PROBABLE Questions & Answers (Part-II)

Subject - ADC

Semester - 5th

Branch - ETC

Module - ALL

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PROBABLE Q&A (ADC 5th ETC) (All Modules)

SHORT QUESTIONS:

1.a State the conditions under which a given signal is fourier transformable. What is the Fourier transform of an eternal sinusoid?

Ans: For a signal to be fourier transformable, the signal has to satisfy Dirichlet's condition given below.

i) The signal is a single valued function with a finite no of maxima & minima & a finite number of discontinuities in any finite time interval.

ii) The function/signal is absolutely integrable. So, if $x(t)$ is a signal, then mathematically,

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

Fourier Transform of An Eternal Sinusoid:

→ Eternal sinusoidal signal means the sinusoidal input signals applied to the system at $n=-\infty$. The fourier transform can be found out by Duality.

$$\delta(t-T) \longleftrightarrow e^{-j\omega T}$$

By duality,

$$e^{-jtT} \longleftrightarrow 2\pi\,\delta(\omega+T)$$

Replacing $T \to \omega_0$

$$\boxed{e^{-j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega+\omega_0)}$$

$$x(t) = x(t+T) = \sum_{K=-\infty}^{\infty} a_K e^{j\frac{2\pi}{T}Kt} \xleftrightarrow{CTFS} \{a_K\}$$

$$x(t) = x(t+T) = \sum_{K=-\infty}^{\infty} a_K e^{j\frac{2\pi}{T}KT} \xleftrightarrow{CTFT} \sum_{K=-\infty}^{\infty} 2\pi a_K\, \delta\left(\omega - \frac{2\pi}{T}K\right)$$

b) Give Examples of two orthonormal functions. Prove these orthonormality.

Ans: Two functions are said to be orthonormal functions if they will satisfy the following criteria.

$$\int_a^b x_1(t)\, x_2(t)\, dt = 0 \rightarrow \text{orthogonal}.$$

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$$\frac{W_0 T_0}{4} = \frac{2\pi}{4} = \pi/2, \quad \frac{n\omega_0 T_0}{4} = \frac{n\pi}{2}$$

$$C_n = \frac{V}{2}\left\{\frac{\sin n\pi/2}{(n\pi/2)}\right\}$$

$C_0 = V/2$

$c_n = 0$, for n = even

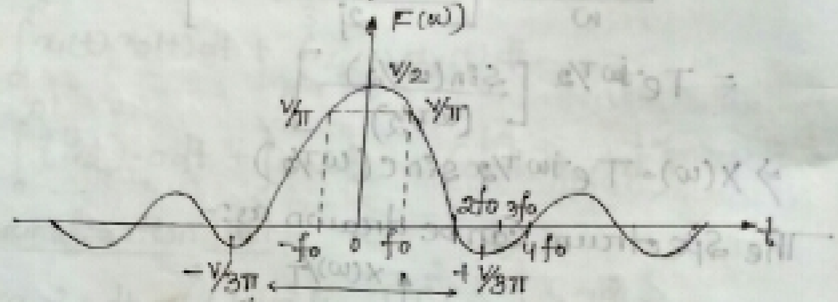
$$\frac{n\omega_0 \tau}{2} = n2\pi f_0 (T_0/2)$$

$$\Rightarrow C_n = \frac{V}{n\pi}(-1)^{n-1/2} \rightarrow n(\text{odd}) = n\pi f_0 T_0$$

Thus, $f(t) = \frac{V}{2}\sum_{n=\infty}^{\infty}\left\{\frac{\sin(n\pi/2)}{(n\pi/2)}\right\} e^{jn\omega_0 t}$

$$= V/2 + \sum (-1)^{n-1/2}\left(\frac{2V}{n\pi}\right)\cos n\omega_0 t$$

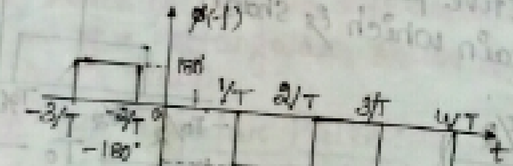
The spectrum $F(\omega) = \sum_{n=-\infty}^{\infty} C_n \,\delta(f - nf_0)$

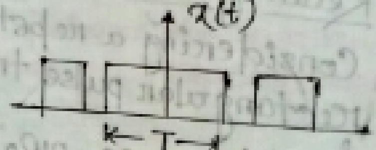


d) Draw the phase spectrum of a rectangular pulse train. What information does it convey?

Ans: Phase Spectrum of a Rectangular pulse Train:

Considering x(t) as a rectangular pulse train showing here, The phase spectrum can be drawn as follows:-





[Phase Spectrum of A rect (t/T)]

→ Regarding this phase spectrum, sinc(fT) is real. However, it can be bipolar, During the interval $m/T < (f) < m+1/T$, with m odd since (fT) is negative as the magnitude spectrum is always positive, negative values of sinc (fτ) are taken care of by marking $\angle X(f) = \pm 180°$ for the appropriate ranges, as shown in the figure above.

e) Give the low pass to band pass transformation of a signal & sketch it.

Ans <u>Low Pass to Band pass Transformation of a signal:</u>

Transformation $\Omega = 0$ (dc) transforms to $\omega = \omega_0$ (geometric center of pass band). Every frequency is transformed into two frequencies whose geometric mean is ω_0 i.e. if there is a peak at a peak in the prototype respone, the transformed response has two peaks at ω_{p_1} & ω_{p_2} where $\boxed{\omega_{p_1}\omega_{p_2} = \omega_0^2}$

$$\frac{s}{\Omega_p} \longleftrightarrow \frac{1}{\omega_b}\,\frac{s^2+\omega_0^2}{s}$$

$$j\Omega \longleftrightarrow j\frac{\Omega_p}{\omega_b}\,\frac{\omega^2-\omega_0^2}{\omega}$$

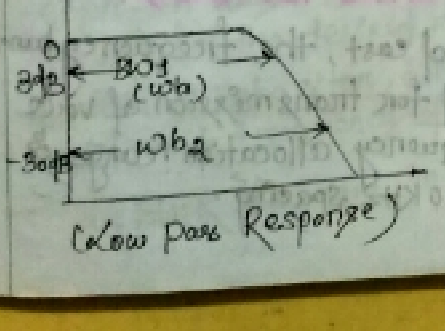
Where, $\omega_b \rightarrow$ Band width, the width of the passband $|\omega_{p_1} - \omega_{p_2}|$ within which the attenuation is less than A_p.

→ Every complex conjugate pole pair $P_r \pm jP_i$ is transformed into two complex conjugate poles pair $P_{r_1} \pm jP_{i_1}$ & $P_{r_2} \pm jP_{i_2}$ both of which have the same quality factor Q. The quality factor of the transformed pole pair increases as the ratio ω_0/ω_b increases.

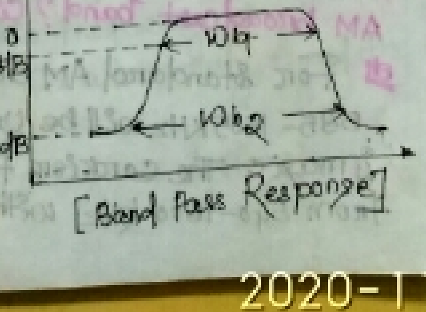
→ Every Real pole 'P' is transformed into complex conjugate pole pair $P_r \pm jP_i$

→ Resulting filter has $N = 2n$ poles. The order is doubled.

→ The sketch of the individual responses can be drawn as follows:



(Low Pass Response)



[Band Pass Response]

f) What is a pilot tone? Why is it used?

Ans: Pilot Tone/Carrier:-

→ A small amount of carrier signal known as pilot carrier is transmitted along with the modulated signal from the transmitter. This small amount of carrier signal is known as pilot carrier.

→ This pilot carrier, separated at the receiver by an appropriate filter is amplified & is used to phase lock the locally generated carrier at the receiver. This phase locking provides synchronization.

g) What is a Balanced in a balanced Modulator?

Ans: Balanced in a balanced Modulator means, two non-linear devices are connected in Balanced mode, so as to suppress the carrier wave. Balanced Modulator is used to represent DSB-SC signal.

h) Is FM a linear Modulation scheme? Justify.

Ans: No, FM is not a linear modulation scheme. Considering the equation of FM signal.

$S(t) = A \cos [2\pi f_c t + k_f \int x(t)\, dt]$

Now, if s(t) is replaced with a sum of signals, the resulting FM wave is not the sum of the individual FM waveforms as the cosine function is not linear.

$\cos(A+B) \neq \cos A + \cos B$

So,

$A \cos (2\pi f_c t + k_f \int [S_1(t) + S_2(t)\, dt]) \neq A\cos[2\pi f_c t + k_f \int S_1(t)dt] + A\cos[2\pi f_c t + k_f \int S_2(t) dt]$

Hence, FM is a non-linear Modulation scheme.

i) Which part of the spectrum is used as the standard AM broadcast band? Cite at least two Reasons.

Ans: For standard AM broadcast, the frequency band 535-1605 kHz will be used for transmission of voice & music. The carrier frequency allocation range is from 540-1600 kHz with 10 KHz spacing.

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The Reason for choosing this is:-
i) Radio station uses Conventional AM for signal transmi-ssion.
ii) Limited Band width of 5KHz.
iii) Economic.

i) What is the physical reason for making a slope detector being used as an FM detector?

A: Slope Detector acts as the simplest form of demodulating FM signals whose principle of operation depends on the slope of the frequency response c/s of a frequency selec-tive n/w. So it consists of a tuned ckt that is tuned to a frequency slightly offset from the carrier of the signal.

Operation:- As the frequency of the signals varies up & down in frequency according to its modulation, so the signal moves up & down the slope of the tuned circuit. This causes the amplitude of the signal to vary within the frequency variations.

LONG QUESTION:-

2. a) Explain the need of an RF amplifier as the front end in a standard AM broadcast receiver with suitable example as it essential?

A: Standard AM Broad Cast Receiver:-

→ The receiver most commonly used in AM radio broad cast is called super heterodyne receiver, which consists of (RF) radio frequency tuned amplifier, a mixer, a local oscillator, an intermediate frequency (IF) amplifier, an envelope det-ector, an audio frequency amplifier & a loud speaker. Tuning for the desired radio frequency is provided by a variable capacitor, which simultaneously tuned the RF amplifier & frequency of LO.

→ The Block diagram can be shown as follows:-

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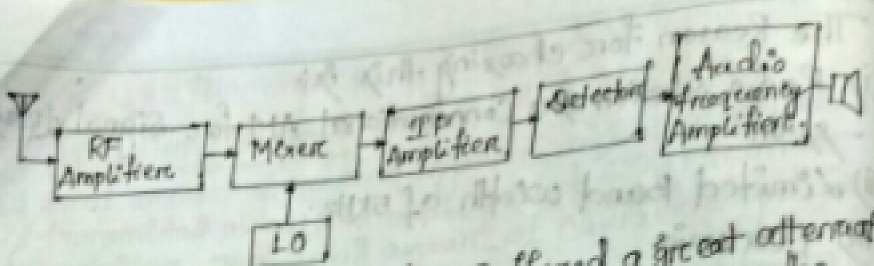

→ Assume that, the signal has suffered a great attenuation during the course of the transmission over the communi-cation channel & hence is in need of amplication. The input to the system might be a signal furnished by a receiving antenna which receives of the received signal is called a radio frequency (RF) carrier & its frequency is the radio frequency fcf. So, the input signal is amplified & passed on to the mixer after amplification from the RF amplifier.

Need of RF Amplifier:-

→ RF amplication is employed whenever the incoming RF signal is very small. This is because of the fact that RF amplifiers, such as, MASERs (Microwave amplification by Simulated emission of Radiation) are low noise device i.e. an RF amplifier can be designed to provide relatively high gain while generating relatively little noise.

→ When RF amplification is not emplyed, the signal is applied directly to the mixer. The mixer provides relatively little gain & generates a relatively large noise power. So. RF amplifier is the essential component in AM standard receiver.

b) An angle Modulated signal has the form $U(t) = 100\cos[2\pi f_c t + 4\sin 2\pi f_m t]$ Where $f_c = 10MHz$, $f_m = 1KHz$. Assuming this to be an FM signal, determine the Modulation index & the transmitting signal bandwidth. Give the expression for the corresponding phase Modulated Signal.

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3. An angle Modulated signal has the form:
$u(t) = 100 \cos [2\pi f_c t + 4 \sin 2\pi f_m t]$
Where, $f_c = 10$ MHz & $f_m = 1$KHz.
This is an FM signal. So,
$u(t) = 100 \cos [2\pi (10\times10^6) t + 4 \sin 2\pi (1\times10^3) t]$
Comparing this with expression of FM signal
$v(t) = A \cos [2\pi f_c t + m_f \sin \omega_m t]$
So, comparing the two equations.
i) Modulation index = $\boxed{m_f = 4}$
ii) Transmitted signal Band width can be found out using Carson's Rule So, $BW = 2(\Delta f + f_m)$
$\Rightarrow BW = 2 f_m (m_f + 1)$
So, putting the values,
$BW = 2\times1(4+1) = 2\times5 = \boxed{10 KHz}$
As, $m_f = 4 \Rightarrow m_f = \frac{\Delta f}{f_m} = 4 \Rightarrow \frac{K_f \cdot |x(t)|_{max}}{f_m} = 4$
$\Rightarrow \frac{K_f \cdot V_m}{f_m} = 4$
For PM signal, $k_p V_m = 4$
So, comparing $K_f V_m = 4 \times f_m = k_p \cdot V_m$.
So, $k_p V_m = 4 \times 1KHz = 4000$.
So, the PM signal can be written as:
$V(t) = A \cos [\omega_c t + k_p V_m \cos \omega_m t]$
$\Rightarrow V(t) = 100 [\cos 2\pi f_c t + 4000 \cos 2\pi f_m t]$
$\Rightarrow V(t) = 100 [\cos 2\pi (10\times10^6) t + 4000 \cos 2\pi (1\times10^3) t]$
So this equation Represents PM signal.

3a) Explain the need for an interlaced scanning

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b) Let a carrier be given by $c(t) = 10\cos(2\pi f_c t)$ & the message signal be $\cos(20\pi t)$. The message signal frequency modulation the carrier with $k_f = 50$ find an expression for the Modulated signal & determine how many harmonics should be selected to contain 99% of the modulated signal power.

Sol: Carrier is given by $c(t) = 10\cos 2\pi f_c t$

Message signal : $\cos(20\pi t)$

$k_f = 50$

So the power content of the carrier signal is given by

$$\boxed{P_c = \frac{A_c^2}{2}} = \frac{10^2}{2} = \frac{100}{2} = 50$$

The modulated signal is represented by,

$$u(t) = 10\cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} \cos(20\pi t)\,dt\right]$$

$$= 10\cos\left[2\pi f_c t + \frac{2\pi \times 50}{20\pi}\sin(20\pi t)\right]$$

$$\Rightarrow u(t) = 10\cos\left[2\pi f_c t + 5\sin(20\pi t)\right]$$

The modulation index is given by

$$\beta = k_f \frac{\max[|m(t)|]}{f_m} = \frac{50 \times 10}{10} = 5 \quad \left[2\pi f_m = 20\pi \Rightarrow f_m = 10\right]$$

So, the FM modulated signal is

$$S(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos\left[2\pi(f_c + n f_m)t\right]$$

$$= \sum_{n=-\infty}^{\infty} 10 J_n(5) \cos[2\pi(f_c + 10n)t]$$

So the frequency content of the message signal is concentrated at frequency of the form $f_c + 10n$ for various n. So, to make sure that at least 99% of the total power is within the effective band width, we have to choose K large enough so that

$$\sum_{n=-K}^{n=K} \frac{100 J_n^2(5)}{2} \geq 0.99 \times 50$$

This is a non-linear equation & to find the solution for 'K' trial & error will be applied & table of Bessel function will be used. Symmetry property of Bessels function may be used which is:

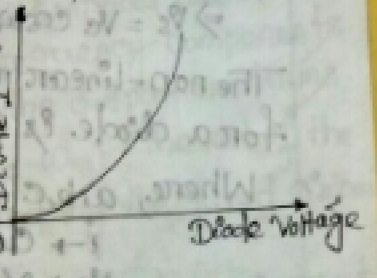
$$50\left[J_0^2(5) + 2\sum_{n=1}^{K} J_n^2(5)\right] \geq 49.5$$

→ So, starting with small values of K & increasing it we can see that the smallest value of K for which the left hand side exceeds the right hand side is K = 8. Therefore taking frequencies $\boxed{f_c \pm 10K}$ for $0 \leq K \leq 8$ guarantees that 99% of the power of the modulated signal has been included & only 1% has been left out.

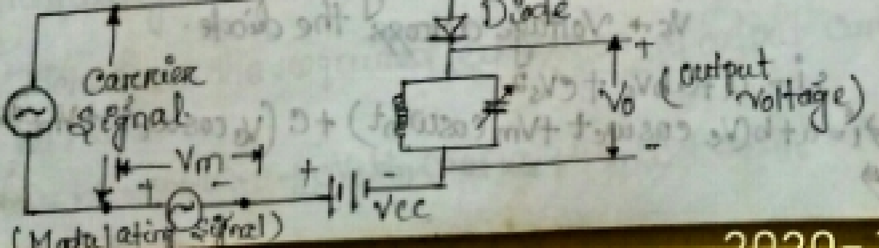
(4.) Compare the Square-Law Modulator & the Switching Modulator used for generating AM signals

Square – Law Modulation :-

→ Square law diode modulation circuit make use of non-linear current voltage c/s of diode. This method is suited at low voltage levels because of the fact that I-V characteristic of a diode is highly non-linear particularly in the low voltage region.



→ The circuit can be drawn as follows:-



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→ Here, carrier & modulating signals are applied across the diode. A dc battery Vdc (Vcc) is connected across the diode to get a fixed operating point on the V-I c/s of diode. The working of this circuit may be explained by considering the fact when two different frequencies are passed through a non linear device, the process of amplitude modulation takes place.

→ Hence when carrier & modulating frequencies are applied at the input of diode, then different frequency terms appear at the output of diode. These different frequency terms are applied across a tuned circuit which is tuned to the carrier frequency & has a narrow BW just to pass two side bands along with the carrier & reject other frequencies. so at the output of tuned circuit, carrier & two side bands are obtained, so AM wave is produced.

Mathematical Analysis:-

→ Carrier voltage may be expressed as: $v_c = V_c \cos \omega_c t$

Where, $\omega_c \rightarrow$ Carrier frequency.

Modulating voltage may be expressed as: $v_m = V_m \cos \omega_m t$

$\omega_m \rightarrow$ Modulating frequency.

→ The total a.c voltage across the diode is given as,

$v_s = v_c + v_m$

$\Rightarrow v_s = V_c \cos \omega_c t + V_m \cos \omega_m t$

The non-linear relationship between voltage & current for a diode is expressed as $\boxed{i = a + b v_s + c v_s^2}$

Where, $a, b, c \rightarrow$ Constants

$i \rightarrow$ Current through the diode.

$v_s \rightarrow$ Voltage across the diode.

$i = a + b v_s + c v_s^2$

$\Rightarrow i = a + b(V_c \cos \omega_c t + V_m \cos \omega_m t) + c(V_c \cos \omega_c t + V_m \cos \omega_m t)^2$

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$\Rightarrow i = a + bV_c\cos\omega_c t + bV_m\cos\omega_m t + c(V_c^2\cos^2\omega_c t + V_m^2\cos^2\omega_m t + 2V_cV_m\cos\omega_c t\cos\omega_m t)$

$\Rightarrow i = a + bV_c\cos\omega_c t + bV_m\cos\omega_m t + cV_c^2\cos^2\omega_c t + cV_m^2\cos^2\omega_m t + 2cV_cV_m\cos\omega_c t\cos\omega_m t$

$\Rightarrow i = a + bV_c\cos\omega_c t + bV_m\cos\omega_m t + \frac{1}{2}cV_c^2(2\cos^2\omega_c t) + \frac{1}{2}cV_m^2(2\cos^2\omega_m t) + cV_cV_m(2\cos\omega_c t\cos\omega_m t)$

$\Rightarrow i = a + bV_c\cos\omega_c t + bV_m\cos\omega_m t + \frac{1}{2}cV_c^2(1+\cos 2\omega_c t) + \frac{1}{2}cV_m^2(1+\cos 2\omega_m t) + cV_cV_m[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$

$\Rightarrow i = a + bV_c\cos\omega_c t + bV_m\cos\omega_m t + \frac{1}{2}cV_c^2 + \frac{1}{2}cV_c^2\cos 2\omega_c t + \frac{1}{2}cV_m^2 + \frac{1}{2}cV_m^2\cos 2\omega_m t + cV_cV_m\cos(\omega_c+\omega_m)t + cV_cV_m\cos(\omega_c-\omega_m)t$

$\Rightarrow i = \underset{①}{(a + \frac{1}{2}cV_c^2 + \frac{1}{2}cV_m^2)} + \underset{②}{bV_c\cos\omega_c t} + \underset{③}{bV_m\cos\omega_m t} + \underset{④}{(\frac{1}{2}cV_c^2\cos 2\omega_c t + \frac{1}{2}cV_m^2\cos 2\omega_m t)} + \underset{⑤}{cV_cV_m\cos(\omega_c+\omega_m)t} + \underset{⑥}{cV_cV_m\cos(\omega_c-\omega_m)t}$

Eq^n- ① → DC term.
② → Carrier signal
③ → Modulating signal
④ → Consists of harmonic of carrier & Modulating signals.
⑤ → Represents the Upper side band.
⑥ → " Lower side band.

→ The load impedance is a tuned circuit which is tuned to the carrier frequency ω_c. So this tuned circuit correspond to a narrow band of frequencies centered about the carrier frequency. Thus the frequency components which are actually developed in the output are terms of frequency ω_c, $(\omega_c+\omega_m)$, $(\omega_c-\omega_m)$. The rest of the frequency components are rejected by the tuned circuit.

→ Therefore, the required expression of output current will be.

$i_o = bV_c\cos\omega_c t + cV_cV_m\cos(\omega_c+\omega_m)t + cV_cV_m\cos(\omega_c-\omega_m)t$

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→ $i_0 = bV_c \cos\omega_c t + cV_cV_m[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$

→ $i_0 = bV_c\cos\omega_c t + 2cV_cV_m\cos\omega_c t\cos\omega_m t$

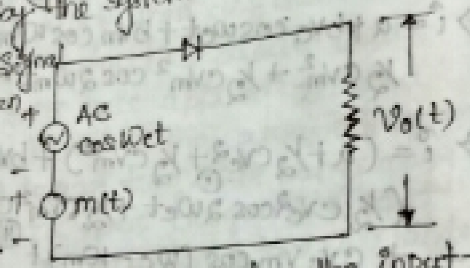
→ $\boxed{i_0 = bV_c(1+m_a\cos\omega_m t)\cos\omega_c t}$

Where, $m_a = \frac{2cV_m}{b}$ is the modulation index.

Switching Modulator/Chopper-type Modulator:

→ Another method of generating an AM modulated signal is means of a switching modulator such modulator can be implemented by the system illustrated here.

The sum of the message signal & the carrier i.e. $v_i(t)$ given by,

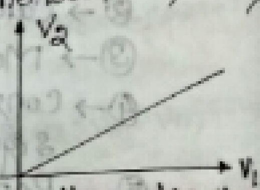


$\boxed{v_i(t) = m(t) + A_c\cos 2\pi f_c t}$

→ This will be applied to diode that has the input-output voltage c/s can be shown as follows:

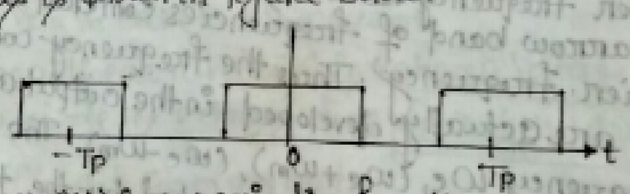
→ Here, $A_c \gg m(t)$. The output across the load resistor is simply:

$v_0(t) = \begin{cases} v_i(t), & c(t)>0 \\ 0, & c(t)<0 \end{cases}$



The switching operation may be viewed mathematically as a multiplication of the input $v_i(t)$ with the switching function $s(t)$ i.e. $v_0(t) = [m(t) + A_c\cos 2\pi f_c t]\,s(t)$

This is shown in figure below:



→ Since $s(t)$ is a periodic function, it is represented in the fourier series as:

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]$$

Hence,

$$v_0(t) = [m(t) + A_c\cos 2\pi f_c t]\,s(t)$$

$\Rightarrow V_0(t) = \frac{A_c}{2}\left[1+\frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t$ + other terms

→ The desired AM signal is obtained by passing $V_0(t)$ through a band pass filter with centre frequency $f = f_c$ & bandwidth $2W$. At its output, we have the desired conventional DSB AM signal.

$$u(t) = \frac{A_c}{2}\left[1+\frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t$$

5. a) Give the fourier transform of a signal give as $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-nT)$. Sketch the magnitude spectrum.

Ans: The signal is given as $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-nT)$

$$= \sum_{l=-\infty}^{\infty} \delta(t-2lT) - \sum_{l=-\infty}^{\infty} (\delta t - T - 2lT)$$

Taking Fourier Transform

$F[x(t)] = X(f)$

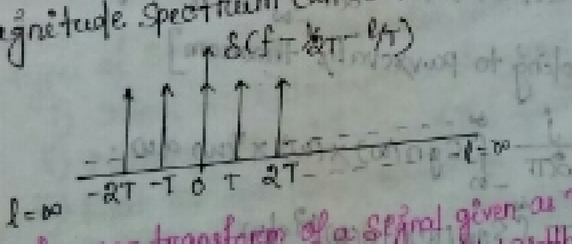
$$\Rightarrow X(f) = \frac{1}{2T}\sum_{l=-\infty}^{\infty} \delta(f - l/2T) - \frac{1}{2T}\sum_{l=-\infty}^{\infty} \delta(f - l/2T) e^{-j2\pi f T}$$

$$= \frac{1}{2T}\sum_{l=-\infty}^{\infty} \delta(f - l/2T) - \frac{1}{2T_s}\sum_{l=-\infty}^{\infty} \delta(f - l/2T) e^{-j2\pi \frac{l}{2T} T}$$

$$= \frac{1}{2T}\sum_{l=-\infty}^{\infty} \delta(f - l/2T) - \frac{1}{2T}\sum_{l=-\infty}^{\infty} \delta(f - l/2T)(-1)^l$$

$$\boxed{X(f) = \frac{1}{T}\sum_{l=-\infty}^{\infty} \delta\left(f - \frac{1}{2T} - \frac{l}{T}\right)}$$

The magnitude spectrum can be drawn as



b) Give the fourier transform of a signal given as $x(t) = \pi(t-3) + \pi(t+3)$ Where $\pi(t)$ is a rectangular pulse of unit width. Sketch the Magnitude spectrum.

Ans: A signal is given as $x(t) = \pi(t-3) + \pi(t+3)$

Where, $\pi(t)$ → Rectangular pulse of unit width.

$F[x(t)] = X(f)$

$\Rightarrow X(f) = F[\pi(t-3) + \pi(t+3)]$

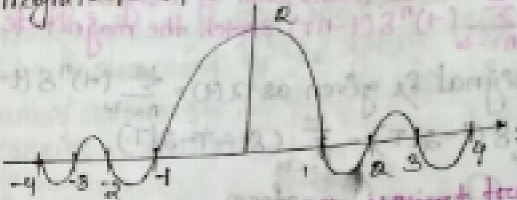
$= \text{Sinc}(f)e^{-j2\pi f3} + \text{Sinc}(f)e^{j2\pi f3}$

$= \text{Sinc}(f)\left[e^{j2\pi f3} + e^{-j2\pi f3}\right]$

$= 2\text{Sinc}(f)\left\{\frac{e^{j2\pi f3} + e^{-j2\pi f3}}{2}\right\}$

$$\boxed{X(f) = 2\text{Sinc}(f)\cos(2\pi 3f)}$$

The magnitude spectrum can be drawn as follows



6) a) Show that a signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ are orthogonal?

Ans: The hilbert transform of a signal $x(t)$, i.e $\hat{x}(t)$ can be expressed as: $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau$

→ Two signals are said to be orthogonal if two signals will be integrated over $-\infty$ to $+\infty$ then the resultant will be 0. i.e $\int_{-\infty}^{\infty} x(t)\hat{x}^*(t)\,dt = 0$

Proof

$$\int_{-\infty}^{\infty} x(t)\hat{x}^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\left\{-j\,\text{sgn}(\omega)X(\omega)\right\}^* d\omega.$$

[According to parseval's theorem]

$$= \frac{j}{2\pi}\int_{-\infty}^{\infty}\text{sgn}(\omega)X(\omega).X^*(\omega)\,d\omega.$$

$$= \frac{j}{2\pi}\int_{-\infty}^{\infty}\text{sgn}(\omega)\,|X(\omega)|^2 d\omega$$

Sgn(ω) is an odd function & the fact that $X(\omega)$ is Hermitian, gives $|X(\omega)|^2$ is an even function. so

$$= \frac{-j}{2\pi}\left[\int_{-\infty}^{0}|X(\omega)|^2 d\omega - \int_{0}^{\infty}|X(\omega)|^2 d\omega\right]$$

Hence, signal & its hilbert transform are orthogonal.

b) Show that the set of orthogonal signals $\{\phi_n(t)\}_{n=-\infty}^{\infty}$ where, $\phi_n = \text{Sinc}(2wt-n)$ represent an orthogonal signal set.

Ans: The set of signal, $\{\phi_n(t)\}_{n=-\infty}^{\infty}$ where, $\phi_n = \text{Sinc}(2wt-n)$. So taking two signal variables as m & n & using parseval's theorem,

$$F = \int_{-\infty}^{\infty} \text{Sinc}(2wt-m)\,\text{sinc}(2wt-n)\,dt$$

Taking fourier transform & applying within the integration.

$$F = \int_{-\infty}^{\infty} F[\text{Sinc}(2wt-m)]\, F[\text{Sinc}(2wt-n)]^{*}\, dt$$

Fourier transform of Sinc function is always rectangular function.

$$= \int_{-\infty}^{\infty} (1/2w)^2\, \pi^2\left(\frac{f}{2w}\right) e^{j2\pi f \frac{m-n}{2w}}\, df$$

$$= \frac{1}{4w^2} \int_{-w}^{w} e^{-j2\pi f \frac{m-n}{2w}}\, df$$

$$= \frac{1}{2w} \delta_{mn}$$

Here, $\delta_{mn} \rightarrow$ kronecker's delta.

This latter represents $\{\text{Sinc}(2wt-m)\}$ form an orthogonal set of signal.

7.a) Compare the three types of sampling. Sketch the sampled waveforms in each case assuming the message signal to be a Sinusoid.

Ans: Three types of sampling techniques are:

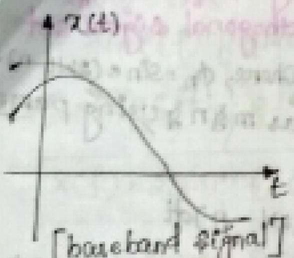
i) Ideal or Instantaneous Sampling.

ii) Natural Sampling / practical Sampling.

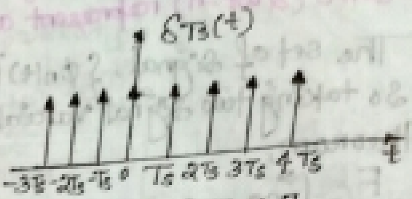
iii) Flat top Sampling / Practical Sampling.

i) Ideal Sampling / Impulse Sampling / Instantaneous Sampling

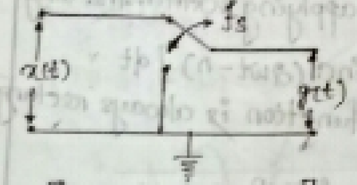
→ The sampling function is a train of impulses. $x(t)$ is the input signal. The circuit to produce instantaneous or ideal sampling is known as switching sampler. The graphical representation is as follow.



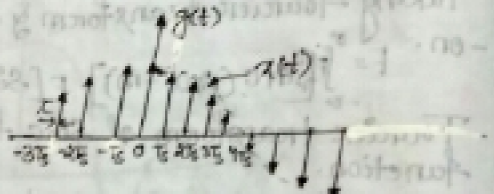
[baseband signal]



[Impulse Train]



[Switching Sampler]



τ → pulse width.
[Sampled Signal]

→ The circuit simply consists of a switch. Mathmatically the train of impulses may be represented as:

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s).$$

This is known as sampling function. The sampled signal can be expressed as the multiplication of $x(t)$ & $\delta_{T_s}(t)$.

So, $g(t) = x(t) \cdot \delta_{T_s}(t)$

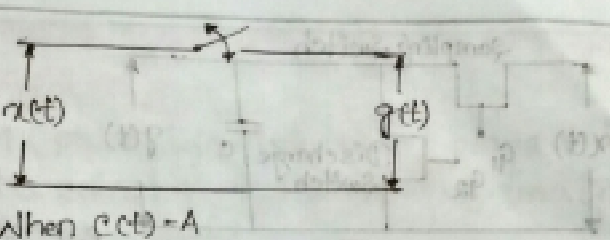
$$= x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\Rightarrow g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s)$$

$$\Rightarrow \boxed{G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)}$$

(ii) Natural Sampling:

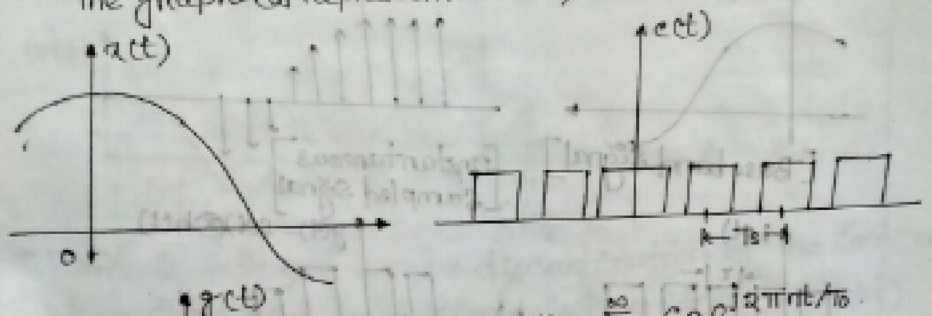
→ Here the pulse has a finite width equal to τ. Considering a continous signal $x(t)$ to be sampled at the rate of f_s Hz where $f_s > 2f_m$. Sampled signal will be obtained by multiplication of sampling function $c(t)$ & input signal $x(t)$. When $c(t)$ goes high, switch 's' is closed & when $c(t)$ goes low switch 's' is open.

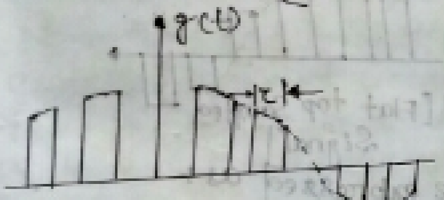


$g(t) = x(t)$ When $c(t) = A$

$g(t) = 0$ When $c(t) = 0$

→ The graphical representation is as follow:





$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt/T_0}$$

$T_0 = T_s = 1/f_s$ [period of c(t)]

· The mathematical expressions are:

$$c(t) = \sum_{n=-\infty}^{\infty} \frac{\tau A}{T_s} \operatorname{sinc}(f n \tau) e^{j2\pi n f s t}$$

$$\Rightarrow \boxed{g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(f n \tau) e^{j2\pi n f s t} x(t)}$$

Similarly, the frequency domain representation will be,

$$G(f) = F\left[\frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(f n \cdot \tau)\left[e^{j2\pi n f s t} \cdot x(t)\right]\right]$$

$$\Rightarrow \boxed{h(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(f n \tau)\, X(f - n f_s)}$$

iii) Flat Top Sampling:

→ Here, the amplitude of each pulse in the sampled signal is kept constant over the duration. This is most popular & widely used since noise can be easily removed.

→ Sample & hold circuit is used here. sampling switch

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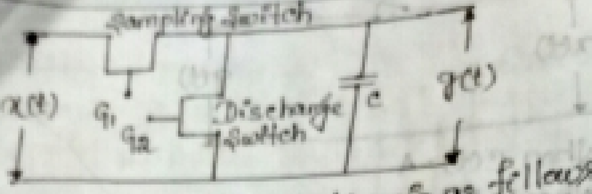
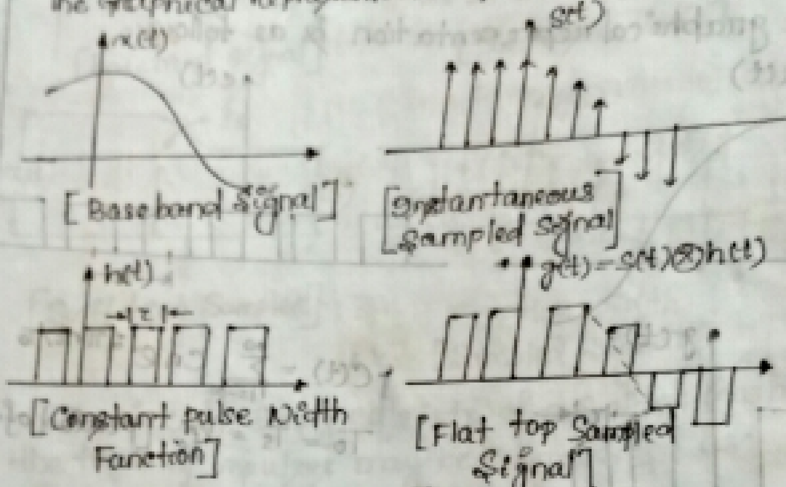


The Graphical representation is as follows:



Mathematically, it can be expressed as:

$g(t) = s(t) \otimes h(t)$

Where $\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

Thus the signal s(t) is obtained by multiplying x(t) with s(t).

$g(t) = \int_{-\infty}^{\infty} s(\tau) h(t-\tau) d\tau$

$\Rightarrow g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s) \delta(t - nT_s) h(t-\tau) d\tau$

$\Rightarrow g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(t - nT_s) h(t-\tau) d\tau$

$\Rightarrow \boxed{g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) h(t - nT_s)}$

The frequency domain representation will be,

$h(f) = F[g(t)]$

$\Rightarrow \boxed{h(f) = f_s \sum_{n=-\infty}^{\infty} x(f - nf_s) \cdot H(f)}$

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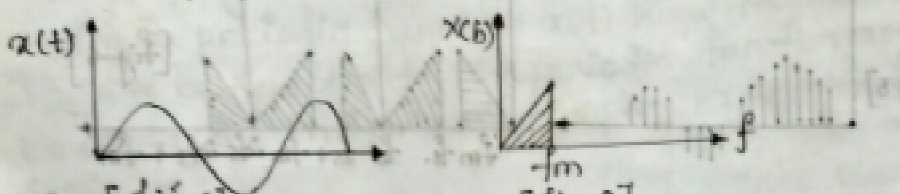
b) How do you Realize a PCM Signal from a PAM Signal? Show the steps very clearly.

Ans The PCM (pulse Code Modulation) plays an important role in the collection, transmission & analysis of measured values.

→ The analog signal is initially amplified & the spectrum representation can be shown as follows:-



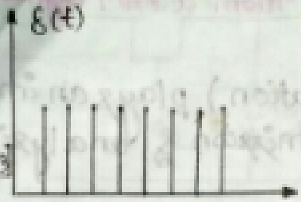
[fig-1] [fig-2]

→ This is followed by the discretization of the continuous measuring signal. An electronic switch (sample & hold) controlled by a clock generator, takes individual samples in time from the signal, where as the pulse amplitude corresponds to the momentary values of the analog input voltage. The output of the electronic switch is a PAM signal.

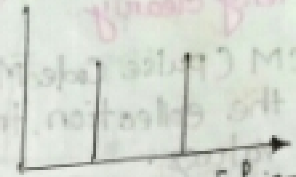
→ In practice, 4 to 5 samples per Hz bandwidth will be taken. The effect of PAM becomes more clearly by viewing the signals in the time & frequency domain. The sampling process produces a sequence of pulses which contains a DC component & sum of sinusoidal voltages, which are integer multiple of the fundamental frequency.

→ In the frequency domain the pulse samples create systematically spectral lines intervals of f_s. At the right & left hand sides of these carriers arises modulation sidebands with USB & LSB at f_s, f_s+f_m, $2f_s+f_m$, $2f_s-f_m$ etc.

[fig-3]

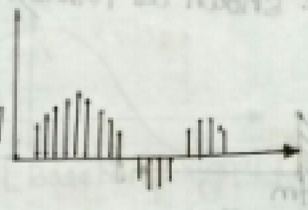


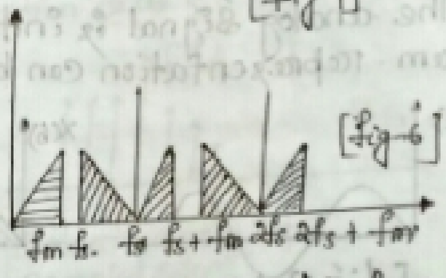
[Dirac Pulse]



[fig-4]

[fig-5]





[fig-6]

→ The pulse amplitude modulated signal in fig-5 is still an analog frame of the input signal. But the samples can be much better processed in digital form. Inorder to finalize the quantization & coding, the PAM signal is fed to a 12 bit A/D converter. The A-D converter converts (quantized) each PAM pulse according to their current amplitude into 12 bit words, a digital resolution of 1024 steps.

→ Therefore a PAM pulse amplitude of 1v is digitized with a resolution of < 1 mv.

→ The digitized PAM signal is called PCM signal. The 12 bit A/D converter follows a parallel/serial converter which converts the 12-bit words into a biserial data stream & can be transferred on a data line or via a fiber optic RF line. To enable the receiver to synchronize on to the serial data stream, synchron bits in front of each data word are transferred as well.

→ On the receiving side the same thing is done & reverse. After the series/parallel conversion, the 12-bit words are converted with the help of D/A converter in PAM signals, & then low pass filtered to smoothed amplitude continous signals. Each signal value is equal to the average of the corresponding quantization interval. After signal amplification according.

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to the level adjustment, the original modulating signal is available again.

8. Write short notes on any two

a) Fourier Transform of periodic signals

Ans: → Fourier transform of periodic signals may be found using the concept of impulse function. So, considering a periodic function $x(t)$ whose fourier transform will be found out. The periodic function $x(t)$ may be expressed in terms of complex exponential fourier series as:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

Where, $\omega_0 = 2\pi/T$ = fundamental frequency.

→ Taking fourier transform of both sides:

$$F[x(t)] = F\left[\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right]$$

$$= \sum_{n=-\infty}^{\infty} C_n F\left[e^{jn\omega_0 t}\right]$$

Using frequency shifting theorem,

$$F\left[1 \cdot e^{jn\omega_0 t}\right] = 2\pi\delta(\omega - n\omega_0)$$

Hence, $$F[x(t)] = \sum_{n=-\infty}^{\infty} C_n 2\pi\delta(\omega - n\omega_0)$$

$$F[x(t)] = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$$

→ Hence the fourier transform of a periodic function consists of a train of equally space samples. These impulses are located at the harmonic frequency of the signal & the strength or area of each impulse is given by $2\pi C_n$. This seems logical because fourier series analysis of periodic functions provides the discrete signal/discrete frequency spectrum, located at harmonic frequencies. Here, also we have got the discrete spectrum by fourier transform which generally provides a continous spectrum.

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b) Balanced Discriminator:

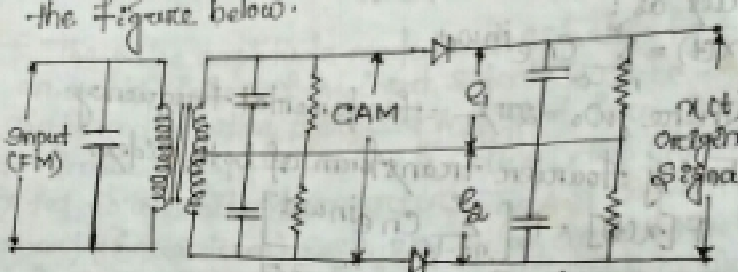
Ans: → The principle of slope detector depends upon the slope of the frequency response c/s of a frequency selective network. The two main FM discriminators are:

i) Single tuned detector/simple slope detector.

ii) Stagger tuned detector/Balanced slope discriminator.

Balanced slope Discriminator:

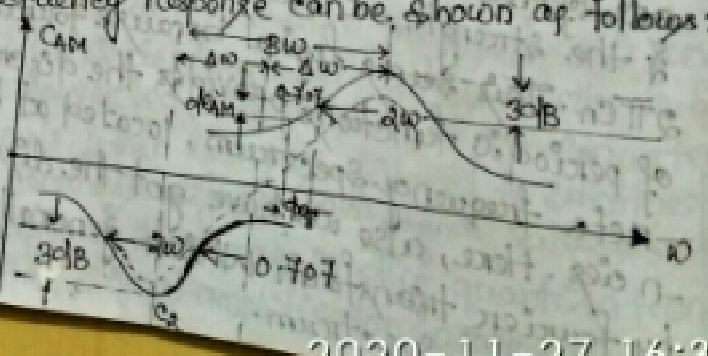
→ The circuit consists of two L-C circuits shown in the figure below.



→ Here, the two tuned circuits are used in the stagger tuned mode i.e one tuned circuit is tuned above the carrier frequency (ω_c) & another tuned circuit is tuned below the carrier frequency represented by curve lines C_1 & C_2 respectively. The resultant curve (C_1+C_2) is linear.

→ The slop $\frac{dE_{AM}}{d\omega}$ is identical at all points on the linear portion of the c/s curve. Hence, no harmonics distortion is caused when the operating is restricted to the linear region.

→ The frequency response can be shown as follows:



DISADVANTAGES:-

The balanced slope detector has few disadvantages as follows :-

i) The linear characteristics is limited to a small frequency deviation $\Delta\omega$. The frequency deviation $\Delta\omega$ for which the resultant c/s in linear depends upon the 3-dB BW of each tuned circuit which is equal to $2W$. The linearity is limited to a range of $3W$ which is shown in the above figure. Thus the deviation $\Delta\omega$ for which is the curve is linear & gives a satisfactory output is given as, $2(\Delta\omega) = 3W$.

$\Rightarrow \boxed{\Delta\omega = 1.5W}$

If the frequency deviation of an input FM wave is more than the one expressed by above equation, the distortion will occur due to the non-linearity of the c/s. Hence the operation of this detector is limited to small deviation only.

ii) The discriminator is depends critically upon the amount of detuning of the resonant circuit.

iii) The tuned circuit o/p is not purely bandlimited & thus the low pass RC filter of the envelope detector introduces distortion.

Advantages:

i) More efficient than simple slope detector

ii) Better linearity

iii) It provides a satisfactory operation.

iv) Keep distortions within tolerable limits.

② COMPANDING

→ Also called companded PCM. Companding is non-uniform quantization it is required to be implemented to improve the SNR of weak signals.

Need:-

→ In uniform quantizer, once the step size is fixed, the quantization noise power remains constant. But signal power is not constant rather it is proportional to the square of the signal amplitude.

→ Hence signal power will be small for weak signals but noise power is constant. Therefore SNR for weak signals is very poor. This will affect the quality of the signal. Hence, Companding is needed.

Companding = Compressing + Expanding

Companding Model:-

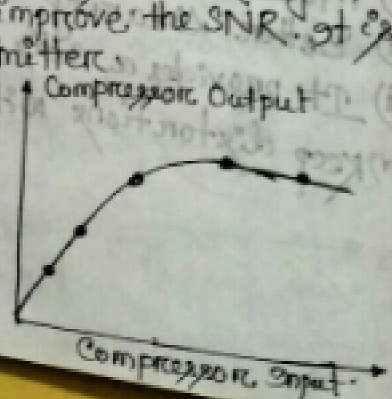
Input → Compressor → Uniform Quantizer → Expander → o/p

→ Here the weak signals are amplified & strong signals are attenuated before applying them to uniform quantizer. This process is called compression.

→ At the receiver exactly opposite is followed which is called expansion & circuit is called expander. So combined process is called Companding.

Compressor Characteristics :-

→ It provides higher gain to the weak signals & smaller gain to strong i/p signals. Thus, weak signals are artificially boosted to improve the SNR. It is included at PCM transmitter.



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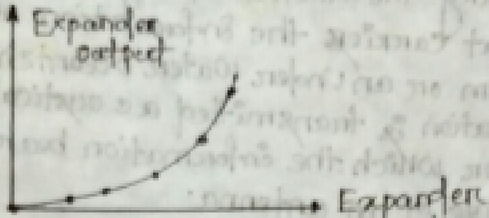
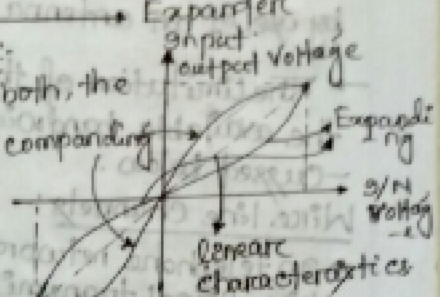
Expander Characteristics:-

→ It is exactly inverse of the compressor c/s. This ensures that all the artificially boosted signals by the compressor are brought back to their original amplitude at the receiver end.



Compander characteristics:-

Due to their inverse nature of both, the overall c/s is a straight line (dotted line) which indicates that all the boosted signals are brought back to their original amplitudes.



Types:-

Linear compression c/s is needed for small amplitude of the input signal & a logarithmic c/s else where. It is achieved by two methods.

i) μ- Law Companding:-

Mathematically,

$$z(x) = (\operatorname{sgn} x)\frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \quad 0 \leq \frac{|x|}{x_{max}} \leq 1$$

Where, $z(x) \rightarrow$ output & x is the input

$\frac{|x|}{x_{max}} \rightarrow$ Normalized value of input w.r.t max. value of x_{max}.

ii) A-law Companding:-

$$\frac{z(x)}{x_{max}} = \begin{cases} \frac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \leq \frac{|x|}{x_{max}} \leq \frac{1}{A} \\ \frac{1+\log_e[A|x|/x_{max}]}{1+\log_e A} & \frac{1}{A} \leq \frac{|x|}{x_{max}} \leq 1 \end{cases}$$

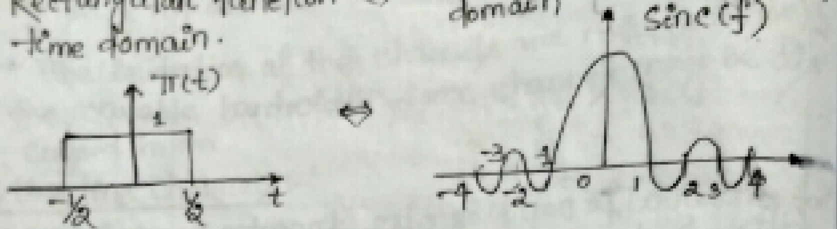
→ Max^m value of μ is 255, A is 87.56.

1 a) Time limited & band limited signal

Ans: Time limited signal means the signal which is dependent on time axis. Whereas as band limited signal means the signal which is limited by frequency.

→ Taking an example of each, rectangular function in time domain and its fourier transform representation is sinc function in frequency domain.

Rectangular function $\overset{F.T}{\Leftrightarrow}$ sinc function in frequency domain
time domain.

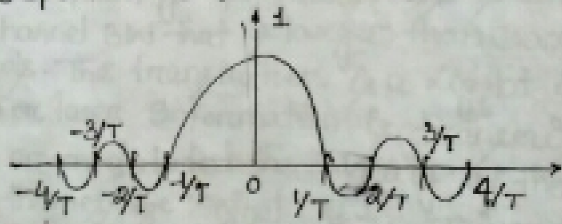


So, $\boxed{F[\pi(t)] = \text{sinc}(f)}$

b) Draw the freq spectrum of sinc function

Ans: $\text{sinc}(f_1 t) = \dfrac{\sin(\pi f_1 t)}{\pi f_1 t}$

The spectrum can be drawn as follows :-

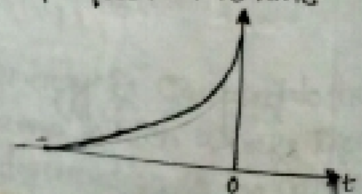


c) Find the FT of the given signal

Ans: The signal $e^{at}u(-t)$ represents an exponential decaying function in the left hand side due to the time reversal property.

So, $x(t) = e^{at}u(-t)$

[time domain Representation]

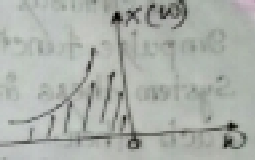


So, in order to represent this signal in terms of its spect-rum, its fourier transform has to be calculated. so,

$F[x(t)] = F[e^{at} v(-t)]$

$= \int_{-\infty}^{\infty} e^{at} v(t) \cdot e^{-j\omega t} dt$

$= \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt = \int_{-\infty}^{0} e^{(a-j\omega)t} dt$



[Spectrum Representa-tion]

$= \frac{e^{(a-j\omega)t}}{(a-j\omega)} \Big|_{-\infty}^{0} = \frac{1}{a-j\omega}(e^{0} - e^{-\infty})$

$= \frac{1}{a-j\omega} = \frac{a+j\omega}{(a-j\omega)} \cdot \frac{1}{(a+j\omega)} = \frac{a+j\omega}{a^2+\omega^2}$

$= \frac{a}{a^2+\omega^2} + j\frac{\omega}{a^2+\omega^2} = \frac{1}{\sqrt{a^2+\omega^2}}$

d) Find F.T. of given signal

Ans. The signal is given as $y(t) = x(2-t)$

Taking the fourier transform, $y(f) = F[y(t)]$

$\Rightarrow y(f) = F[x(2-t)]$

$= \int_{-\infty}^{\infty} x(2-t) e^{-j\omega t} dt$

Let $(2-t) = y \Rightarrow -dt = dy \Rightarrow dt = -dy \Rightarrow dt = dy$

$-t = -y - 2 \Rightarrow t = -(y+2) \Rightarrow t = y+2$

So, replacing the respective values,

$\Rightarrow y(f) = \int_{-\infty}^{\infty} x(y) e^{j\omega(2+y)}(+dy) = +\int_{-\infty}^{\infty} x(y) e^{+j\omega 2} \cdot e^{-j\omega \cdot y}$

$= \int_{-\infty}^{\infty} x(-y) e^{-j\omega y} dy \; e^{-2j\omega}$

$= \int_{-\infty}^{\infty} x(y) e^{-j\omega(y)} (-dy) \; e^{-2j\omega}$

$= -y(-f) e^{-2j\omega}$.

So, $\boxed{y(b) = -x(f) e^{-2j\omega}}$ (Ans)

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e) Write short LTI system with example

Ans: Two continous LTI system are connected in cascade, its impulse function is given as $\frac{\sin(at)}{at} = \text{sinc}(at)$. In LTI System these impulse function will be convolved with each other.

→ Convolution in time domain results in multiplication in frequency domain. So, taking the fourier transform of sinc(at) will give rectangular function.

→ Multiplication of two rectangular functions will give triangular functions.

f) Define Selectivity, Sensitivity & Fidelity

Ans: For a distortion less reception, the receiver must have following feature :-

i) Selectivity :- It is the receiver's ability to distinguish betn two adjacent carrier (radio) frequencies and reject others.

ii) Sensitivity :- Ability of a receiver to detect the weakest possible signal.

iii) Fidelity :- Ability of a receiver to reproduce faithfully all frequency components present in the base band signal.

g) Difference between AM & NBFM Signal :

Ans:

AM	NBFM
i) Modulation index can be greater than or less than or equal to 1.	i) Modulation index is Mandatorily less than one.
ii) Here, the Band width is finite.	ii) It has infinite no of side band in addition to a single carrier.

h) Can SSB-SC used for audio broadcasting Suggest.

Ans: Yes, SSB-SC can be used as a Modulation Scheme for audio broadcasting as it will consume less amount

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of BW & simultaneously carrying the same amount of information as DSB-SC.

→ Again for audio broadcasting, frequency discrimination is used for demodulation which is compatible with SSB-SC system.

i) Advantages of DM?

Ans: i) The DM (Delta Modulation) transmits only one bit for sample, therefore the signaling rate & transmission channel bandwidth is quite small.

ii) The transmitter & receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulator.

j) Difference between DM & DPCM System?

Ans:

DM (Delta Modulation)	DPCM [Differential pulse Code Modulation]
→ It uses only one bit per sample.	→ Bits can be more than one.
→ Step size is fixed & cannot be varied	→ Fixed Number of levels are used.
→ Slope overload distortion & granular noise are present	→ Slope overload distortion & quantization noise is present
→ Lowest bandwidth is required	→ Bandwidth required is lower
→ Feedback exists in transmitter	→ Feedback exists
→ Simple implementation	→ Here also simple implementation

2.a)

Ans: The signal is given as $x(t) = \begin{cases} \cos\pi t, & -1/2 \leq t \leq 1/2 \\ 0, & \text{otherwise} \end{cases}$

The expression for fourier transform is:

$$X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\Rightarrow X(\omega) = \int_{-1/2}^{1/2} \cos(\pi t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1/2}^{1/2} \cos\pi t\,[\cos\omega t - j\sin\omega t]\, dt$$

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$$= \int_{-1/2}^{1/2} \cos\pi t \cos\omega t \, dt - j\int_{-1/2}^{1/2} \cos\pi t \sin\omega t \, dt$$

$$= \int_{-1/2}^{1/2} \cos\pi t \cos 2\pi f t \, dt - j\int_{-1/2}^{1/2} \cos\pi t \sin 2\pi f t \, dt$$

→ Here the second integral is zero since $\cos\pi t$ is even while $\sin 2\pi f t$ is odd.

So, $$x(\omega) = \int_{-1/2}^{1/2} \cos\pi t \cos 2\pi f t \, dt$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} 2\cos\pi t \cos 2\pi f t \, dt$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} \{\cos\pi(2f+1)t + \cos\pi(2f-1)t\} \, dt$$

$$= \frac{1}{2}\left[\frac{\sin\pi(2f+1)}{\pi(2f+1)}\Bigg|_{-1/2}^{1/2} + \frac{\sin\pi(2f-1)}{\pi(2f-1)}\Bigg|_{-1/2}^{1/2}\right]$$

$$= \frac{1}{2}\left[\frac{2\sin\pi(f+\frac{1}{2})}{2\pi(f+\frac{1}{2})} + \frac{2\sin\pi(f-\frac{1}{2})}{2\pi(f-\frac{1}{2})}\right]$$

b) Find F.T of $x(t)$:

Ans $x(t) = \frac{1}{1+t^2}$

To find the fourier transform of this signal, Fourier transform pair can be used.

Using the fourier transform pair,

$$e^{-\alpha|t|} \xrightarrow{F} \frac{2\alpha}{\alpha^2+(2\pi f)^2} = \frac{2\alpha}{4\pi^2}\,\frac{1}{\frac{\alpha^2}{4\pi^2}+f^2}$$

Using the duality property of fourier transform

$$X(f) = F[x(t)] \Rightarrow x(-b) = F[X(t)]$$

So, $$\left(\frac{2\alpha}{4\pi^2}\right) F\left[\frac{1}{\frac{\alpha^2}{4\pi^2}+t^2}\right] = e^{-\alpha|b|}$$

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With $\alpha = 2\pi$, the desired result will be

$$F\left[\frac{1}{1+t^2}\right] = \pi e^{-2\pi|b|}$$

3.a) Find F T of given signal.

Ans: i) The signal is given as $x(t) = e^{-3t}u(t-2)$.

The fourier transform can be found as:

$$X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} e^{-3t}u(t-2)\, e^{-j\omega t}\,dt$$

$$u(t-2) = \begin{cases} 1, & t \geq 2 \\ 0, & \text{otherwise} \end{cases}$$ [Property of shifted unit step function]

Now applying this property,

$$X(\omega) = \int_{2}^{\infty} e^{-3t} e^{-j\omega t}\,dt$$

$$= \int_{2}^{\infty} e^{-(3+j\omega)t}\,dt$$

$$= \frac{1}{-(3+j\omega)} e^{-(3+j\omega)t}\Big|_{2}^{\infty} = \frac{1}{-(3+j\omega)}\left[e^{-\infty} - e^{-(3+j\omega)2}\right]$$

$$= \frac{1}{3+j\omega}\left[e^{-(6+2j\omega)} - 0\right]$$

$$X(\omega) = \frac{e^{-(6+2j\omega)}}{3+j\omega}$$

ii) The signal is given as $x(t) = 2t\exp(-2t)u(t)$
$= 2t e^{-2t}u(t)$

The fouriere transform of the signal can be found by

$$X(\omega) = F[x(t)]$$

$$= \int_{-\infty}^{\infty} 2t e^{-2t}u(t) = \int_{0}^{\infty} 2(t) e^{-2t}$$

Now applying the property of fourier transform

$$t\,x(t) \xleftrightarrow{F} j\frac{d}{d\omega}X(\omega)$$

$$\text{So, } X(\omega) = j\frac{d}{d\omega}\left[\frac{1}{2+j\omega}\right] = \frac{1}{(2+j\omega)^2}$$

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b) Compute the energy contained in the signal $x(t) = 4\,\text{sinc}(40t)$

Sol: The signal is given as $x(t) = 4\,\text{sinc}(40t)$

The energy of the signal is given as $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

Using Parseval's theorem, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

$E = \int_{-\infty}^{\infty} (4\,\text{sinc}\,40t)^2 dt$

$= 16 \int_{-\infty}^{\infty} (\text{sinc}\,40t)^2 dt = 16 \int_{-1}^{1} \frac{1}{40}\,\text{rect}(\omega/40)\,d\omega$

$= \frac{16}{40} \cdot 1 = \frac{16}{40}$ [Area acquired by a rectangular function is one]

So, $E = \frac{16}{40} = \frac{2}{5}$

4. a) Given a signal $x(t) = A\pi(t/T)\cos(\omega_c t + \theta)$ find

i) Its analytic signal

ii) Spectrum of the analytic signal and the

iii) Complex envelope.

Sol: A signal is given as $x(t) = A\pi(t/T)\cos(\omega_c t + \theta)$.

i) Its analytic signal will be represented as a sum of the original signal & its hilbert transformed signal $\hat{x}(t)$.

Analytic signal, $\boxed{z(t) = x(t) + \hat{x}(t)}$

The signal $x(t) = A\cos(\omega_c t + \theta)\,\pi(t/T)$

Taking $z_1(t) = A\cos(\omega_c t + \theta)$

$z_2(t) = \pi(t/T)$

So, its hilbert transform $h[z_1(t) z_2(t)] = \hat{z}(z_1 t) z_2(t)$

$= h[z_2(t)]\, z_1(t)$

hilbert transform of $A\cos(\omega_c t + \theta) = A\sin(\omega_c t + \theta)$

So, $\hat{x}(t) = A\sin(\omega_c t + \theta)\,\pi(t/T)$.

So Analytic signal $z(t) = x(t) + \hat{x}(t)$

$\Rightarrow z(t) = A\pi(t/T)\cos(\omega_c t + \theta) + A\pi(t/T)\sin(\omega_c t + \theta)$

$\Rightarrow z(t) = A\pi(t/T)[\cos(\omega_c t + \theta) + \sin(\omega_c t + \theta)]$

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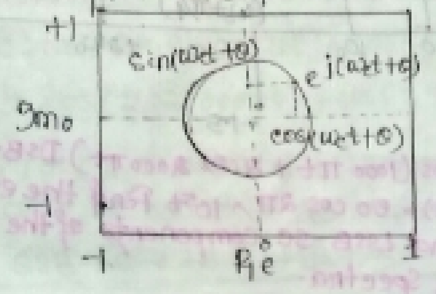
(ii) The analytical signal can be represented as:

$$z(t) = A\pi(t/T)\, e^{j(\omega_c t+\theta)}$$

(iii) The complex envelope can be written as



b) Express the signal $x(t) = 2 + \sin\omega_0 t - 3\cos(\omega_0 t+\pi/4) + 2\cos 2\omega_0 t$ as the sum of complex exponentials and plot the magnitude spectrum.

Sol: The signal is given as

$x(t) = 2 + \sin\omega_0 t - 3\cos(\omega_0 t+\pi/4) + 2\cos 2\omega_0 t$.

Representing the signal in terms of complex exponentials

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$\cos(\omega_0 t+\pi/4) = \frac{1}{2}\left[e^{j(\omega_0 t+\pi/4)} + e^{-j(\omega_0 t+\pi/4)}\right]$$

$$\cos 2\omega_0 t = \frac{1}{2}\left[e^{2j\omega_0 t} + e^{-2j\omega_0 t}\right]$$

So,

$$x(t) = 2 + \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] - \frac{3}{2}\left[e^{j(\omega_0 t+\pi/4)} + e^{-j(\omega_0 t+\pi/4)}\right] + \frac{2}{2}\left[e^{2j\omega_0 t} + e^{-2j\omega_0 t}\right]$$

$$= 2 + \frac{1}{2}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right] - \frac{3}{2}\left[e^{j(\omega_0 t+\pi/4)} + e^{-j(\omega_0 t+\pi/4)}\right] + \left[e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right]$$

The magnitude spectrum can be represented as,

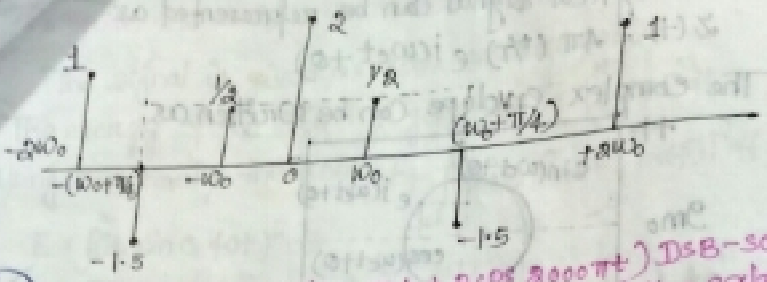
So, at origin the amplitude is 2

at $\pm\omega_0$, the amplitude is $\frac{1}{2}$ [$\sin\omega_0 t$]

at $\pm(\omega_0+\pi/4)$, the amplitude is $-\frac{3}{2}$ $\cos(\omega_0 t+\pi/4)$

at $\pm 2\omega_0$, the amplitude is 1 [$\cos 2\omega_0 t$]

So drawing the magnitude spectrum.



5.a) A signal $x(t) = \cos(1000\pi t + 2\cos 2000\pi t)$ DSB-SC Modulation & the carrier $c(t) = 50\cos 2\pi \times 10^5 t$ Find the expression for the USSB-SC and LSSB-SC components of the Modulated signal & Sketch the spectra.

Ans: The modulating signal is given as;

$x(t) = \cos 1000\pi t + 2\cos 2000\pi t$

carrier signal is given as: $c(t) = 50\cos 2\pi \times 10^5 t$.

→ For representation of SSB signal the hilbert transform of modulating signal $x_h(t)$ is needed.

So, $x_h(t) = x(t)$ shifted by $\pi/2$ phase

$= \cos(1000\pi t - \pi/2) + 2\cos(2000\pi t - \pi/2)$

$= \sin 1000\pi t + 2\sin 2000\pi t$

The expression for SSB signal can be written as.

$x_{SSB}(t) = 0.5\,x(t)\,c(t) \pm 0.5\,x_h(t)\,c(t)$

$= 0.5\,x(t)\cos\omega_c t \pm 0.5\,x_h(t)\sin\omega_c t$

$= 0.5[\cos 1000\pi t + 2\cos 2000\pi t]\cos(2\pi \times 10^5 t)\,50 \pm$

$0.5[\sin 1000\pi t + 2\sin 2000\pi t]\sin(2\pi \times 10^5 t)\,50$

$= 25[\cos 1000\pi t \cos(2{,}00{,}000\pi t) + 50\cos 2000\pi t \cos(200{,}000\pi t)] \pm$

$= 25[\sin 1000\pi t \sin(2{,}00{,}000\pi t) + 50\sin 2000\pi t \sin[2{,}00{,}000\pi t)]$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

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For SSB-SC USB (USSB - SC)

$25\left[\left\{\frac{1}{2}\left\{\cos(2{,}00{,}000\pi + 1000\pi)t + \cos(200000\pi - 1000\pi)t\right\}\right\}\right] -$

$50\left[\frac{1}{2}\cos(2{,}00{,}000\pi + 2000\pi)t + \cos(2{,}00{,}000\pi - 2000\pi)t\right]$

$= 25\left[\frac{1}{2}\left\{\cos(2{,}01{,}000\pi t) + \cos(1{,}99{,}000\pi t)\right\}\right] -$

$50\left[\frac{1}{2}\left\{\cos(2{,}02{,}000\pi t) + \cos(1{,}98{,}000\pi t)\right\}\right]$

$= 12.5\cos(2{,}01{,}000\pi t) + 12.5\cos(1{,}99{,}000\pi t) -$

$25\cos(2{,}02{,}000\pi t) - 25\cos(1{,}98{,}000\pi t)$

For SSB-SC LSB (LSSB - SC):

$25\left[\frac{1}{2}\left\{\cos(2{,}00{,}000\pi - 1000\pi)t - \cos(2{,}00{,}000\pi + 1000\pi)t\right\}\right] +$

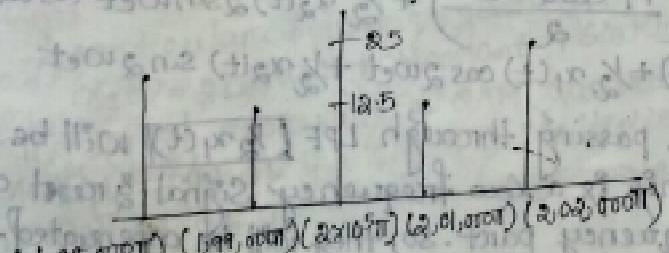
$50\left[\frac{1}{2}\left\{\cos(2{,}00{,}000\pi - 2000\pi)t - \cos(2{,}00{,}000\pi + 2000\pi)t\right\}\right]$

$= 25\left[\frac{1}{2}\left\{\cos(1{,}99{,}000\pi t) - \cos(2{,}01{,}000\pi t)\right\}\right] +$

$50\left[\frac{1}{2}\left\{\cos(1{,}98{,}000\pi t) - \cos(2{,}02{,}000\pi t)\right\}\right]$

$= 12.5\cos(1{,}99{,}000\pi t) - 12.5\cos(2{,}01{,}000\pi t) + 25\cos(1{,}98{,}000\pi t) - 50\cos(2{,}02{,}000\pi t)$

• Taking the equation of upper side band to draw the spectrum,



[Magnitude Spectrum]

→ Frequency spectrums are :-

$(2{,}000\,00\pi \pm 1000\pi)$ & $(2{,}00{,}000\pi \pm 2000\pi)$

(b) Discuss quadrature Amplitude Multiplexing.

Ans: Quadrature Amplitude Multiplexing (QDM):-

→ QAM is similar to DSB-SC but sends two message signal over the same spectrum. 1st, a modulating signal $x_1(t)$ is sent in phase i.e by multiplying it with $\cos\omega_c t$ & another modulating signal $x_2(t)$ is sent in quadrature by multiplying it with $\sin\omega_c t$ & finally adding these two signals.

→ So, a QAM signal is thus represented as,

$$\boxed{x_{QAM}(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t}$$

→ The demodulation uses coherent detection & Multiplication by carrier & then baseband low pass filtering. So, if $x_{QAM}(t)$ is multiplied by $\cos\omega_c t$, $x_1(t)$ will be obtained & if it is multiplied by $\sin\omega_c t$, $x_2(t)$ will be obtained. This can be shown further.

Mathematical Representation:-

$x_{QAM}(t)\cos\omega_c t$

$= (x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t)\cos\omega_c t$

$= x_1(t)\cos^2\omega_c t + x_2(t)\sin\omega_c t\cos\omega_c t$

$= x_1(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) + \frac{1}{2}x_2(t)\, 2\sin\omega_c t\cos\omega_c t$

$= \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos 2\omega_c t + \frac{1}{2}x_2(t)\sin 2\omega_c t$

So, after passing through LPF $\boxed{\frac{1}{2}x_1(t)}$ will be obtained as this is a low frequency signal & rest are high frequency part. So, they will be attenuated. Similarly,

$x_{QAM}(t)\sin\omega_c t$

$= (x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t)\sin\omega_c t$

$= x_1(t)\cos\omega_c t \cdot \sin\omega_c t + x_2(t)\sin^2\omega_c t$

$= \frac{1}{2}x_1(t)\, 2\sin\omega_c t\cos\omega_c t + x_2(t)\left(\frac{1-\cos 2\omega_c t}{2}\right)$

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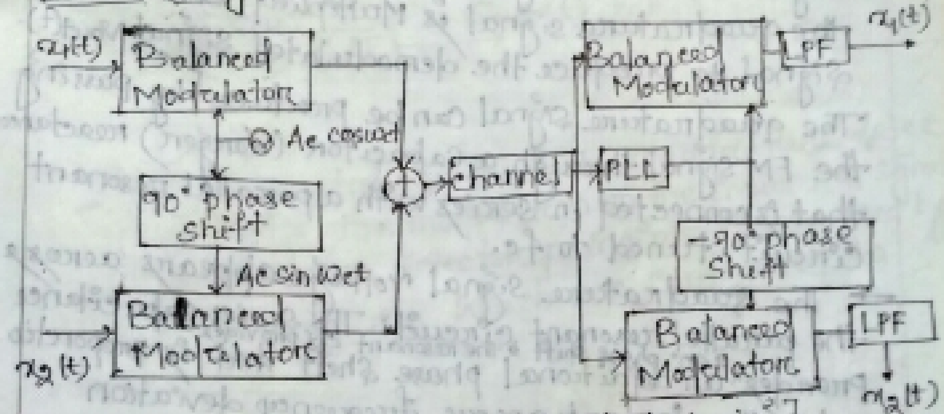
$= \frac{1}{2} x_1(t) \sin 2\omega_c t + \frac{1}{2} x_2(t) - \frac{1}{2} x_2(t) \cos 2\omega_c t$

So, after passing through LPF, $\boxed{\frac{1}{2} x_2(t)}$ will be obtained by the same concept.

→ The disadvantage of QAM is that, it gives channel interference if local carrier has some phase error. Block diagram can be shown as follows:

Block Diagram:



[Quadrature Amplitude Multiplexing]

6.a) A particular signal is given by $x_c(t) = 2\cos\omega_c t + 0.4 \cos\omega_m t \sin\omega_c t$. Comment on the nature/type of Modulation.

Ans. A particular signal is given as:

$$x_c(t) = 2\cos\omega_c t + 0.4\cos\omega_m t + \sin\omega_c t$$

The signal represents a QAM (Quadrature Amplitude Multiplexing) as the signal is multiplied by $\cos\omega_c t$ as well as its quadrature i.e $90°$ phase shift of it i.e $\sin\omega_c t$

→ Generally QAM signal is,

$$X(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$$

So, this equation resembles with the above equation so, this is a QAM signal.

Hence, a QAM signal.

Here, we can take: $x_1(t) = 2$

$x_2(t) = 0.4\cos\omega_m t$

One part is multiplied with $\cos \omega_c t$ & the other part is multiplied with $\sin \omega_c t$.

(b) Discuss the quadrature FM detector with the help of appropriate expressions & sketches.

Ans: Quadrature FM Detector:-

→ A quadrature signal is first obtained from the FM signal. Then through the use of a product detector the quadrature signal is multiplied with the FM signal to produce the demodulated signal $V_{out}(t)$.

The quadrature signal can be produced by passing the FM signal through a capacitor (larger reactance) that is connected in series with a parallel resonant circuit tuned to f_c.

→ The quadrature signal voltage appears across the parallel resonant circuit. The series capacitance provides a 90° phase shift & the resonant ckt provides an additional phase shift that is proportional to the instantaneous frequency deviation (from f_c) of the FM signal.

→ The FM signal can be written as:-

$$S(t) = V_L \cos[\omega_c t + \theta(t)] \quad —①$$

Where, $\theta(t) = K_f \int^t x(t_1)\, dt_1$

$K_f \rightarrow$ A constant known as frequency selectivity (Hz/volt)

The quadrature signal can be written as:-

$$S_{quad}(t) = K_1 V_L \sin\left[\omega_c t + \theta(t) + K_2 \frac{d\theta(t)}{dt}\right] \quad —②$$

Where, K_1 & K_2 are constants depending on component values used for the series capacitor & in the parallel resonant circuit.

→ Equations ① & ② are multiplied together by a product detector to produce the o/p signal.

$$S_{out}(t) = \frac{1}{2} K_1 V_L^2 \sin\left[K_2 \frac{d\theta(t)}{dt}\right]$$

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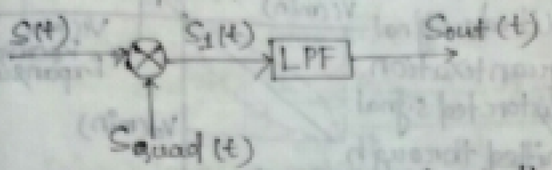
Where the sum-frequency term is eliminated by the LPF

→ For K_2 sufficiently small, $\sin\theta \simeq \theta$ & from the value of $\theta(t)$ written above, the output becomes,

$$\boxed{S_{out}(t) = \frac{1}{2} K_1 K_2 V_m^2 K_f\, m(t)}$$

→ The product relator can be shown as follows:



→ This demonstrates that the quadrature detector detects the modulation on the input FM signal. The quadrature detector principle is also used by phase-locked loops which are configured to detect FM.

7/ a) Elucidate the companding process. Would you apply it to a sinusoid of peak to peak amplitude of 5V ? Justify.

\# Companding:

→ The quantization error depends upon the step size & as the steps are uniform in size, the small amplitude signals will have a poorer amplitude signals, because in both the cases the denominator (quantization noise) is same where as the numerator is small for small amplitude signal & large for large amplitude signal.

→ As fixed no. of quantization levels will be used, the only way to have uniform SNR is to adjust the step size in such a way that the ratio remains constant. Thus, the step size should be small for small amplitude signals & large for large amplitude signals.

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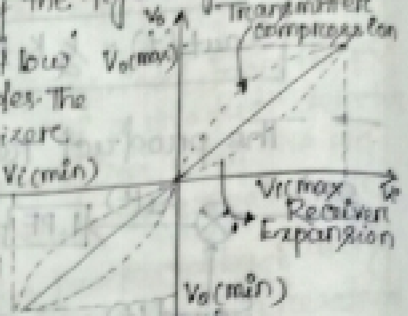
→ The effect of adaptive size may also be achieved in a more feasible way by distorting the signal before quantization. An inverse distortion has to be introduced at the receiver to make the overall transmission distortion -less. This can be explained by the figure given here:-



→ The output is enhanced more at low amplitudes than at high amplitudes. The output is than applied to quantizer. Thus the low amplitude signal will carry more quantization levels than the undistorted signal.

→ A signal transmitted through a non linear n/w with the c/s showned by dotted line will have its extremes compressed. Hence such a n/w is called a compressor. At the receiver side, an inverse operation is to be performed to receiver the original signal. This is achieved by an expander, connected between the decoder & holding ckt. [Companding] The combination of compressor & expander is known as compander which performs the companding operation.

→ A sinusoid of peak-to-peak amplitude of 5V can be applied to a compander, as it has linear compression c/s for small amplitude of i/p signal. There are two laws of companding i.e (i) μ-law (ii) A-law.

→ From these, μ-law can be applied to the 5V sinusoid signal as its c/s is continous. Here again, for smaller values of i/p signal, the approximation is linear. Mathmatically, this law can be written as:-

$$z(x) = (\text{sgn}\,x)\,\frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \qquad 0 \leq \frac{|x|}{x_{max}} \leq 1.$$

Where, $z(x) \rightarrow$ o/p, $x \rightarrow$ i/p

$|x|/x_{max} \rightarrow$ Normalized value of i/p w.r.t maxm value of x_{max}.

$\text{sgn}\,x \rightarrow$ either +1 or -1, depending on amplitude of signal

$\mu \rightarrow$ 0-255 (Range of value)

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b) Establish the rel^n ship bet^n the SQNR and the number of bits used to represent a given sample.

Ans Relation Between SQNR & No of Bits:

Let, $v \to$ No. of bits used.

SNR can be given as:-

$$\frac{S}{N} = \frac{\text{Normalized Signal Power}}{\text{Normalized Noise power}}$$

→ Normalized power has been calculated as $\frac{\Delta^2}{12}$

Where, $\Delta \to$ step size & can be represented as

$$\Delta = \frac{VH - VL}{M} = \frac{(x_{max}) - (-x_{max})}{M} = \frac{2x_{max}}{M}$$

$M \to$ No. of quantization levels & $\boxed{M = 2^v}$

→ Therefore, $\frac{S}{N} = \frac{\text{Normalized Signal Power}}{(\Delta^2/12)}$ — ①

Substituting the values

$$\frac{S}{N} = \frac{\text{Normalized Signal power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \cdot \frac{1}{12}}$$

→ Let the normalized signal power be denoted as P.

Thus, $\frac{S}{N} = \frac{P}{\frac{4x^2_{max}}{2^{2v}} \times \frac{1}{12_3}} = \frac{3P}{x^2_{max}} \cdot 2^{2v}$

$$\Rightarrow \boxed{\frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2^{2v}}$$

This expression shows that, SNR of quantizer increases exponentially with increasing bits/sample. Now assuming input $x(t)$ is normalized. i.e. $x_{max} = 1$

Then, $SNR = 3 \times 2^{2v} \times P$

Again, if destination signal power 'p' is normalized.

$$\boxed{P \le 1 \Rightarrow S/N \le 3 \times 2^{2v}}$$

Expressing the SNR in dB, we get

$$(S/N)_{dB} = 10\log_{10}(S/N)\,dB \leqslant 10\log_{10}[3 \times 2^{2v}]$$

$$\leqslant 10\log_{10}(3) + 10\log_{10}(2^{2v})$$

$$\leqslant 4.8 + 20\log_{10} 2^{v}$$

$$\boxed{SNR \leqslant (4.8 + 6v)\,dB}$$

So, the equation represents the relationship betn samp & the no. of bits used to represent a given sample.

8.a) Discuss any two line codes place your comparisons in a tabular format justifying your answers in each step.

Ans: Line codes / Transmission Codes:-

→ The digital data (0's & 1's) are transmitted over the line by means of line codes / data transmission codes / Modulation codes which give electric representation of symbol 0 & 1.

→ Any two line codes that can be discussed is:

i) UNRZ (Unipolar Non-return to zero) code.

ii) BNRZ (Bipolar Non-return to zero) code.

i) UNRZ:

Here the wave form has a single polarity. The waveform can have +5v or +12v when high. It is simply ON-OFF

→ When a symbol '1' is to transmitted the signal has 'A' v for full duration. When '0' is to be transmitted the signal has '0' v for complete duration of symbols.

So, $x(t) = \begin{cases} A, & 0 \leqslant t \leqslant T_b \ (\text{Symbol-1}) \\ 0, & 0 \leqslant t \leqslant T_b \ (\text{Symbol-0}) \end{cases}$

ii) BNRZ

This is also called AMI (Alternate Mark Inversion). Here the symbol '1' is represented by +ve polarity & '0' is represented by -ve polarity. These polarities are maintained over complete pulse duration.

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So, $x(t) = \begin{cases} +A/2 & \text{for } 0 \leq t \leq T_b \text{ (symbol-1)} \\ -A/2 & \text{for } 0 \leq t \leq T_b \text{ (symbol-0)} \end{cases}$

→ Now this two line Codes can be Compared as follo

Line Codes	Minimum Bandwidth	Average DC	Clock Recovery	Error Detecti
UNRZ	$1/2T$	$A/2$	poor	No
BNRZ	$1/2T$	0	poor	No

b). Discuss a typical Monochrome