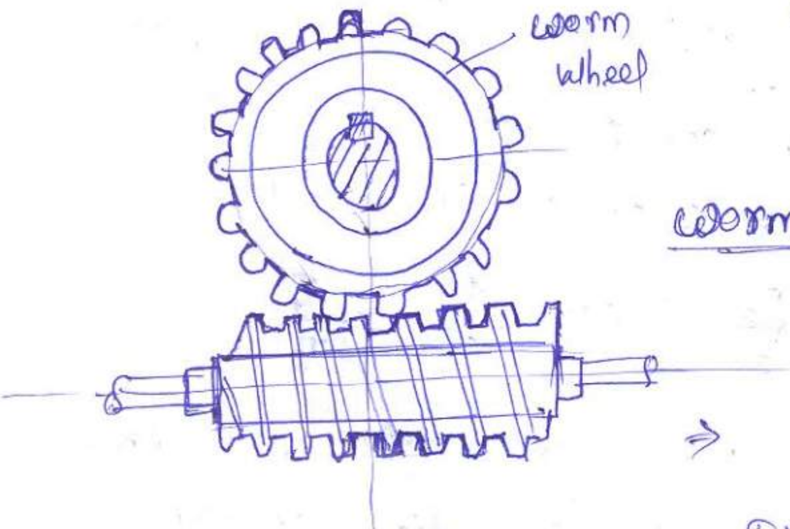
* Gear Material :-

Metallic → cast iron, steel (high strength gear), phosphor bronze (worms).

Non-Metallic → wood, synthetic resins like nylon, raw hide, compressed paper especially used for reducing noise.

* condition for constant velocity Ratio of toothed wheels - Law of gearing :-

Worm gear




If thread makes a complete turn, the result is a worm & the mating gear is called worm wheel.

worm & worm wheel

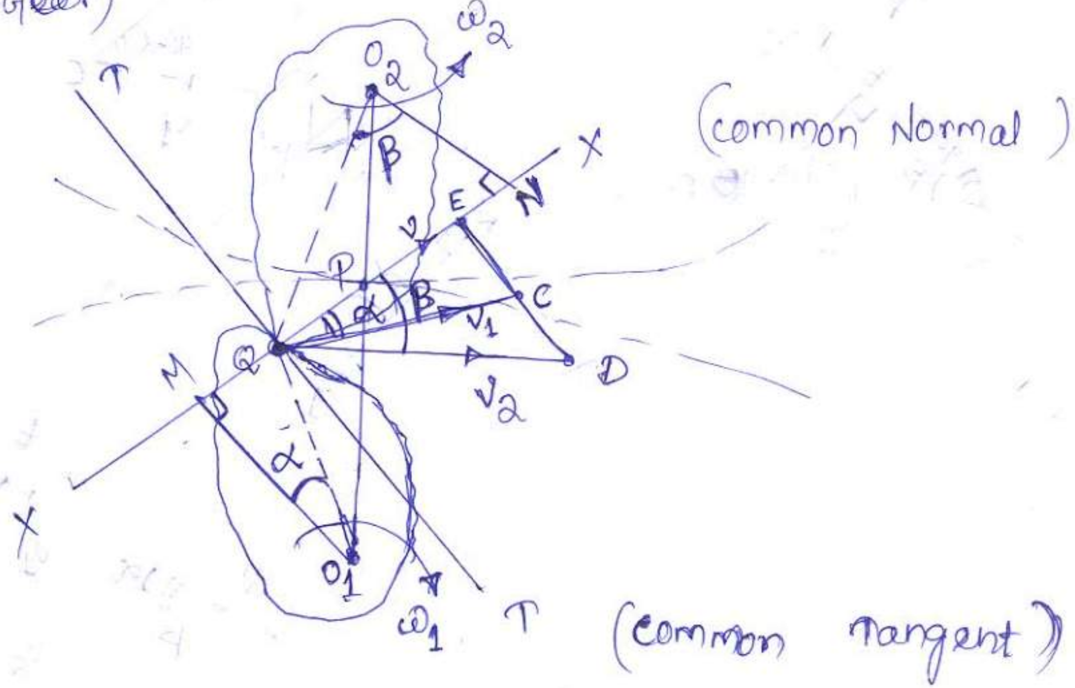
⇒ Higher velocity ratio

Ex :- Lathe, Drill, Milling

Tooth space :- It is the width of space betn the two adjacent teeth measured along the Pitch circle.

# * Condition for constant velocity Ratio of toothed wheel – Law of gearing

wheel-1 (Pinion)
wheel-2 (Gear)




* Consider the portions of the two teeth, one on the wheel 1 (Pinion) & other on the wheel-2 (gear).

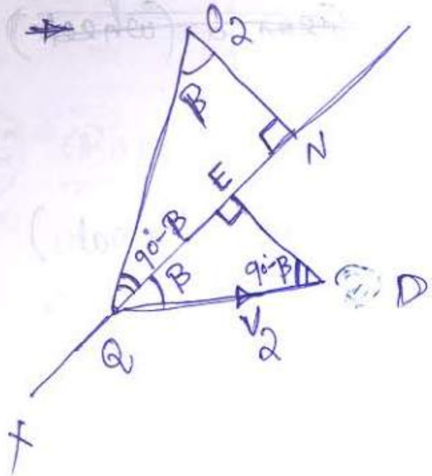
* Let two teeth come in contact at point Q & the wheel rotate in the directions as shown in fig

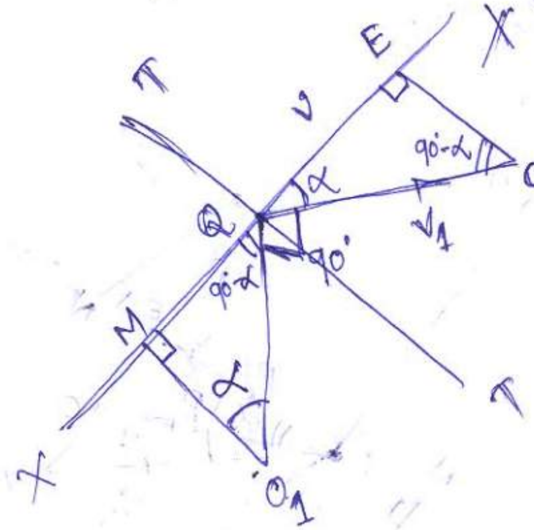
→ Let TT be the common tangent and XX be the common normal to the curves at the point of contact Q.

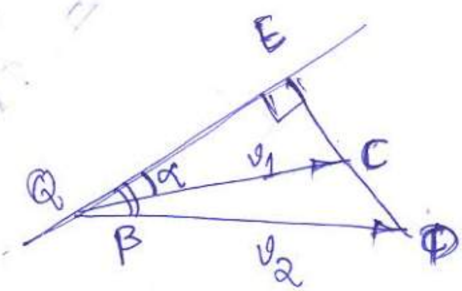
→ from the centres $O_1$ & $O_2$, draw $O_1M$ and $O_2N$ perpendicular to XX.

→ A little consideration will show that the point Q moves in the dirn. QC when considered as a point on wheel 1, and in the dirn. QD when considered as a point on wheel 2.

→ Let $v_1$ and $v_2$ be the velocities of the point 'Q' on the wheels 1 and 2 respectively.




→ If the teeth are to remain in contact, then the components of these velocities along the common normal XX must be equal.

$\Rightarrow v_1 \cos\alpha = v_2 \cos\beta$

$\Rightarrow (\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$

$\Rightarrow (\omega_1 \times O_1Q) \times \frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \times \frac{O_2N}{O_2Q}$

In Δ QEC, $\cos\alpha = \frac{QE}{QC}$

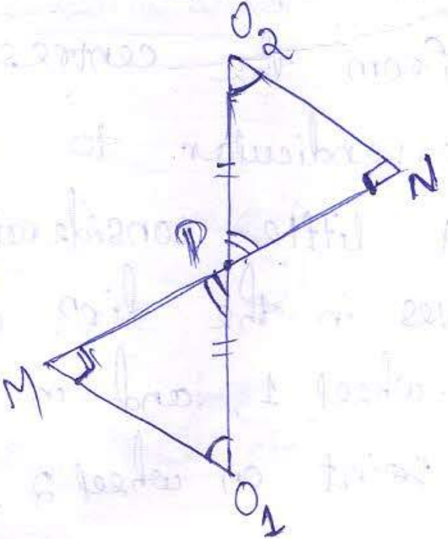
$\Rightarrow QE = QC \cos\alpha$

In Δ QED, $\cos\beta = \frac{QE}{QD}$

$\Rightarrow QE = QD \cos\beta$

$\Rightarrow \omega_1 \times O_1M = \omega_2 \times O_2N.$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \quad \text{---} (1)$$




Also from similar triangles

$O_1MP$ & $O_2NP$,

$$\Rightarrow \boxed{\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}} \quad \text{---} (2)$$

from eqn. (1) & (2) we get

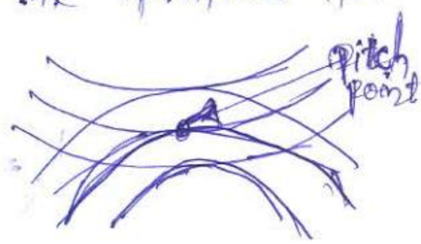
$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P} = \frac{O_2 N}{O_1 M}} \quad -(3)$$

From above, we show that the angular velocity ratio is inversely proportional to the ratio of the distances of the point 'P' from the centres $O_1$ and $O_2$

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point 'P' must be fixed point (pitch point) for the wheels (The law of gearing states that for constant angular VR of the two gears,

Law of gearing :- the common normal at the point of contact betn a pair of teeth must always pass through the pitch point.

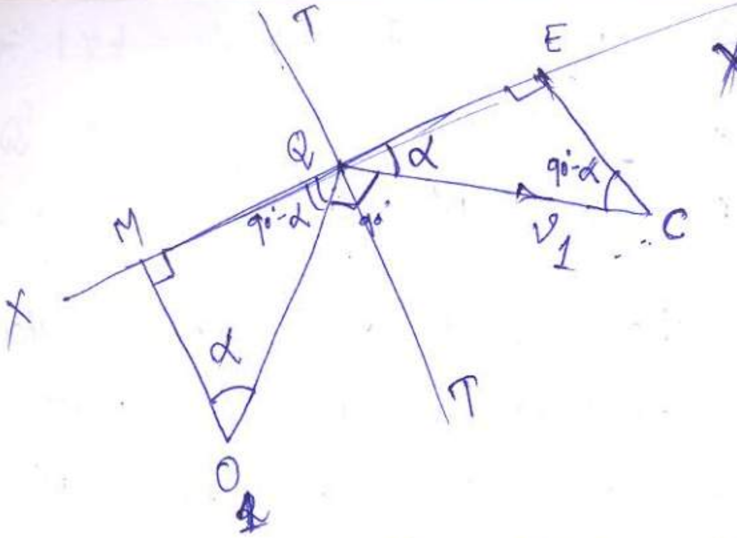
This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels.




* velocity of sliding of teeth :-

"The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact".

The sliding betn a pair of teeth in contact at 'Q' occurs along the common tangent "TT" to the tooth curves as shown in fig.




The velocity of point Q, considered as a point on wheel 1, along the common tangent TT is represented by EC.

from similar triangles QEC & $QMO_1$

$$\Rightarrow \frac{EC}{QC} = \frac{QM}{QO_1}$$

$$\Rightarrow EC = \frac{QC}{QO_1} QM$$

$$\Rightarrow \boxed{EC = \omega_1 \; QM}$$

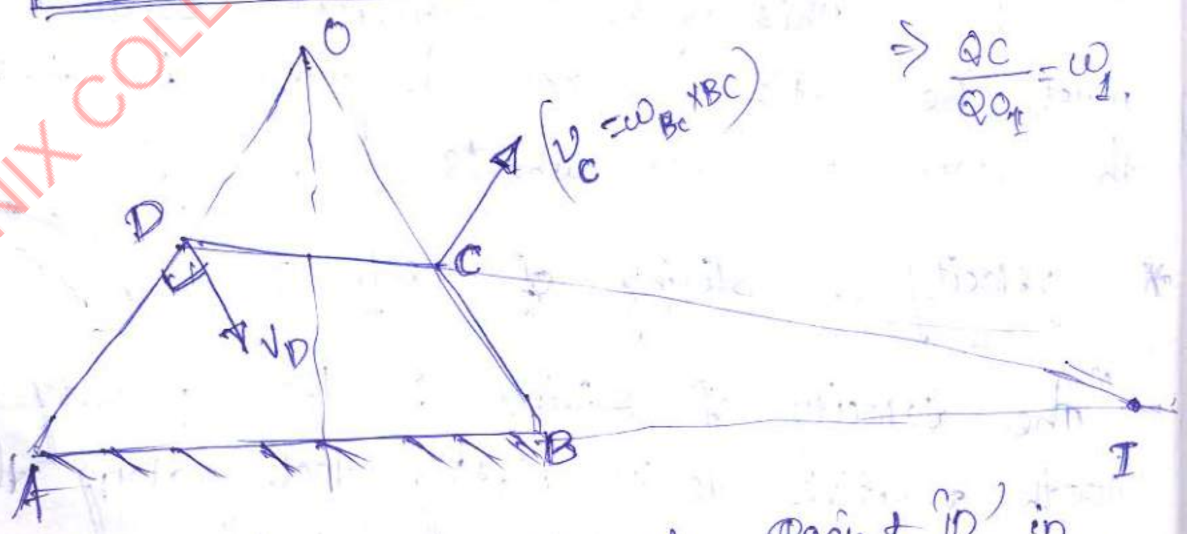
$v_1 = \omega_1 O_1Q$

$QC = v_1$

$O_1Q = r$

$\Rightarrow QC = \omega_1 QO_1$

$\Rightarrow \frac{QC}{QO_1} = \omega_1$.

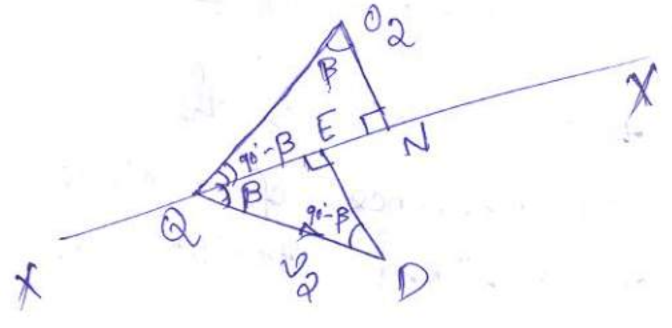



$V_D$ = linear velocity at point 'D' in the dirn. of ⊥ar to AD

$$V_D = \omega_{AD} \times AD$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent TT is represented by ED.

In Similar Δ $O_2QN$ & QED




$$\frac{ED}{QD} = \frac{QN}{O_2Q}$$

$$\Rightarrow ED = \frac{QD}{O_2Q} QN$$

$$= \frac{v_2}{O_2Q} QN$$

$$\boxed{ED = \omega_2 \, QN}$$

$\{ v_2 = \omega_2 \, QO_2$

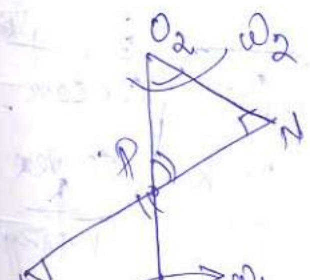
Let $v_s$ = velocity of sliding at 'Q'

$$v_s = ED - EC = \omega_2 QN - \omega_1 QM$$

$$= \omega_2 (QP + PN) - \omega_1 (MP - QP)$$

$$= \omega_2 QP + \omega_2 PN - \omega_1 MP + \omega_1 QP$$

$$= QP(\omega_1 + \omega_2) + \omega_2 PN - \omega_1 MP$$




Since in Similar Δ we get

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$$

$$\Rightarrow \omega_1 \cdot MP = \omega_2 \cdot PN$$

$$\boxed{v_s = QP(\omega_1 + \omega_2)}$$

Note (V_s) is proportional to the distance of the point of contact from the pitch point.

(2) velocity of sliding at pitch point in involute profile is zero.

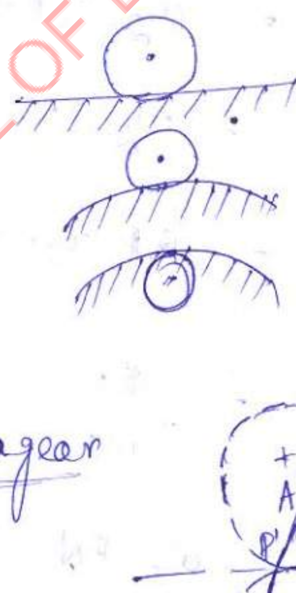
## * Forms of Teeth (common forms of teeth that also satisfy the law of gearing)

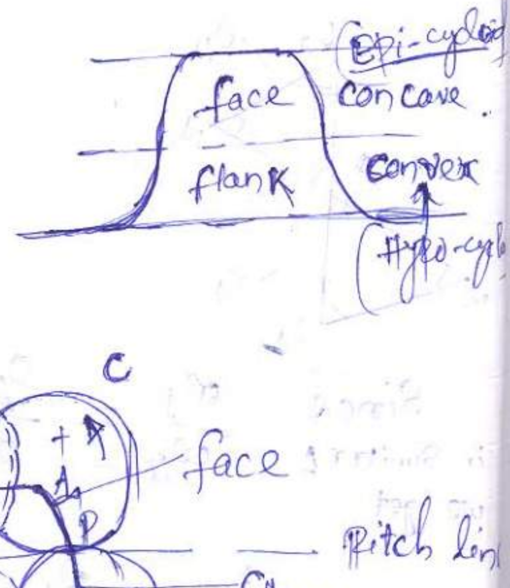
1. cycloidal teeth
2. Involute teeth

## * cycloidal teeth :-

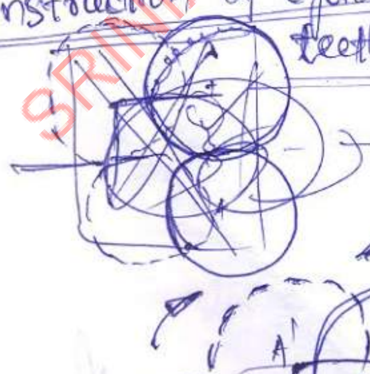
* A cycloid is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line.
* When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as epi-cycloid.
* When a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called hypo-cycloid.

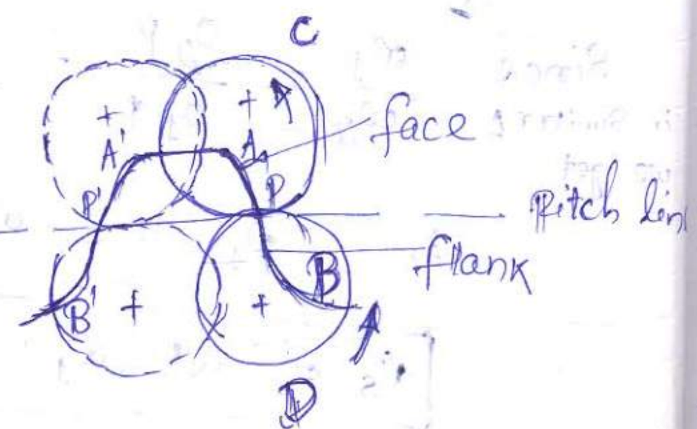
→ cycloid
→ Epi-cycloid
→ Hypo-cycloid

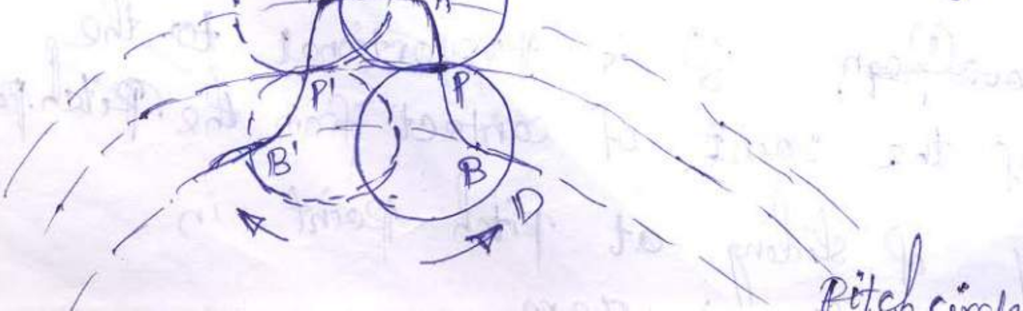




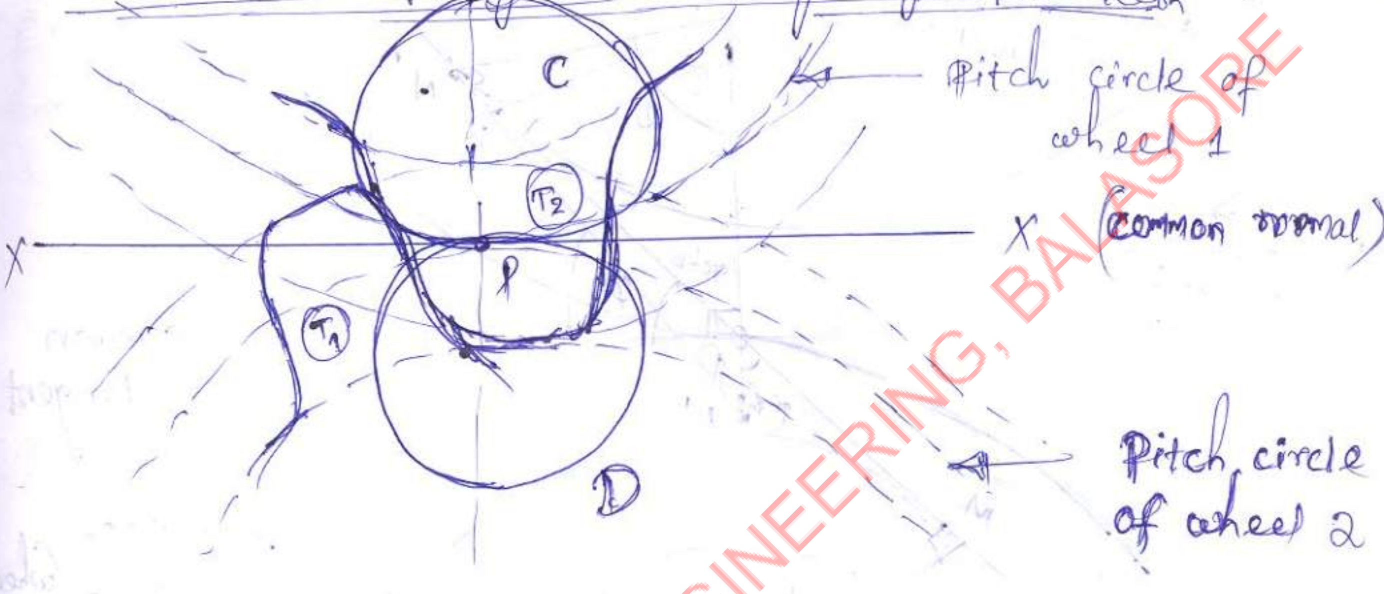

### construction of cycloidal teeth of a gear





## Construction of two mating cycloidal teeth



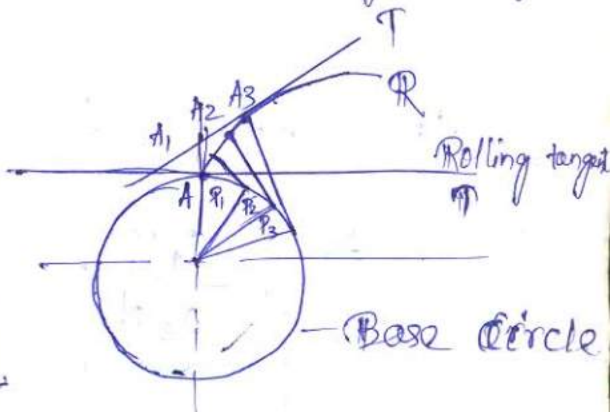

## * Involute Teeth :-

An involute is a curve generated by a point on a straight edge as the straight edge is rolled on a cylinder.

Point (A) - Starting point

→ Normal at any point of an involute is a tangent to the base circle

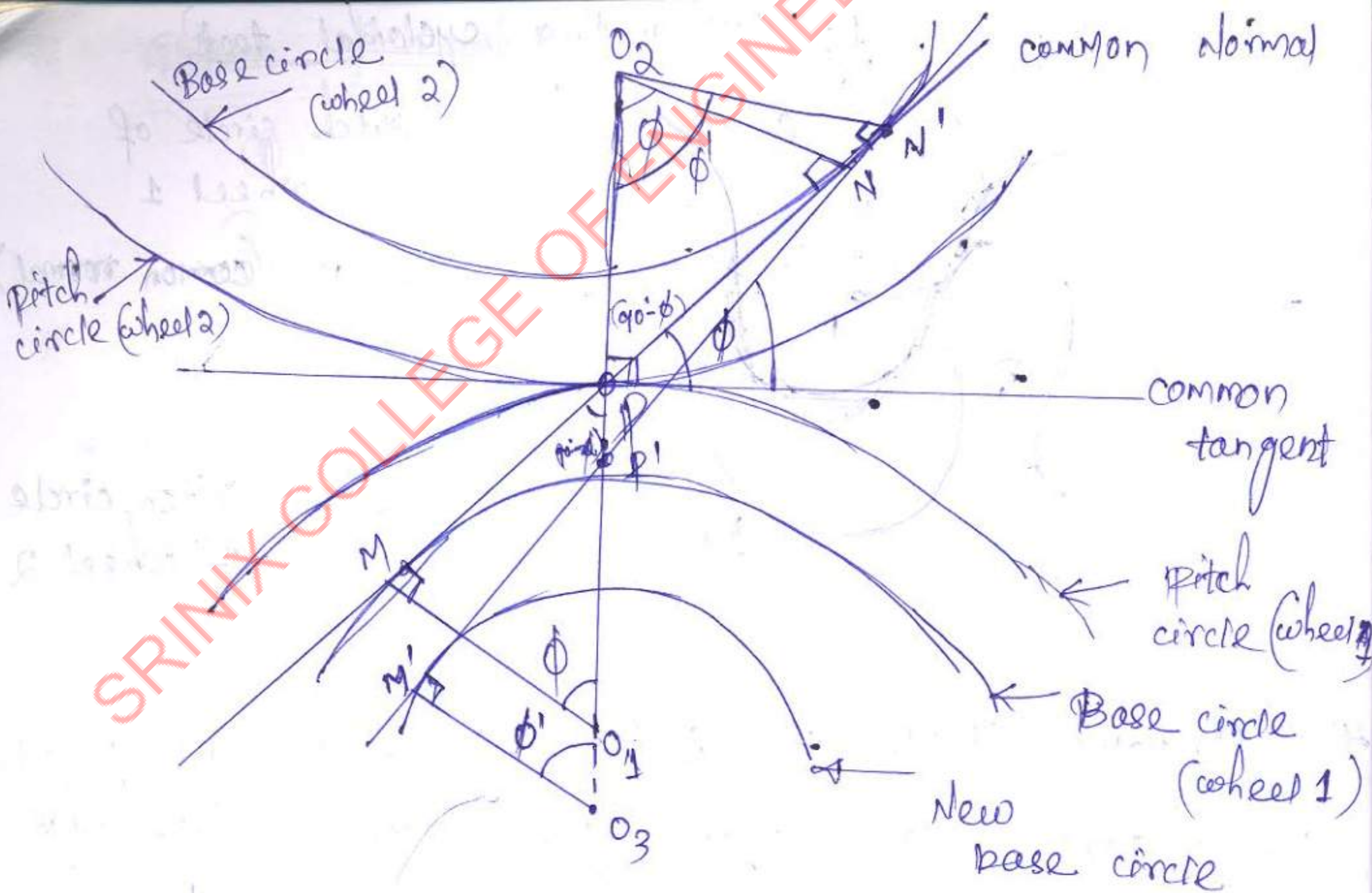

$P_1A_1 = AP_1$
$P_2A_2 = AP_2$
$P_3A_3 = AP_3$




Construction of Involute

→ An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut (tight) string which is unwrapped from a reel as shown in fig.

## * Effect of Altering the centre Distance on the velocity Ratio for Involute teeth gears :-





In similar $\Delta$ $O_2PN$ & $O_1MP$

$$\Rightarrow \frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1}$$

In similar triangle $O_2P'N'$ & $O_3P'M'$

$$\Rightarrow \frac{O_3M'}{O_2N'} = \frac{O_3P'}{O_2P'}$$

But $O_2N = O_2N'$ & $O_1M = O_3M'$

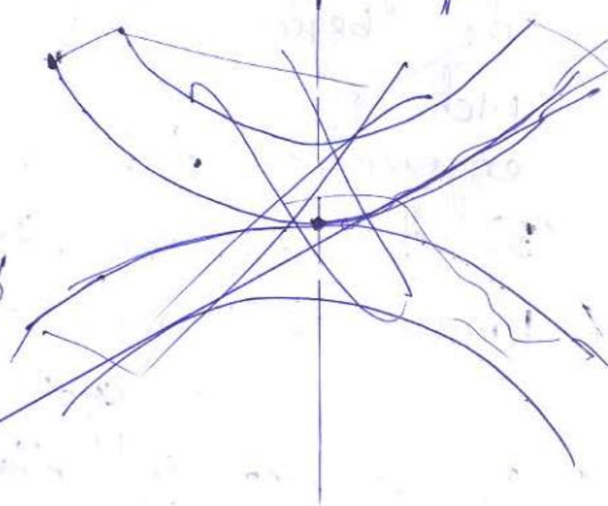
$$\Rightarrow \boxed{\frac{O_1P}{O_2P} = \frac{O_3P'}{O_2P'} = \frac{\omega_2}{\omega_1}}$$

If we increase the centre distance upto a certain limit of one gear then there will no affect of the velocity ratio and power transmission, However the pressure angle increases from $\phi$ to $\phi'$ with the increase in the centre distance.

When the power is being transmitted, the max^m tooth pr. (neglecting friction at the tooth) is exerted along the common normal through the pitch point

$$\sin\phi = \frac{F_R}{F}$$

$$\cos\phi = \frac{F_T}{F}$$



If 'F' is the max^m tooth pr.

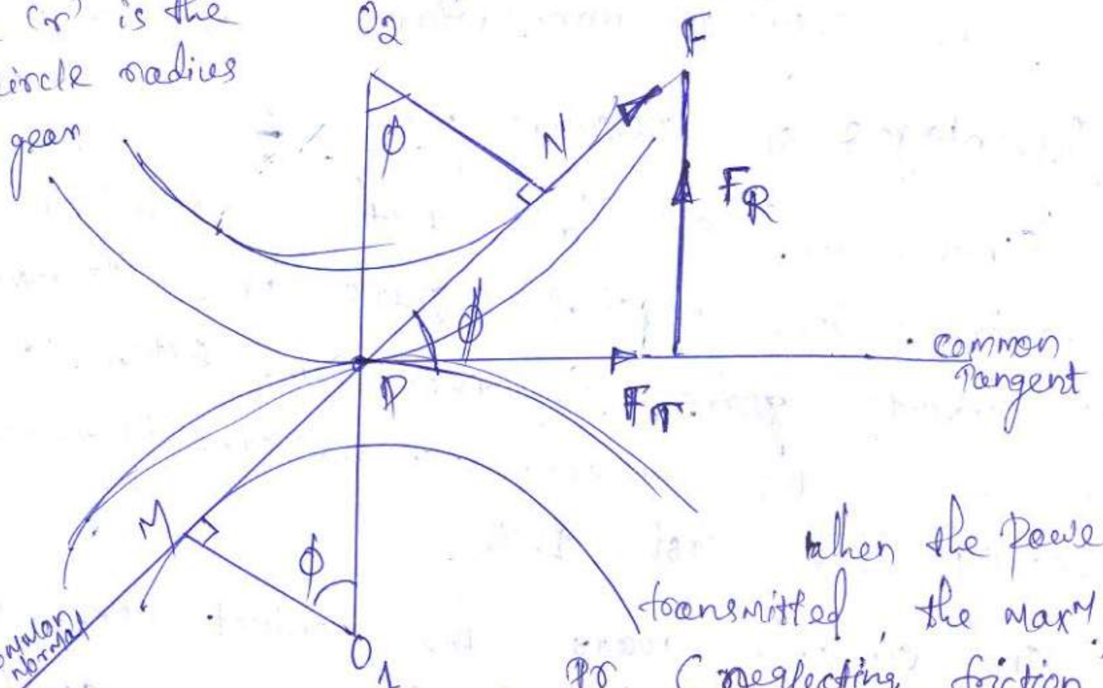
Tangential force $(F_T) = F\cos\phi$

Radial or normal force $(F_R) = F\sin\phi$

Torque exerted on the gear shaft

$T = F_T \times r$

where 'r' is the pitch circle radius of the gear




→ The tangential force provides the driving torque and the radial or normal force produces radial deflection of the rim and bending of the shaft

When the power is being transmitted, the max^m tooth pr. (neglecting friction) is exerted along the common normal through the pitch point and bending of the shaft

# * Advantages of Involute gears :-

(1) The most important advantages of involute gears is that the centre distance for a pair of involute gears can be varied within limits without changing the velocity Ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained

(2) In involute gears, the pressure angle from the start of the engagement of teeth to the end of the engagement remains constant. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maxm at the beginning of engagement, reduces to zero at pitch point and again becomes maxm. at the end of engagement. This result in less smooth running of gears.
(3) The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e epi-cycloid & hypo-cycloid) are required for the face and flank respectively. Thus involute teeth are easy to manufacture than cycloidal teeth.

* Advantages of cycloidal gears :-

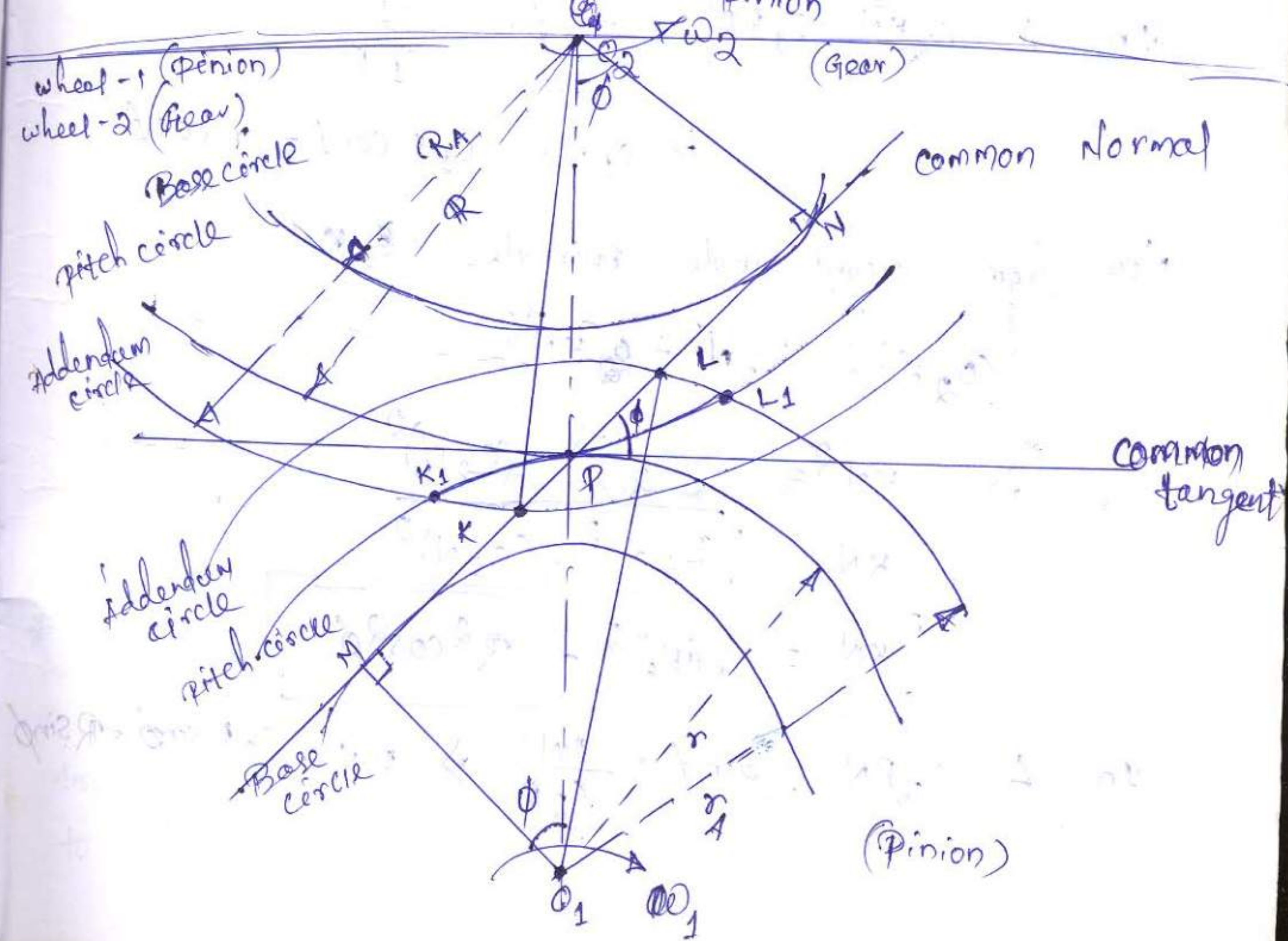
(1) Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

(2) In cycloidal gears, the contact takes place both a convex flank and concave surface, whereas in involute gears the convex surfaces are in contact. This condition results in less wear in cycloidal teeth as compared to involate teeth.

(3) In cycloidal gears, the interference does not occur at all, but in involate gear the interference well occurs.

* **Length of path of contact (LPC) :-**

It is the length of the common normal cut-off by the addendum circles of the wheel & pinion.



$O_2$
$\omega_2$
wheel-1 (Pinion)
wheel-2 (Gear)
(Gear)
$\phi$
Base circle
RA
R
common Normal
N
pitch circle
Addendum circle
L
L1
$\phi$
common tangent
K1
P
K
Addendum circle
pitch circle
M
Base circle
r
$r_A$
$\phi$
(Pinion)
$O_1$
$\omega_1$

Let $r_A = O_1L$ = Radius of addendum circle of pinion
$R_A = O_2K$ = " " " " " Gear
$r = O_1P$ = " " pitch circle of pinion
$R = O_2P$ = " " " " " Gear.

from fig. the length of path of contacts KL which is the sum of the parts of contacts KP & PL.

The part of the path of contact KP is known as path of approach & the part of the path of contact PL is known as path of recess.

In $\Delta$ $O_2PN$, $\cos\phi = \frac{O_2N}{O_2P}$

$\Rightarrow O_2N = O_2P\cos\phi = R\cos\phi$

In $\Delta$ $O_1PM$, $\cos\phi = \frac{O_1M}{O_1P}$

$\Rightarrow O_1M = O_1P\cos\phi = r\cos\phi$

Now from right angle triangle $O_2KN$

$(O_2K)^2 = (O_2N)^2 + (KN)^2$

$\Rightarrow KN^2 = (O_2K)^2 - (O_2N)^2$

$\Rightarrow KN = \sqrt{(O_2K)^2 - (O_2N)^2}$

$$\boxed{KN = \sqrt{(R_A)^2 - R^2\cos^2\phi}}$$

In $\Delta$ $O_2PN$, $\sin\phi = \frac{PN}{O_2P}$ $\Rightarrow PN = O_2P\sin\phi = R\sin\phi$

∴ length of the part of the path of contact, or the path of approach $(KP) = KN - PN = \sqrt{R_A^2 - R^2\cos^2\phi}$

$- R\sin\phi$

Similarly from right angle triangle $O_1ML$

$$(O_1L)^2 = (O_1M)^2 + (ML)^2$$

$$\Rightarrow \quad ML = \sqrt{(O_1L)^2 - (O_1M)^2}$$

$$\boxed{ML = \sqrt{r_A^2 - r^2\cos^2\phi}}$$

In $\Delta\ O_1MP$, $\sin\phi = \frac{MP}{O_1P} \Rightarrow MP = O_1P\sin\phi$

$MP = r\sin\phi$

∴ length of the part of the path of contact, or the path of recess $(PL) = ML - MP$

$$= \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ length of path of contact (LPC)

$$KL = KP + PL = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$+ \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$KL = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - \sin\phi(R + r)$$

* **Length of Arc of contact (LAC) :-**

LAC is defined as the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.

Length of Arc of contact = $\frac{\text{Length of path of contact}}{\cos\phi}$

* **Contact Ratio (No. of pairs of teeth in contact)**

Contact Ratio is defined as the ratio of the length of the arc of contact to the circular pitch.

Mathematically $(C.R) = \frac{LAC}{P_c} = \frac{LAC}{\pi m}$

$$P_c = \frac{\pi D}{T} = \pi m \quad \left(\frac{D}{T} = m\right) : \quad m = \text{module}$$

(1) Path of contact :- It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

(2) Length of path of contact :- It is the length of the common normal cut-off by the addendum circles of the gear & pinion.

(3) Arc of contact :- It is the path traced by the point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts i.e

→ Arc of approach

→ Arc of recess

→ Arc of approach is the portion of the path of contact from the beginning of engagement to the pitch point.

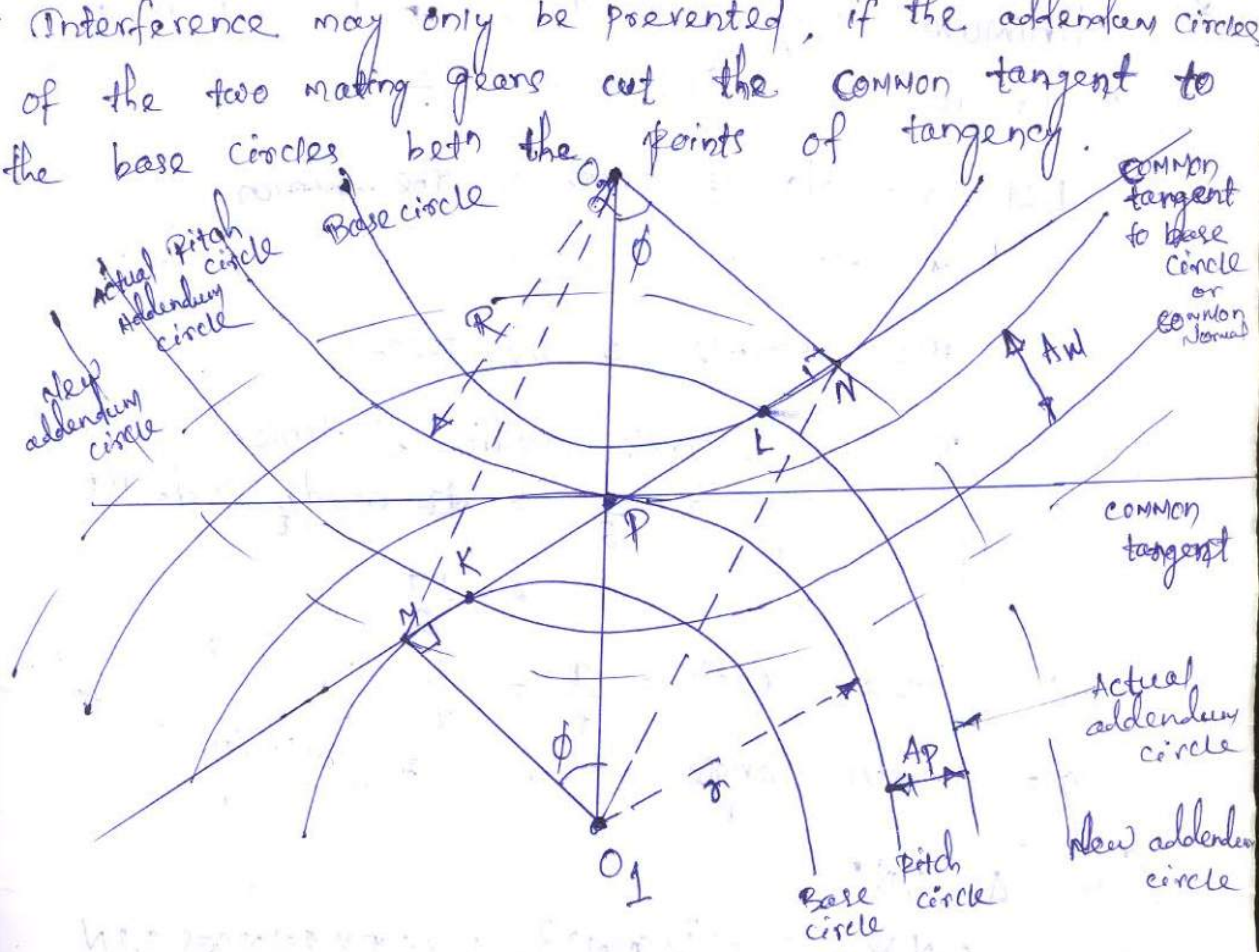
→ Arc of recess is the portion of the path of contact from the pitch point to the end of engagement of a pair of teeth

* Interference in Involute Gears :-

→ The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference and occurs when the teeth are being cut.

→ The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

→ Interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles betn the points of tangency.



When interference is just avoided, the max^m. length of path of contact is MN when the max^m. addendum circles for pinion & wheel pass through the point of tangency N and M respectively.

Max^m. length of path of approach $MP = O_1 P \sin\phi$
$= r \sin\phi$

Max^m. length of path of recess $PN = O_2 P \sin\phi$
$= R \sin\phi$

Max^m. length of path of contact

$$MN = MP + PN = r\sin\phi + R\sin\phi = (r+R)\sin\phi$$

$$\text{Max}^m.\ LAC = \frac{\text{Max}^m\ LPC}{\cos\phi} = \frac{(r+R)\sin\phi}{\cos\phi} = (R+r)\tan\phi$$

path of approach $KP = \frac{1}{2}MP$, path of recess $PL = \frac{1}{2}NP$

$$\Rightarrow KP + PL = \frac{1}{2}MP + \frac{1}{2}NP = \frac{r\sin\phi}{2} + \frac{R\sin\phi}{2}$$

$$= \frac{(r+R)\sin\phi}{2}$$

## * Minimum Number of Teeth on the Pinion in order to Avoid Interference :-

Let $t$ = No. of teeth on the pinion

$T$ = No. " " " wheel (gear)

$m$ = module of the teeth

$r$ = pitch circle radius of pinion

$$r = \frac{d}{2} \quad \& \quad m = d/t \Rightarrow d = mt$$

$$r = \frac{mt}{2}$$

$$G = \text{Gear ratio} = \frac{T}{t} = \frac{R}{r}$$

$\phi$ = pressure angle or angle of obliquity.

from $\Delta\ O_1NP$

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \times \cos O_1PN$$

$$= r^2 + R^2\sin^2\phi - 2rR\sin\phi\cos(90^\circ + \phi)$$

In $\Delta\ O_2PN$ $\quad \sin\phi = \frac{PN}{O_2P}$

$\Rightarrow PN = O_2P\sin\phi = R\sin\phi$

$$\Rightarrow (O_1N)^2 = r^2 + R^2\sin^2\phi + 2rR\sin^2\phi \quad \left(\because \cos(90+\phi) = -\sin\phi\right)$$

$$= r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$\Rightarrow (O_1N)^2 = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

∴ limiting radius of the pinion addendum circle

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi}$$

$$\boxed{O_1N = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}}$$

Let $A_p \cdot m$ = Addendum of the pinion where $A_p$ is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know addendum of the pinion = $O_1N - O_1P$

$$A_p \cdot m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{mt}{2}$$

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$A_p = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$\Rightarrow \boxed{t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$$

**Line of Action or pressure line** :- The force which the driving tooth exerts on the driven tooth is along a line from the pitch point to the point of contact of the two teeth. This line is also known as the common normal at the point of contact of the mating gears & is known as the line of action or the pr. line

* Min^m No. of teeth on the wheel in order to avoid Interference :-

Let $T$ = min^m. no. of teeth required on the wheel inorder to avoid interference.

$A_\omega \cdot m$ = Addendum of the wheel, where $A_\omega$ is a fraction by which the standard addendum for the wheel should be multiplied in order to avoid interference

from fig: In $\Delta$ $O_2MP$

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \times \cos \angle O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2R \cdot r \sin\phi \cos(90° + \phi)$$

In $\Delta$ $O_1MP$ $\quad \sin\phi = \frac{PM}{O_1P} \Rightarrow PM = O_1P\sin\phi = r\sin\phi$

$$= R^2 + r^2 \sin^2\phi + 2R \cdot r \sin^2\phi$$

$$= R^2 \left[1 + \frac{r^2 \sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right]$$

$$= R^2 \left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

∴ limiting radius of wheel addendum circle

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin 2\phi}$$

$$= \frac{mT}{2}\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin 2\phi}$$

we know that addendum of the wheel

$$= O_2M - O_2P$$

$$= \frac{mT}{2}\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} - \frac{mT}{2}$$

to z…
again…
of …
sma…
(ii) …
the …
this …
(iii) …
N…
(iv) …
do…
(v) I…
has…
res…

$$\Rightarrow A_\omega \cdot m = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$\boxed{T = \frac{2A_\omega}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1}}$$

$$\boxed{T = \frac{2A_\omega}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}}$$

Note ① Angle through which the pinion turns

$$= \frac{LAC \times 360^\circ}{\text{circumference of pinion } (\pi d)} = \frac{LAC \times 360^\circ}{2\pi r}$$

② Angle through which the gear turns

$$= \frac{LAC \times 360^\circ}{\text{circumference of gear } (\pi D)} = \frac{LAC \times 360^\circ}{2\pi R}$$

③ Rolling velocity $(V_R) = \omega_1 \cdot r = \omega_2 R$

| * Cycloidal Teeth | Involute Teeth |
|---|---|
| (i) Pr. angle varies from maxm. at the beginning of engagement, reduces to zero at the pitch point & again increases to maxm. at end of engagement resulting in less smooth running of the gears. | (i) pr. angle is constant throughout the engagement of teeth. This results in smooth running of the gears. |
| (ii) It involves double curve for the teeth, epicycloid & hypocycloid. This complicates the manufacturer. | (ii) It involves single curve for the teeth resulting in simplicity of manufacturing & of tools. |
| (iii) These are costlier to manufactured. | (iii) These are cheaper to manufactured |
| (iv) Phenomenon of interference does not occur at all. | (iv) Interference can occur if the condition of minm. number of teeth on a gear is not followed. |
| (v) In this, a convex flank always has contact with a concave face resulting less wear. | (v) Two convex surfaces are in contact and thus there is more wear. |

| Forms of teeth | Min$^m$. No. of teeth on pinion to avoid interference |
|---|---|
| ⇒ $14\frac{1}{2}°$ composite | 12 |
| ⇒ $14\frac{1}{2}°$ full Depth Involute | 32 |
| ⇒ 20° Full Depth Involute | 18 |
| ⇒ 20° stub involute | 14 |

$$t = \frac{2}{\sin 2\phi}$$

Q Two gears meshing with each other and the no. of teeth in larger gear & smaller gear are 40 & 30. calculate the no. of teeth in contact with each other by considering Module 10 MM & pressure angle 20°.

Ans:- Given data

No. of teeth on wheel ($T_W$) = 40 (Larger)

No. of teeth on pinion ($t_p$) = 30 (smaller)

Module (M) = 10 MM.

pressure angle($\phi$) = 20°

$$\text{Gear Ratio} = \frac{T_W}{t_p} = \frac{40}{30} = 4:3 = 1.34$$

$$\text{circular Pitch } (P_c) = \frac{\pi D}{T} = \pi M = (\pi \times 10)\text{ MM}$$

$$\text{Module } (M) = \frac{D}{T}$$

$$\Rightarrow M = \frac{2R}{T}$$

$$\Rightarrow R = \frac{TM}{2} = \frac{40 \times 10}{2}$$

Radius of pitch circle ⇒ $R = 200$ MM

$$m = \frac{2r}{t} \Rightarrow r = \frac{10 \times 30}{2} = 150 \text{ MM}$$

$$\text{Contact Ratio} = \frac{LAC}{P_c} = \frac{LPC}{\cos\phi\,\pi M}$$

$$LPC = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

$R_A = R + \text{addendum}$ (Always Module = 1)

$= 200 + 10 = 210$ MM

$r_A = 150 + 10 = 160$ MM

$$LPC = \sqrt{210^2 - 200^2\cos^2 20^\circ} + \sqrt{160^2 - 150^2\cos^2 20^\circ} - (200+150)\sin 20^\circ$$

$$\Rightarrow LPC = (93.7) + (75.7) - (119.7)$$

$$= \Rightarrow \boxed{LPC = 49.7\,mm}$$

$$\text{contact Ratio} = \frac{LPC}{\cos\phi \pi M} = \frac{49.7}{\cos 20 \times \pi \times 10} = 1.6 \simeq 2$$

$\boxed{CR = 2}$

(*) Minm. No. of teeth — Two gears Meshing with each other having Module 20 MM & no. of teeth on both the gear 60 & 40. check whether the interference will occur both gear or not. If the addendum is assume to be 10 MM. & pr. angle is 12°. If interference will occur what should be the pr. angle inorder to avoid the interference.

$$LPA = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$> R\sin\phi$

LPC → LPA, APR

Module (m) = 20 MM

$KP > MP \rightarrow$ Interference occur

$(T_a) = 60$

$(t_p) = 40$

Adendum = 10 MM

$\phi = 12^\circ$

We know length of path of approach (LPA)

$$= \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi$$

where $R_A$ = Radius of addendum circle on Gear wheel

= R + addendum = 600 + 10 = 610 MM

⇒ radius of addendum circle on pinion = $r_A$ = 400 + 10 = 410 MM

R = Radius of pitch circle on Gear wheel

We know Module (m) = $\frac{D}{T} = \frac{2R}{T}$

$$\Rightarrow R = \frac{Tm}{2} = \frac{60 \times 20}{2} = 600 \text{ MM}$$

r = Radius of pitch circle on pinion

$$(m) = \frac{2r}{t}$$

$$\Rightarrow r = \frac{mt}{2} = \frac{20 \times 40}{2} = 400 \text{ MM}$$

$$LPA = \sqrt{610^2 - 600^2 (\cos 20°)^2} - 600 \sin 20°$$

$$= 167 - 125 = 42 \text{ MM}$$

length of path of approach < R sin φ. So Interference will not occur.

$r\sin\phi = 84$

Q. (*) The number of teeth on each of the ~~wheel~~ two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.

Ans:- Pressure Angle ($\phi$) = 20° Involute tooth profile.

Module (m) = 6 mm ($T_w = t_p = 40$)

Arc of contact = 1.75 $P_c$

$P_c$ = circular pitch

We know circular pitch ($P_c$) = $\pi m$ = $\pi \times 6$ = 18.9 mm

$\therefore$ Length of Arc of contact = $1.75 P_c$ = $1.75 \times 18.9$ = 33 mm

$\therefore$ Length of path of contact = length of arc of contact $\times \cos\phi$

= $33 \times \cos 20$ = 31 mm

Let $R_A = r_A$ = radius of the addendum circle of each wheel

radius of pitch circle of each wheel $R = r = \frac{mT}{2} = \frac{6 \times 40}{2} =$ 120 mm.

We know length of path of contact

$$\Rightarrow 31 = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

$$\Rightarrow 31 = \sqrt{R_A^2 - 120^2(\cos 20)^2} + \sqrt{r_A^2 - 120^2(\cos 20)^2} - 120\sin 20$$

$$\Rightarrow 72^2 = R_A^2 - 12715 + r_A^2 - 12715$$

$(r_A = R_A)$

$$\Rightarrow 2R_A^2 = 30615$$

$\Rightarrow R_A$ = 124 mm.

$R_A$ = R + Addendum

$\Rightarrow$ Addendum = $R_A - R$ = 124 − 120 = 4 mm

Q. A pinion having 30 teeth drives a gear having 80 teeth.
(*) The profile of the gears is involute with 20° ps. angle, 12 MM Module and 10 MM addendum. Find the length of path of contact, arc of contact & contact ratio.

Ans:- Given data No. of teeth on pinion ($t_p$) = 30

No. of teeth on Gear ($T_\omega$) = 80

Involute tooth profile

Module (m) = 12 MM

addendum = 10 MM

pressure angle ($\phi$) = 20°

① LAP = ? ② LAC = ? ③ contact ratio = ?

⇒ Length of path of contact :-

let radius of pitch circle of pinion = $r$

we know $m = \frac{D}{t} = \frac{2r}{t}$

$\Rightarrow r = \frac{mt}{2} = \frac{12 \times 30}{2} = 180$ MM

and radius of pitch circle of gear = $R$

$\Rightarrow R = \frac{mT}{2} = \frac{12 \times 80}{2} = 480$ MM

∴ Radius of addendum circle of gear

$R_A = R +$ addendum

$= 480 + 10 = 490$ MM

∴ Radius of addendum circle of pinion

$r_A = r +$ addendum

$= 180 + 10 = 190$ MM.

∴ Length of path of contact

$$= \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

$$= \sqrt{490^2 - 480^2(\cos 20)^2} + \sqrt{190^2 - 180^2(\cos 20)^2} - (480 + 180)\sin 20$$

$$= 191.4 + 86.5 - 226 = 52 \text{ MM}$$

**LPC = 52 MM**

⇒ Length of Arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{52}{\cos 20}$$

**LAC = 55.66 MM**

⇒ contact ratio $= \dfrac{LAC}{P_c} = \dfrac{LAC}{\pi M} = \dfrac{55.66}{\pi \times 12} = 1.5$ say 2

**contact ratio = 2**

Q. Two involute gears of 20° pr. angle are in mesh. The no. of teeth on pinion is 20 & gear ratio is 2. If the pitch expressed in module is 5 MM and the pitch line speed is 1.2 M/s, assuming addendum as standard and equal to one module find.

1. The angle turned through by pinion when one pair of teeth is in mesh.
2. The Maxm. velocity of sliding.

Ans:- Given data. No. of involute gear = 2

pressure angle ($\phi$) = 20°

No. of teeth on pinion ($t_p$) = 20

Gear ratio(G) = $\frac{T}{t} = 2$

Module (m) = 5 $\Rightarrow 2t = T \Rightarrow (T_w) = 40$

pitch line speed (v) = 1.2 M/s . addendum = 1 Module = 5 MM

1. The angle turned through by pinion when one pair of teeth is in mesh :-

Let radius of pitch circle of pinion = r

$$r = \frac{mt}{2} = \frac{5 \times 20}{2} = 50 \text{ MM}$$

Let radius of pitch circle of gear = R

$$R = \frac{MT}{2} = \frac{5 \times 40}{2} = 100 \text{ MM}$$

Let radius of Addendum circle of Gear $= R_A$

$\Rightarrow R_A = R + \text{addendum} = 100 + 5 = 105$ mm

Let radius of addendum circle of gear $= r_A$

$\Rightarrow r_A = r + \text{addendum} = 50 + 5 = 55$ mm

We know Length of path of contact

$$LPC = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

$$= \sqrt{105^2 - 100^2(\cos 20^\circ)^2} + \sqrt{55^2 - 50^2(\cos 20^\circ)^2} - (100+50)\sin 20$$

$$= 47 + 29 - 52 = 24 \text{ mm}$$

Length of arc of contact

$$LAP = \frac{LPC}{\cos\phi} = \frac{24}{\cos 20} = 26 \text{ MM}$$

We know that angle turned through by pinion

$$= \frac{LAC \times 360°}{\text{circumference of Pinion}}$$

$$= \frac{26 \times 360}{2\pi \times 50} = 30°$$

2. Max^M. velocity of sliding

Let $\omega_1$ = Angular velocity of pinion

$\omega_2$ = " " " gear.

We know pitch line speed

$$v = \omega_1 . r = \omega_2 R$$

$$\Rightarrow \omega_1 = V/r = \frac{1200}{50} = 24 \text{ rad/sec}$$

$$\Rightarrow \omega_2 = \frac{V}{R} = \frac{1200}{100} = 12 \text{ rad/s}$$

∴ Max^M. velocity of sliding

$$V_s = (\omega_1 + \omega_2) PL = (24 + 12) 12.65 = 455.4 \text{ MM/s}$$

Length of path of approach $(LPA) = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$$= 12.65 \text{ mm}$$

$$\boxed{V_s = 455.4 \text{ MM/s}}$$

Q. Two Mating gears have 20 and 40 involute teeth of Module 10 MM and 20° pr. angle. The addendum on each wheel is to be made of such a length that the line of contact on each of the pitch point has half the Maxm. possible length. Determine the addendum height for each gear wheel, length of path of contact, arc of contact and contact ratio.

Ans:- Given data :- No. of teeth on pinion ($t_p$) = 20

No. of teeth on gear ($T_w$) = 40

Module (m) = 10 MM

Pressure angle ($\phi$) = 20°.

⇒ Addendum height for each gear wheel :-

Let r = Radius of pitch circle of pinion

We know ⇒ $r = \frac{mt}{2} = \frac{10 \times 20}{2} = 100$ MM

Let R = Radius of pitch circle of gear.

⇒ $R = \frac{MT}{2} = \frac{10 \times 40}{2} = 200$ MM

Let $R_A$ = Radius of addendum circle of pinion

$r_A$ = " " " " " gear

Since the addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point (i.e. path of approach & path of recess) has half the maxm. possible length therefore path of approach $KP = \frac{1}{2} MP$ $(MP = r\sin\phi)$

$\Rightarrow \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{r\sin\phi}{2}$

$\Rightarrow \sqrt{R_A^2 - (200)^2\cos^2 20} - 200\sin 20 = \frac{100\sin 20}{2}$

$\Rightarrow \sqrt{R_A^2 - 35320.8} - 68.40 = 17.1$

$\Rightarrow R_A^2 - 35320.8 = 85.5^2$

$\Rightarrow R_A^2 = 85.5^2 + 35320.8$

$\Rightarrow \boxed{R_A = 206.5 \text{ MM}}$

Now path of recess (PL) $= \frac{1}{2} PN$

$\Rightarrow \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$

$\Rightarrow r_A^2 - r^2\cos^2\phi = \left(r\sin\phi + \frac{R\sin\phi}{2}\right)^2$

$\Rightarrow r_A^2 = 100^2 \cos^2 20 + \left(100 \sin 20 + \frac{200 \sin^2 20}{2}\right)^2$

$\Rightarrow r_A^2 = 8830.2 + 4679.1 = 13509.3$

$\Rightarrow \boxed{r_A = 116.2 \text{ MM}}$

Addendum height for larger gear

$= R_A - R$

$= 206.5 - 200 = 6.5 \text{ MM}$

Addendum height for smaller gear i.e pinion

$= r_A - r$

$= 116.2 - 100 = 16.2 \text{ MM}$

$\Rightarrow$ Length of path of contact :-

$LPC = KP + PL = \frac{1}{2} MP + \frac{1}{2} PN$

$= \frac{1}{2}(MP + PN) = \frac{1}{2}(r\sin\phi + R\sin\phi)$

$LPC = \frac{(R+r)\sin\phi}{2} = \frac{(200+100)\sin 20}{2} = 51.3 \text{ MM}$

$\boxed{LPC = 51.3 \text{ MM}}$

$\Rightarrow$ Length of arc of contact :-

$LAC = \frac{LPC}{\cos\phi} = \frac{51.3}{\cos 20^\circ} = 54.6 \text{ MM}$

$\boxed{LAC = 54.6 \text{ MM}}$

$\Rightarrow$ contact Ratio :-

$CR = \frac{LAC}{P_c} = \frac{LAC}{\pi M} = \frac{54.6}{\pi \times 10} = 1.74$ say 2

$\boxed{CR = 2}$

Q. A pair of involute spur gears with 16° pr. angle & pitch of Module 6 MM is in Mesh. The no. of teeth on pinion is 16 and its rotational speed is 240 rpm. when the gear ratio is 1.75, find in order that interference is just avoided. 1. Addenda on pinion and gear wheel. 2. length of path of contact. 3. Maxm. velocity of sliding of teeth on either side of the pitch point.

Ans :- Given data $\phi = 16^\circ$,

$m = 6\ mm$, $(t_p) = 16$ & $N_1 = 240\ rpm$,

$G = 1.75 = \frac{T}{t}$.

1 → Addenda on pinion $= \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1$

$$= \frac{6 \times 16}{2}\left(\sqrt{1 + 1.75\,(1.75+2)\sin^2 16} - 1\right)$$

$$= 48 \times 0.224 = 10.76\ mm$$

2 → Addenda on Gear $= \frac{mT}{2}\left(\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right)$

$$= \frac{6 \times 28}{2}\left(\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16} - 1\right)$$

$$= 4.56\ mm$$

$T = 1.75t$
$= 1.75 \times 16$
$= 28$

2. Length of path of contact :-

Let radius of pitch circle of gear = R

$\Rightarrow R = \frac{MT}{2} = \frac{6 \times 28}{2} = 84$ MM

radius of pitch circle of pinion = r

$\Rightarrow r = \frac{Mt}{2} = \frac{6 \times 16}{2} = 48$ MM

Let $R_A$ = Radius of addendum circle of gear

$R_A = R +$ addendum $= 84 + 10.76 = 94.76$ MM

$r_A$ = radius of addendum circle of pinion

$r_A = r +$ addendum $= 48 + 4.56 = 52.56$ MM

we know length of path of approach (KP)

$$KP = \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi$$

$$= \sqrt{94.76^2 - 84^2 \cos^2 16} - 84 \sin 16$$

$$\boxed{KP = 26.45 \text{ MM}}$$

Length of path of recess (PL)

$$PL = \sqrt{r_A^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{52.56^2 - 48^2 \cos^2 16} - 48 \sin 16 = 11.94 \text{ MM}$$

∴ Length of path of contact (LPC) :-

$KL = KP + PL = 26.45 + 11.94 = 38.39\,mm$

$$\boxed{KL = 38.39\,mm}$$

3. Max^m. velocity of sliding of teeth on either side of pitch point :-

$\omega_1$ = Angular velocity of pinion

$\omega_2$ = " " " gear

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.13\ rad/s.$$

we know that $\frac{\omega_1}{\omega_2} = \frac{T}{t} \Rightarrow \omega_2 = \frac{\omega_1 t}{T} = \frac{25.13 \times 16}{28}$

$\Rightarrow \omega_2 = 14.36\ rad/s.$

∴ Max^m. velocity of sliding of teeth on the left side of pitch point i.e at point K.

$$= (\omega_1 + \omega_2) KP = (25.13 + 14.36)\, 26.45$$
$$= 1043\ mm/s$$

∴ Max^m velocity of sliding of teeth on the right side of pitch point i.e at point L.

$$= (\omega_1 + \omega_2) PL = (25.13 + 14.36)\, 11.94$$
$$= 471\ mm/s.$$

Q. A pair of spur gears with involute teeth is to give a gear ratio of 4:1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pr. is 14.5°. Find. 1. least number of teeth that can be used on each wheel & 2. the addendum of the wheel in terms of the circular pitch?

Ans :- Given data $G = 4:1 = \frac{T}{t}$

$\phi = 14.5°$

1. Least number of teeth on each wheel :-

Let $t$ = least no. of teeth on the smaller wheel i.e pinion

$T$ = least no. of teeth on the larger wheel i.e gear.

$r$ = pitch circle radius of pinion.

We know Max$^m$ length of arc of approach

$$= \frac{\text{Max}^m.\ \text{length of path of approach}}{\cos\phi}$$

$$= \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

& circular pitch $(P_c) = \pi M = \pi \times \frac{2r}{t} = \frac{2\pi r}{t}$

$(\because r = \frac{Mt}{2})$

Since the arc of approach is not to be less than circular pitch.

therefore $r\tan\phi = \frac{2\pi r}{t}$

$$\Rightarrow \tan\phi = \frac{2\pi}{t} \Rightarrow t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5^\circ}$$

$$\Rightarrow t = 24.3 \text{ say } 25$$

No. of teeth of pinion $\Rightarrow$ $\boxed{t = 25}$

No. of teeth of gear $T = Gt = 4 \times 25 = 100$

$\boxed{T = 100}$

2. The addendum of the wheel in terms of circular pitch

$$= \frac{mT}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1$$

$$= \frac{m \times 100}{2}\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5^\circ} - 1$$

$$= 50m \times 0.017 = 0.85m = 0.85 \times \frac{P_c}{\pi}$$

$(\because \pi m = P_c)$

$\boxed{A_w = 0.27 P_c}$

Q. The following data relate to a pair of 20° involute gear in mesh. Module = 6 MM, NO. of teeth on pinion = 17, ($T_w$) = 49, Addenda on pinion & gear = 1 Module. Find 1. No. of pairs of teeth in contact. 2. Angle turned through by the pinion & gear when one pair of teeth is in contact. 3. Ratio of sliding to rolling motion when the tip of a tooth on the larger wheel (i) is just making contact (ii) is just leaving contact (iii) is at the pitch point.

Ans:-

Let $r$ = radius of pitch circle of pinion

we know $\Rightarrow r = \frac{mt_p}{2} = \frac{6 \times 17}{2} = 51$

Let $R$ = radius of pitch circle of gear

$\Rightarrow R = \frac{mT}{2} = \frac{6 \times 49}{2} = 147$

Let $R_A$ = Radius of addendum circle of gear

$\Rightarrow R_A = R +$ addendum $= 147 + 6 = 153$ MM

Let $r_A$ = radius of addendum circle of pinion

$\Rightarrow r_A = r +$ addendum $= 51 + 6 = 57$ MM.

$LPC = KL = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$

$= \sqrt{153^2 - 147^2(\cos 20)^2} + \sqrt{57^2 - 51^2(\cos 20)^2} - (147 + 51)\sin 20^\circ$

$= 65.78 + 30.99 - 67.72 = 29$ MM

Length of Arc of contact $(LAC) = \frac{LPC}{\cos\phi}$

$\Rightarrow LAC = \frac{29}{\cos 20} = 30.86$ MM

contact ratio $= \frac{LPC}{P_c} = \frac{LPC}{\pi m} = \frac{29}{\pi \times 6} = 1.53 \simeq 2$

$\therefore$ No. of pairs of teeth in contact is 2.

2. Angle turned through pinion $= \frac{LAC \times 360^\circ}{2\pi \times 51} = \frac{30.86 \times 360}{2\pi \times 51} = 34.6^\circ$

Angle turned through gear $= \frac{LAC \times 360}{2\pi \times 147} = \frac{30.86 \times 360}{2\pi \times 147} = 12.02^\circ$

3. Ratio of Sliding to rolling Motion :-

Let $\omega_1$ = Angular velocity of pinion
$\omega_2$ = " " " gear wheel.

We know $\frac{\omega_1}{\omega_2} = \frac{T}{t} = \frac{49}{17} \Rightarrow \omega_2 = \frac{17\omega_1}{49} = 0.347\omega_1$

& rolling velocity $V_R = \omega_1 r = \omega_2 R$
$= \omega_1 \times 51 = 51\,\omega_1$ mm/s

(i) At the instant when the tip of a tooth on the larger wheel is just making contact with its mating teeth (i.e when the engagement commences), the sliding velocity $(V_s) = (\omega_1 + \omega_2)\,KP$

$= (\omega_1 + 0.347\omega_1) \times 15.5$

$= 20.88\,\omega_1$ mm/s

$\therefore \frac{V_s}{V_R} = \frac{20.88\omega_1}{51\omega_1} = 0.41$

(ii) when engagement terminates i.e disengagement

$V_s = (\omega_1 + \omega_2)\,PL$
$= (\omega_1 + 0.347\omega_1) \times 13.41 = 18.1\,\omega_1$ mm/s.

$\therefore \frac{V_s}{V_R} = \frac{18.1\,\omega_1}{51\,\omega_1} = 0.355$

(iii) Since at pitch point sliding velocity zero,
So ratio of $V_s / V_R$ is zero.

... involute teeth of