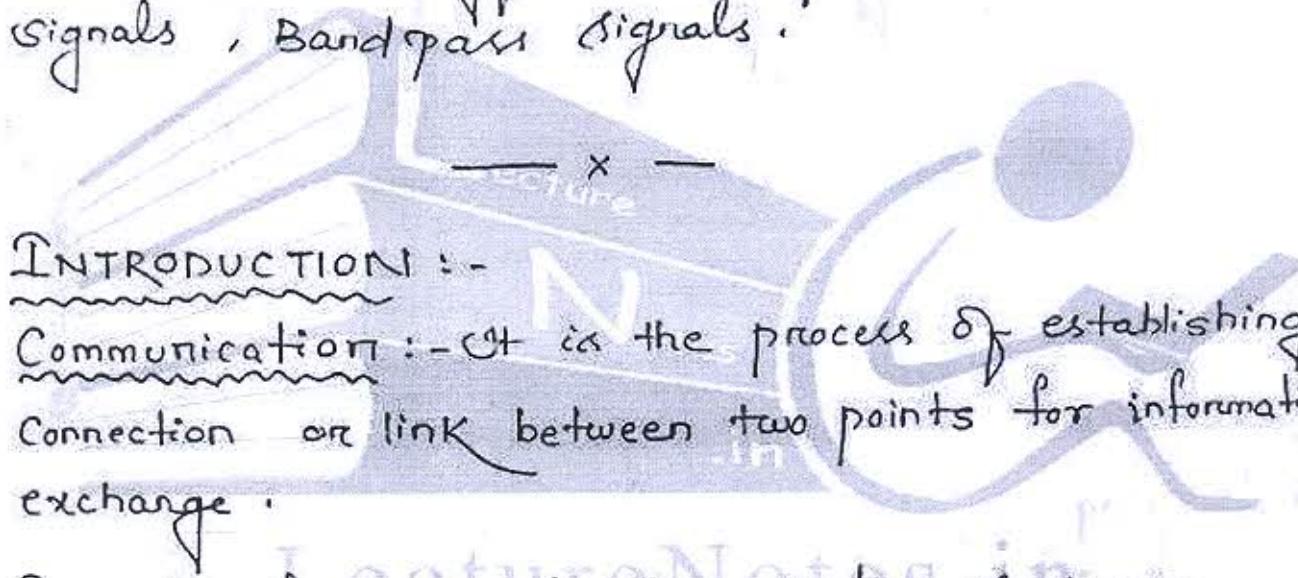


Communication Engineering MODULE - I

INTRODUCTION :- Elements of an Electrical Communication System, Communication channels and their characteristics, Mathematical Model for Communication channels.

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FREQUENCY DOMAIN ANALYSIS OF SIGNALS AND SYSTEMS :- Fourier Series, Fourier Transform, Power and Energy, Sampling and Bandlimited signals, Bandpass signals.



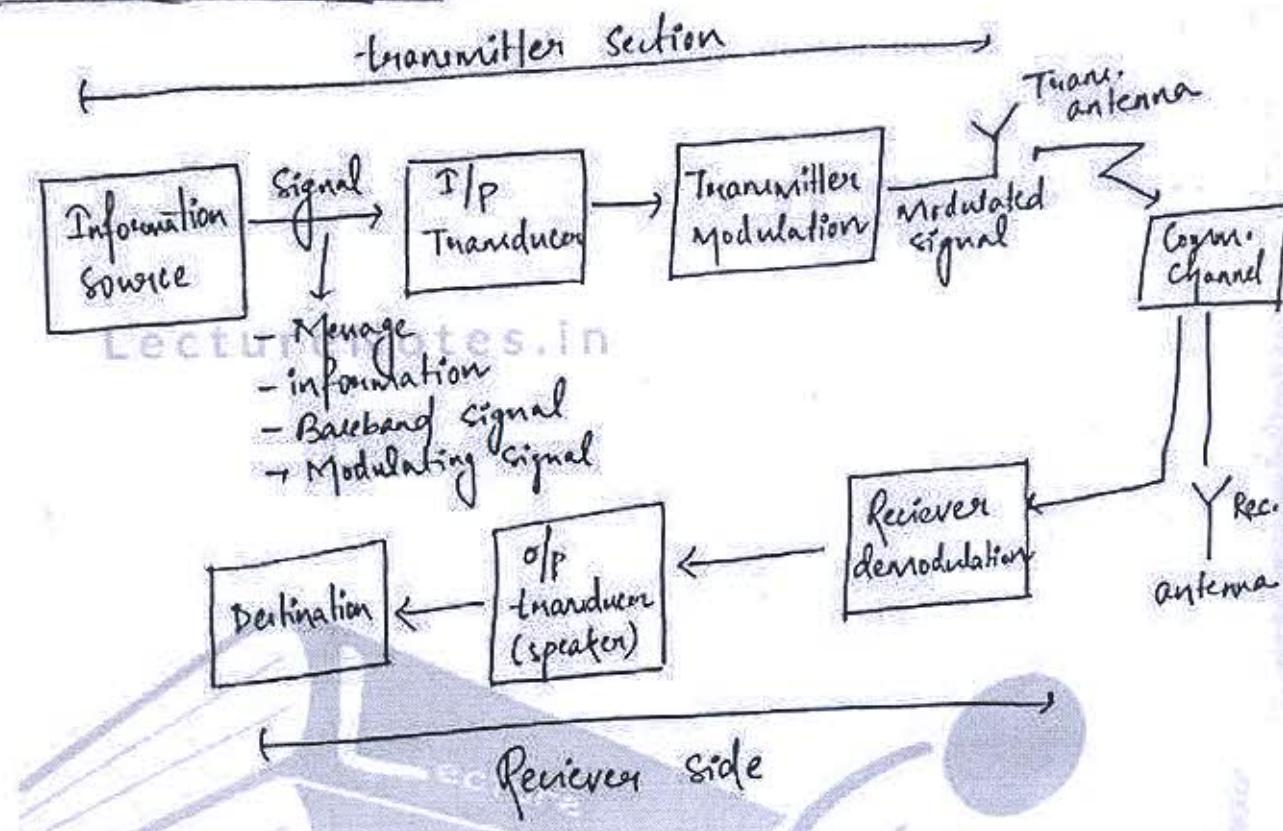
Elements of electrical Communication System :-

- 1) Information Source
 - 2) Input transducer
 - 3) Transmitter
 - 4) Communication channel
- } transmitter Section.

At receiver side :-

- Receivers demodulator
- Output Transducer.
- Destination.

BLOCK DIAGRAM :-



⇒ Information Source :- It generates the message signal or information signal which are present in analog form.

Ex:- speech signal / video signal.

⇒ Input transducer :- It converts the information to be transmitted to its electrical equivalent message signal.

⇒ Transmitter :- The transmitter modulates or changes some parameter of this signal like amplitude, frequency and phase. It also multiplexes or puts a no. of signals in a common pool and

Types of Communication :-

Two types of Communication - analog Communication
- Digital Communication.

→ Analog Communication :- In this communication, the message is in electrical form and is considered to be a continuously varied signal ~~but in digital~~

→ It doesn't require analog to digital converters for naturally occurring signals. So here conversion errors are less.

→ Required bandwidth is relatively less as well as cost components also.

Digital Communication :- The message is discrete in nature.

→ It can be used for long distance communication.
→ It is less affected by noise.

Communication Channels and its Characteristics:-

→ channel is a medium over which the information is passed from transmitter to receiver.

Depending on modes of transmission, we have

→ Guided propagation channel

→ free space propagation channel.

starts transmission of these input signals.

⇒ Channel :- channel is the media by which information is sent and it can be wired type lines such as copper wire or wireless like atmosphere.

The channel is affected by Noise, distortion and Attenuation.

Then the signal is received at receiver side
And at receiver side the elements present are:-

⇒ Receiver demodulator :- It receive the signal and extracts the intended information from it.
It also amplifies and remove the noise.

⇒ Output transducer :- It converts the electrical input into the form of the message as required by the User. The message can be speech, video, audio, image signal etc.

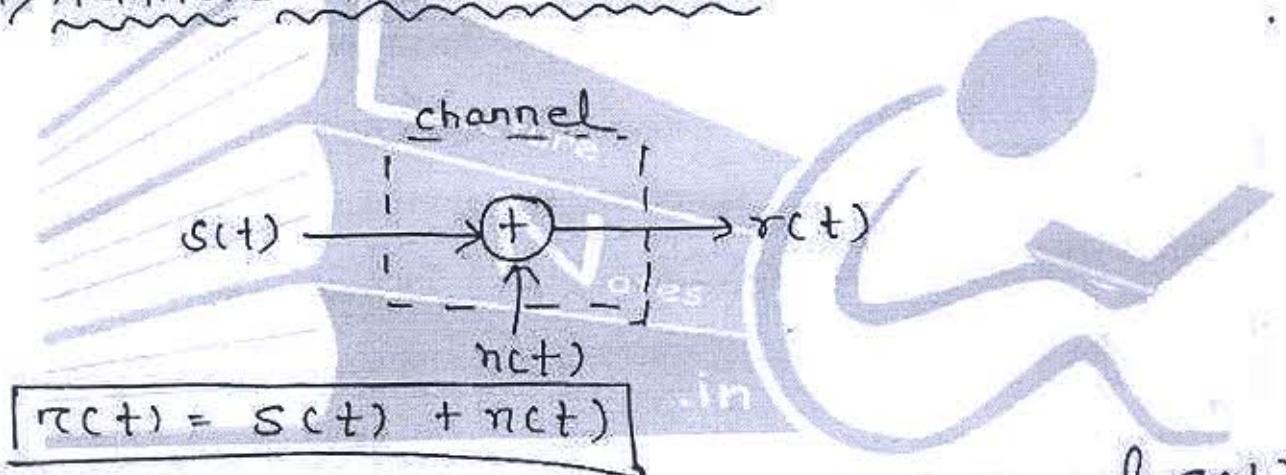
⇒ Destination :- It is the final stage which is used to convert an electrical message to its original form.

Mathematical Model for Communication channel :-

→ The mathematical models for different types of communication channels are required to design proper channel encoder and modulator at transmitter side and demodulator at receiver side.

3 Models :-

i) Additive Noise channel :-



→ In this model, the transmitted signal $s(t)$ is corrupted by an additive random noise process $n(t)$. The additive noise arises from electronic components and amplifiers at the receiver end or from the interference encountered in the transmission.

Here the noise is Additive White Gaussian Noise.

Factors affecting the channel :-

- i) Power required to achieve the desired signal to noise ratio (SNR).
- ii) Bandwidth of channel.
- iii) Amplitude and phase response of a channel.
- iv) Types of channel.
- v) Effect of external interference on the channel.

Guided Propagation channel :-

① Open wire lines :- for ex. Telephone, Telegraph line, twisted pair, Coaxial Cables, optical fibre cables.

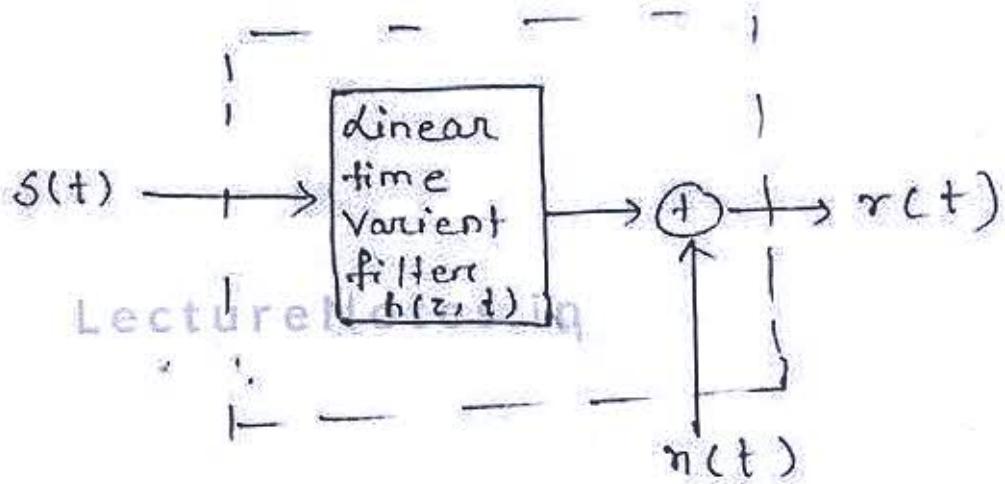
- In guided propagation electrical interferences are avoided.
- Easily available, as well as low cost.

Unguided Propagation channels :-

- Radio Wave :- It is a wireless propagation where radio frequency signals are generated and radiated into free space.
- Wave guide :- This is a hollow conductor cross-section, typically feeding or receiving signal from transmitter.

It offers very high bandwidth.

- Radio frequency spectrum is also a freespace propagation channel.



$$r(t) = s(t) * h(z, t) + n(t)$$

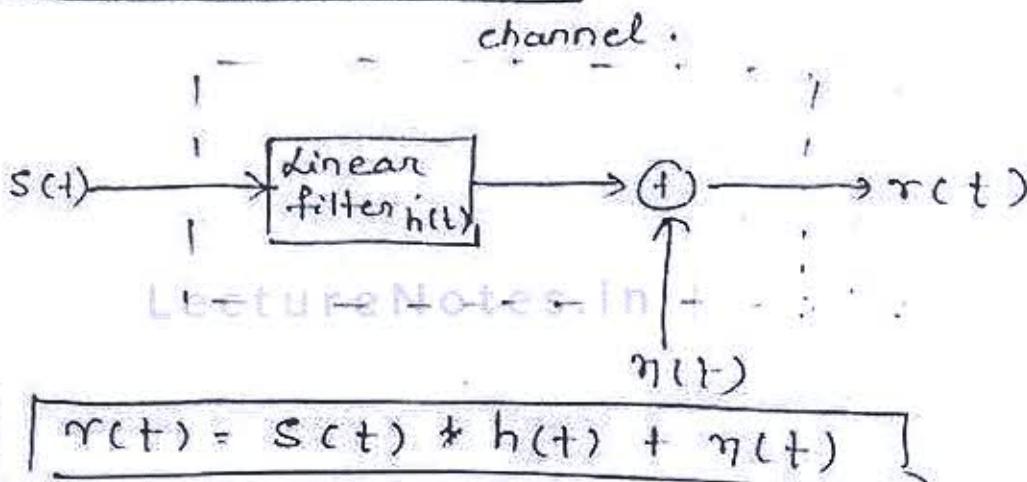
ORTHOGONALITY

Two function $f_1(t)$ and $f_2(t)$ are said to be orthogonal over time interval t_1 and t_2 if $\int_{t_1}^{t_2} f_1(t) f_2(t) dt = 0$.

Significance :- If two function are orthogonal then they are said to be mutually exclusive i.e. one function doesn't contain any component of the other function or we can say that f_1 and f_2 are two independent functions.

Orthonormal $\Rightarrow \int_{t_1}^{t_2} f_1(t) f_2(t) dt = 1$

i) Linear - filter channel



In wireline telephone channels, filters are used to ensure that the transmitted signals do not exceed the specified bandwidth and do not interfere with one another. These channels are generally characterized as linear filter channel with additive noise.

iii) Linear time Variant filter Communication

Channel Model :

→ In this channel, an effect is introduced which is popularly known as multipath propagation.

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→ The multipath propagation is characterised by a linear time Variant filters.

→ example : wireless channel and under water aquatic channel .

Q. Test the orthogonality of following set of functions :

i) $\cos m\omega_0 t$

ii) $\cos n\omega_0 t$

$t_1 = t_0$

$t_2 = t_0 + T$

$T = \text{Time period}$

$= 2\pi/\omega_0$

$$\int_{t_1}^{t_2} \cos m\omega_0 t \cos n\omega_0 t dt$$

$$= \int_{t_0}^{t_0+T} \frac{1}{2} 2 \cos m\omega_0 t \cos n\omega_0 t dt$$

$$= \frac{1}{2} \int_{t_0}^{t_0+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t] dt$$

$$= \frac{1}{2} \left[\int_T^T \cos(m-n)\omega_0 t dt + \int_T^T \cos(m+n)\omega_0 t dt \right]$$

$$= \frac{1}{2} \left[\frac{\sin(m-n)\omega_0 T}{(m-n)\omega_0} + \frac{\sin(m+n)\omega_0 T}{(m+n)\omega_0} \right]$$

$$= 0 \quad \text{if } m \neq n. \quad \left[\frac{\sin(m-n)\omega_0 \times 2\pi}{(m-n)\omega_0} \times \frac{2\pi}{\omega_0} = 0 \right]$$

$$\frac{1}{2} \left[0 + \frac{\sin 2m\omega_0 T}{(m+n)\omega_0} \right] = \frac{1}{2} [0+0] = 0$$

If $m \neq n$ $\cos m\omega_0 t$ & $\cos n\omega_0 t$ are orthogonal.

Limitation of Communication System :-

→ 3 types of limitations :-

① Noise limitation

② Bandwidth limitation

③ Equipment limitation

Remember :-

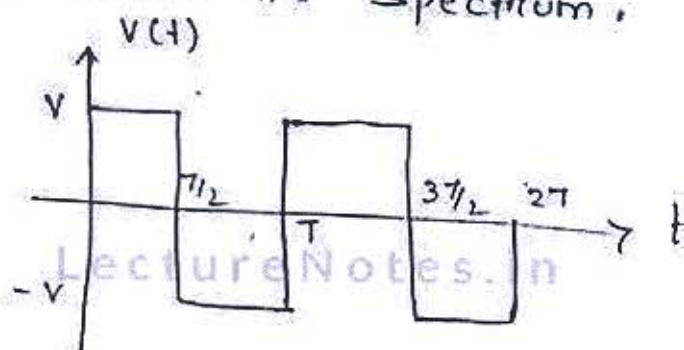
→ The highest modulating frequency used in AM broadcast system is 5 kHz.

→ Major communication medium are water, wire. But free space is not a major communication medium.

⇒ The process of transmitting two or more information signal simultaneously over the same channel is called Multiplexing.

⇒ Noise is an unwanted signal which tend to interfere with required signal.

Q. Find the Fourier series of given signal
and find its spectrum.



$$V(t) = \begin{cases} V & 0 < t < T/2 \\ -V & T/2 < t < T \end{cases}$$

$$\text{Soln. } a_0 = \frac{1}{T} \int_0^T V(t) dt$$

$$a_0 = \frac{1}{T} \left[\int_0^{T/2} V dt - \int_{T/2}^T V dt \right]$$

$$= \frac{1}{T} [V(T/2 - T/2)]$$

$$a_0 = 0$$

$$a_n = \frac{2}{T} \int_0^T V(t) \cos n\omega_0 t dt$$

$$= \frac{2V}{T} \left[\int_0^{T/2} \cos n\omega_0 t dt - \int_{T/2}^T \cos n\omega_0 t dt \right]$$

$$= \frac{2V}{T} \left[\left[\frac{\sin n\omega_0 t}{n\omega_0} \right]_0^{T/2} - \left[\frac{\sin n\omega_0 t}{n\omega_0} \right]_0^T \right]$$

$$= \frac{2V}{T} \left[\frac{\sin n\pi}{n\omega_0} - \frac{\sin 2n\pi - \sin n\pi}{n\omega_0} \right]$$

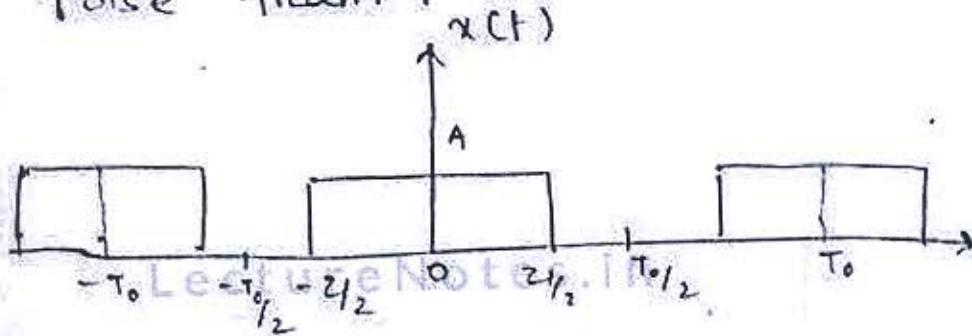
$$= \frac{2V}{T} \left[\frac{n\sin n\pi - \sin 2n\pi}{n\omega_0} \right]$$

$$= 0$$

$$\begin{aligned}
 b_n &= \frac{2V}{T} \int_0^T v(t) \sin n\omega_0 t \, dt \\
 &= \frac{2V}{T} \left[\int_0^{T/2} V \sin n\omega_0 t \, dt - \int_{T/2}^T \sin n\omega_0 t \, dt \right] \\
 &= \frac{2V}{T} \left[\left\{ \frac{-\cos n\omega_0 t}{n\omega_0} \right\} \Big|_0^{T/2} + \left\{ \frac{\cos n\omega_0 t}{n\omega_0} \right\} \Big|_{T/2}^T \right] \\
 &= \frac{2V}{T} \left[-\frac{\cos n\pi}{n\omega_0} + 1 + \frac{\cos 2n\pi - \cos n\pi}{n\omega_0} \right] \\
 &= \frac{2V}{T} \left[\frac{1 - 2\cos n\pi + \cos 2n\pi}{n\omega_0} \right] \\
 &= \frac{2V}{2\pi n} [2 - 2\cos n\pi] \\
 &= \frac{2V}{n\pi} (1 - \cos n\pi) \\
 b_1 &= \frac{4V}{\pi}, \quad b_2 = 0, \quad b_3 = \frac{4V}{3\pi}, \quad b_4 = 0 \\
 b_8 &= \frac{4V}{5\pi}
 \end{aligned}$$

$$\begin{aligned}
 v(t) &= c_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \\
 &= \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \\
 &= b_1 \sin \omega_0 t + b_3 \sin 3\omega_0 t + b_5 \sin 5\omega_0 t + \dots \\
 &= \frac{4V}{\pi} \sin \omega_0 t + \frac{4V}{3\pi} \sin 3\omega_0 t + \dots \\
 &= \frac{4V}{\pi} [\sin \omega_0 t + \sin 3\omega_0 t + \dots]
 \end{aligned}$$

② Find the Fourier series expansion of rectangular Pulse train.



$$x(t) = \begin{cases} A & -\frac{T_0}{2} \leq t \leq \frac{T_0}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$A_0 = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) dt = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A dt = A \frac{T_0}{2}$$

$$\boxed{A_0 = \frac{A T_0}{2}}$$

$$A_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cos n \omega_0 t dt$$

$$= \frac{2}{T_0} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A \cos n \omega_0 t dt = \frac{2A}{T_0} \left[\frac{\sin n \omega_0 t}{n \omega_0} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{4A}{T_0} \frac{\sin n \omega_0 \frac{\pi}{2}}{n \omega_0} = \frac{4A}{2\pi n} \sin n \frac{2\pi}{T_0} \frac{\pi}{2}$$

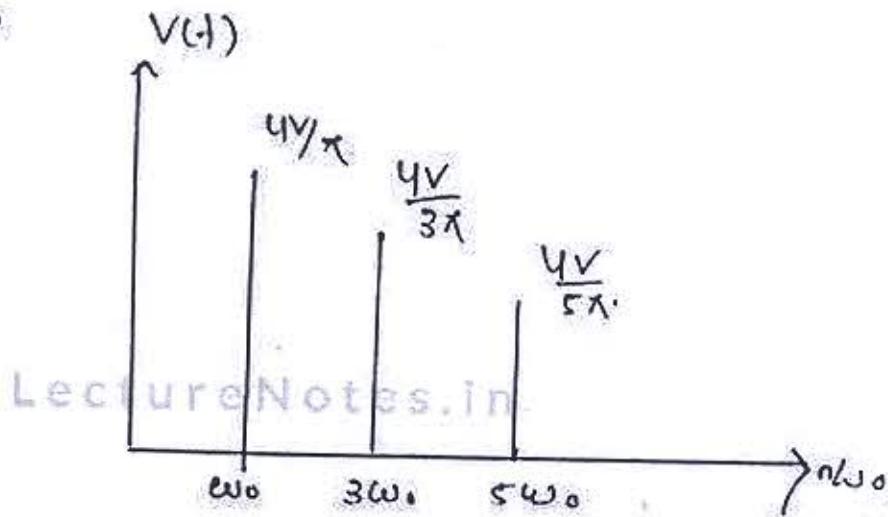
$$= \frac{2A}{\pi n} \sin \left(\frac{n\pi}{T_0} \right)$$

$B_n = 0$ as even symmetric.

Now

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos n \omega_0 t$$

$$= A \frac{T_0}{2} + \sum_{n=1}^{\infty} 2A \frac{\pi}{T_0} \sin \left(\frac{n\pi}{T_0} \right) \cos n \omega_0 t$$



Q. Find the Fourier Series of $\cos 3t - 10 \sin(\frac{\pi}{5}t)$ given.

$$\omega T = 2\pi \Rightarrow T = 2\pi/\omega_0$$

$$T_1 = \frac{8\pi}{3}, \quad T_2 = \frac{2\pi \times 5}{\pi}$$

$$T_1 = 2\pi/3, \quad T_2 = 10$$

$$\frac{\omega_1}{\omega_2} = \frac{3}{\pi} \times 5 = \frac{15}{\pi} \text{ (not rational)}$$

So, time period can't be found.

Since the signal is not periodic its Fourier Series can't be found.

Q. find the exponential Fourier series coefficients and harmonics present in the given signal.

$$x(t) = -10 \cos 15t - 14 \sin\left(\frac{3}{5}t - 30^\circ\right) + 15 \sin(0.3t)$$

Sol: Find fundamental time period.

$$\omega_1 = 15 \quad \omega_2 = \frac{3}{5} \quad \omega_3 = 0.3$$

$$\frac{\omega_1}{\omega_2} = \frac{15}{\frac{3}{5}} = 25 \quad (\text{rational})$$

$$\frac{\omega_2}{\omega_3} = \frac{3}{5 \times 0.3} = 2 \quad (\text{rational})$$

If rational, then fundamental time period can be found.

If not rational, then can't be found.

$$\text{fundamental frequency } \omega_0 = \frac{\text{GCD}(15, 3, 3)}{\text{LCM}(1/5, 1/10)}$$

$$\omega_0 = \frac{3}{10} = 0.3$$

Now to find exponential Fourier series we have to convert or express $x(t)$ to exponential form.

$$x(t) = -10 \underbrace{\left(e^{j15t} + e^{-j15t} \right)}_{2} - 14 \underbrace{\left[e^{j(\frac{3}{5}t - 30^\circ)} - e^{-j(\frac{3}{5}t - 30^\circ)} \right]}_{2j} + 15 \left(e^{j0.3t} - e^{-j0.3t} \right)$$

$$x(t) = -5e^{j15t} - 5e^{-j15t} + \frac{7}{j} e^{j(\frac{3}{5}t - 30^\circ)} +$$

$$\frac{7}{j} e^{-j(\frac{3}{5}t - 30^\circ)} + \frac{7.5}{j} e^{j0.3t} - \frac{7.5}{j} e^{-j0.3t}$$

fundamental frequency

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so harmonic

$\omega = 3\pi$

$$V_0 = 0$$

$$V_1 = \frac{7.5}{j}$$

$$V_{-1} = -\frac{7.5}{j}$$

$$V_2 = -\frac{7}{j} e^{j30^\circ}$$

$$V_{-2} = \frac{7}{j} e^{-j30^\circ}$$

$$V_3 = V_4 = V_5 = \dots = 0$$

$$V_{-3} = V_{-4} = V_{-5} = \dots = 0$$

Now we get

$$V_0 = 0 \quad V_{50} = -5 \quad V_{-50} = -5$$

$$V_2 = -\frac{7}{j} e^{-j30^\circ}, \quad V_{-2} = \frac{7}{j} e^{-j30^\circ}$$

$$V_1 = \frac{7.5}{j} \quad V_{-1} = -\frac{7.5}{j}$$

Harmonics are $n = 1, 2, 50$

Properties of Fourier Series :-

i) Linearity Property :-

If $x(t) \longleftrightarrow C_{xn}$. $C_{xn} \Rightarrow$ Fourier Series

and $y(t) \longleftrightarrow C_{yn}$ Coefficient

then $LectureNotes.in$

$z(t) = a x(t) + b y(t) \longleftrightarrow C_{zn}$

$$C_{zn} = a C_{xn} + b C_{yn}$$

ii) Time Shifting Property :-

It states that if $x(t) \longleftrightarrow C_{xn}$

then $x(t - t_0) \longleftrightarrow e^{-j\omega_0 t_0} C_{xn}$

iii) Frequency Shifting Property :-

If $x(t) \longleftrightarrow C_{xn}$

then $e^{j\omega_0 t} x(t) \longleftrightarrow C_{x(n-n_0)}$

iv) Time scaling Property :-

Let consider a signal $x(t)$ having Period T_0

$$x(t) \rightarrow T_0$$

then a time scaled signal is represented

as $x(\alpha t) \rightarrow \frac{T_0}{\alpha}$

α : +ve real no known

as time scaled Parameter.

Note :- With time scaling property only time period changes not the fourier coefficients.

$$x(t) \longleftrightarrow C_{xn}$$

$$x(\omega t) \longleftrightarrow C_{zn}$$

v) Time differentiation Property :-

If states that if $x(t) \longleftrightarrow C_{xn}$

then $\frac{d}{dt} x(t) \longleftrightarrow \frac{j2\pi n}{T_0} C_{xn}$

vi) Time Integration Property :-

If $x(t) \longleftrightarrow C_{xn}$

$$\int_0^t x(t) dt \longleftrightarrow \frac{C_{xn}}{\frac{j2\pi n}{T_0}} = \frac{T_0}{j2\pi n} C_{xn}$$

vii) Convolution Property :-

If states that if $x(t) \longleftrightarrow C_{xn}$

and $y(t) \longleftrightarrow C_{yn}$

then $z(t) = x(t) * y(t) \longleftrightarrow C_{zn}$

$$C_{zn} = T_0 C_{xn} C_{yn}$$

Note:- we can observe that the Convolution between two signals in time domain is equivalent to multiplication of two signals in frequency domain.

Viii) Multiplication Property :-

$$\text{If } x(t) \longleftrightarrow C_{xn}$$

$$y(t) \longleftrightarrow C_{yn}$$

$$z(t) \longleftrightarrow x(t) y(t) = C_{zn} = T_o [C_{xn} \otimes C_{yn}]$$

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ix) Symmetry Property :-

It states that if input signal $x(t)$ is a real signal then

$$C_n^* = C_{-n}$$

$$\text{If imaginary signal } \Rightarrow C_n^* = -C_{-n}.$$

x) Time Integration Property :-

$$\int x(t) dt = \int \sum_{n=-\infty}^{\infty} V_n e^{jn\omega_0 t} dt$$

$$= \sum_{n=-\infty}^{\infty} V_n \int e^{jn\omega_0 t} dt$$

$$= \sum_{n=-\infty}^{\infty} V_n \frac{1}{jn\omega_0} e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} V'_n e^{jn\omega_0 t}$$

$$V'_N = \frac{V_N}{jn\omega_0}$$

xi) Time derivation Property :-

$$\frac{d x(t)}{dt} = \frac{d}{dt} \sum_{n=-\infty}^{\infty} V_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{d}{dt} V_n e^{jn\omega_0 t}$$

$$\sum_{n=-\infty}^{\infty} V_n e^{jn\omega_0 t} \cdot (j n \omega_0)$$

$$= \sum_{n=-\infty}^{\infty} (V_n j n \omega_0) e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} V'_n e^{jn\omega_0 t}$$

$$\boxed{V'_n = V_n j n \omega_0}$$

FOURIER TRANSFORM :-

Fourier representation of a signal in exponential form, we have

$$x(t) = \sum_{n=-\infty}^{\infty} V_n e^{jn\omega_0 t} \quad \text{--- (1)}$$

$$V_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

Fourier Series

→ Fourier series applied to periodic signals.

Spectrum is
→ Discontinuous in nature

Fourier transform

→ Fourier transform can be applicable for non-periodic signals

Spectrum is
→ Continuous in nature