Revision Notes

HEAT TRANSFER

Module - II

[Convection Heat Transfer]

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Chapter -1

Forced Convection Heat Transfer

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-: CONVECTION :-

Thermal convection is the mode of heat transfer betn a solid surface and the adjacent liquid or gas that is in motion or by the circulation/mixing of fluid medium (gas, liquid or powdery substance)

→ The faster the motion/circulation/mixing of fluid, the greater the convection heat transfer.

→ Convection is possible only in a fluid medium and is directly linked with the transport of medium itself

Mechanisms Of Convection Heat Transfer :

The convection heat transfer mode consists of TWO mechanisms : (i) Random motion (diffusion) of the molecules, and (ii) Bulk or Macroscopic motion of the fluid.

→ This fluid motion is due to the fact that, at any instant, a large number of molecules are moving collectively or as aggregates. Such motion, in the presence of a temperature gradient, will give rise to heat transfer

→ Since the molecules in the aggregate retain their random motion, the total heat transfer is

Convection {
- due to superposition of the energy transports by random motion of the molecules, and
- by the bulk motion of the fluid.

Advection : transport owing to bulk fluid motion only

Newton's Law Of Cooling : The rate equation for the convective heat transfer betn a surface and an adjacent fluid is described/expressed by Newton's Law of cooling as

Rate of heat tranfer by Convection, $\boxed{\dot{Q} = hA_s(t_s - t_f)}$

where h = convective heat transfer coefficient or heat transfer co.efficient or film co.efficient or film conductance, $W/m^2 \cdot ^\circ C$ or $W/m^2 \cdot K$

A_s = surface area exposed to convective heat transfer

t_s and t_f are surface temp and fluid temp (at far distance from the solid surface

* Convection heat transfer strongly depends on the fluid properties dynamic viscosity (μ), thermal conductivity (K), density (ρ), and specific heat (C_p) as well as fluid velocity (V), geometric and roughness of the solid surface, types of fluid flow (i.e, streamlined or turbulent).

Convective Heat Transfer Co-efficient (h):

It can be defined as the rate of heat transfer bet^n a solid surface and a fluid per unit surface area per unit temp. difference.

$$h = \frac{\dot{Q}_{conv}}{A_s(t_s - t_f)} \;,\; W/m^2\text{-}K \text{ or } W/m^2\text{-}^\circ C$$

⇒ The value of 'h' depends upon the following factors:

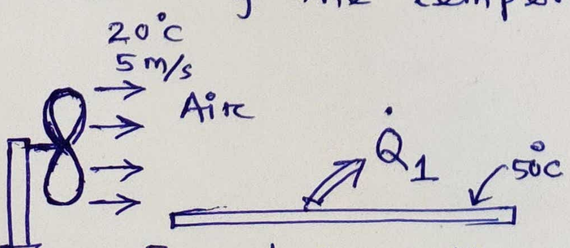
- Thermodynamic and transport properties (eg., viscosity, density, specific heat, etc.)
- Nature of fluid flow
- Geometry and roughness of the surface
- Prevailing thermal conditions

There are two types of convections ~~are~~ distinguished:

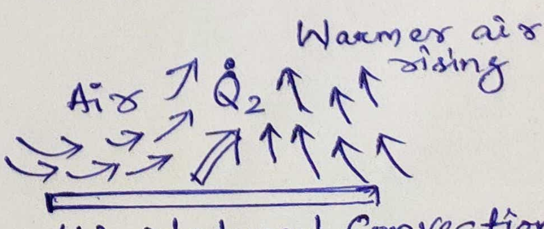
1. Forced Convection
2. Free or Natural Convection.

- **Forced Convection:** Convection is called forced convection, if the fluid is forced to flow over the surface by external means such as fan, pump, blower or the wind.

- **Free or Natural Convection:** Convection is called free or natural convection, if the fluid motion is caused by buoyancy forces that are induced by the density differences due to variation of the temperature in the fluid.



(a) Forced Convection



(b) Natural Convection

Classification Of Convection Heat Transfer

1. Based on Geometry :
   a) External Flow , b) Internal Flow
2. Based on Driving Mechanism :
   a) Natural Convection, b) Forced Convection
3. Based on Nature of flow :
   a) Laminar , b) Turbulent

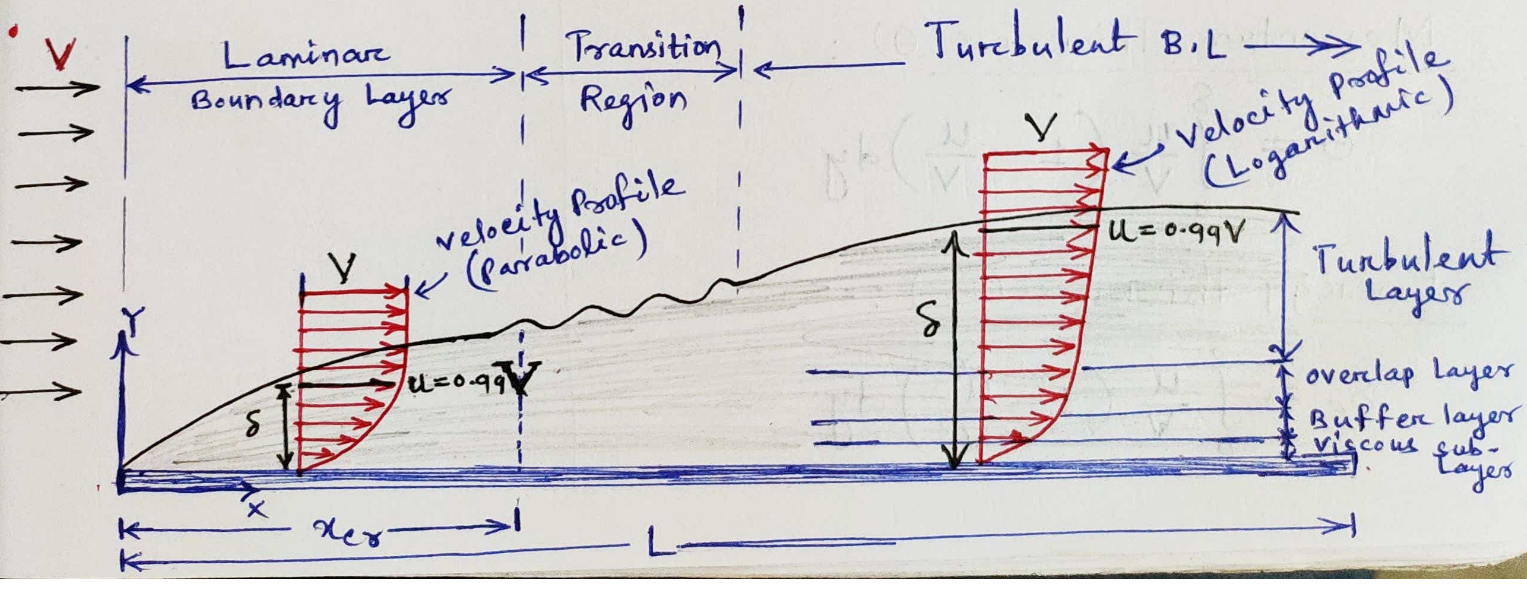
Hydrodynamic or Velocity Boundary Layer:

- Consider the parallel flow of a fluid over a flat plate. The fluid approaches the plate in the x-direction with a uniform velocity V (usually called as free stream velocity or approach velocity over the flat plate away from the plate surface)
- As the fluid consists of adjacent layers piled on top of each other, the velocity of the particles in the first fluid layer adjacent to the plate becomes zero because of the no-slip condition. This motionless layer slows down the particles of the neighbouring fluid layer as a result of friction betn the particles of two adjoining fluid layers at different velocities. This fluid layer then slows down the molecules of the next layer and so on.
- The region of the flow above the plate surface in which the effects of the viscous shearing forces caused by the fluid viscosity are realised/felt is called the hydrodynamic or velocity boundary layer.

OR Hydrodynamic or Velocity B.L. is a region of a fluid flow, near a solid surface, where the flow patterns are directly influenced by viscous drag from the surface wall.

- Hydrodynamic or Velocity boundary layer thickness, δ is the normal distance from the surface where the local fluid velcity, u reaches 99 % of the free stream velocity, V

i.e., $$u\Big|_{y=\delta} = 0.99V.$$



- The hypothetical line of $u = 0.99\,V$ divides the flow over a plate into two regions:
  - → The boundary layer region, in which the viscous effects and the velocity changes are significant.
  - → The irrotational flow region, in which the frictional effects are negligible and the velocity remains essentially constant (i.e., equal to V).
- The thickness of boundary layer (δ) increases in the flow direction.
- The velocity profile in the laminar B.L. and turbulent B.L. are parabolic and logarithmic respectively.
- The velocity profile in turbulent flow is much fuller than in laminar flow, with a sharp drop near the surface.
- The turbulent

\# For greater accuracy, the boundary layer thickness (δ) can be defined in terms of following terms:

1. Displacement thickness (δ^*)
2. Momentum thickness (θ)
3. Energy thickness (δ_e)

Displacement Thickness (δ^*):

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy$$

Momentum Thickness (θ):

$$\theta = \int_0^\delta \frac{u}{V}\left(1 - \frac{u}{V}\right) dy$$

Energy Thickness (δ_e):

$$\delta_e = \int_0^\delta \frac{u}{V}\left(1 - \frac{u^2}{V^2}\right) dy$$

* Shape Factor, $H = \frac{\delta^*}{\theta}$

Boundary layer thickness (δ) :

The velocity within the boundary layer increases from zero at the boundary surface to the velocity of the main stream asymptotically. Therefore, the thickness of the boundary layer is arbitrarily defined as *that distance from the boundary in which the velocity reaches 99 per cent of the velocity of the free stream* ($u = 0.99U$). *It is denoted by the symbol* δ. *This definition, however, gives an approximate value* of the boundary layer thickness and hence δ is generally termed as **nominal thickness** of the boundary layer.

The boundary layer thickness for *greater accuracy* is defined in terms of certain mathematical expressions which are the measure of the boundary layer on the flow. The commonly adopted definitions of the boundary layer thickness are :

1. Displacement thickness (δ^*)
2. Momentum thickness (θ)
3. Energy thickness (δ_e).

Displacement thickness (δ^*) :

The *displacement thickness* can be defined as follws :

"*It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.*"

Or

"*It is an additional "wall thickness" that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation.*"

The displacement thickness is denoted by δ^*.

Let fluid of density ρ flow past a stationary plate with velocity U as shown in Fig. 7.2. Consider an elementary strip of thickness dy at a distance y from the plate.

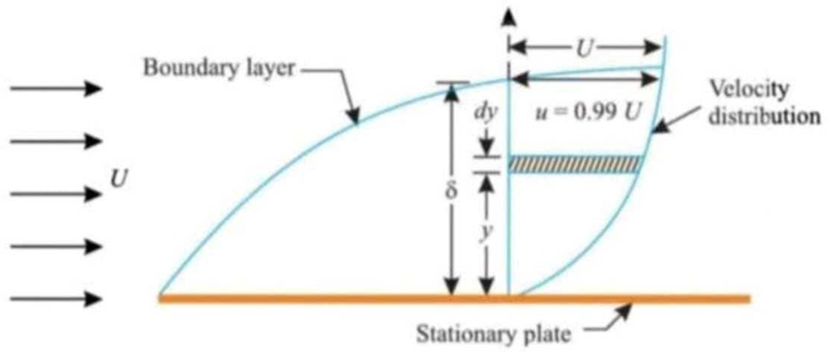


Fig. 7.2. Displacement thickness.

Assuming *unit width*, the mass flow per second through the elementary strip

$$= \rho u dy \quad \text{...(i)}$$

Mass flow per second through the elementary strip (unit width) if the plate were not there

$$= \rho\, U.dy \quad \text{...(ii)}$$

Reduction of mass flow rate through the elementary strip

$$= \rho\,(U - u)dy$$

[*The difference* $(U - u)$ *is called* **velocity of defect**]

Total reduction of mass flow rate due to introduction of plate

$$= \int_0^\delta \rho (U - u)\, dy \quad \text{...(iii)}$$

(if the fluid is incompressible)

Let the plate is displaced by a distance δ^* and velocity of flow for the distance δ^* is equal to the main/free stream velocity (*i.e.*, U). Then, loss of the mass of the fluid/sec. flowing through the distance δ^*

$$= \rho U \delta^* \quad \text{...(iv)}$$

Equating eqns. (*iii*) and (*iv*), we get

$$\rho U \delta^* = \int_0^\delta \rho (U - u)\, dy$$

or,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \quad \text{...(7.1)}$$

Momentum thickness (θ) :

"Momentum thickness" is defined as the distance through which the total loss of momentum per second be equal to if it were passing a stationary plate. It is denoted by θ.

It may also be defined as the *distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for reduction in momentum of the flowing fluid on account of boundary layer formation.*

Refer Fig. 7.2. Mass of flow per second through the elementary strip = $\rho\, u\, dy$

Momentum/sec of this fluid inside the boundary layer = $\rho\, u\, dy \times u = \rho\, u^2 dy$

Momentum/sec of the same mass of fluid before entering boundary layer = $\rho u\, U dy$

Loss of momentum/sec = $\rho u U dy - \rho u^2 dy = \rho u (U - u)\, dy$

$\therefore$ Total loss of momentum/sec.

$$= \int_0^\delta \rho u (U - u)\, dy \qquad ...(i)$$

Let θ = Distance by which plate is displaced when the fluid is flowing with a constant velocity U.

Then loss of momentum/sec of fluid flowing through distance θ with a velocity U

$$= \rho\, \theta\, U^2 \qquad ...(ii)$$

Equating eqns. (*i*) and (*ii*), we have

$$\rho \theta U^2 = \int_0^\delta \rho u (U - u)\, dy$$

or,

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad ...(7.2)$$

The momentum thickness is useful in *kinetics*.

Energy thickness (δ_e) :

"Energy thickness" is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in K.E. of the flowing fluid on account of boundary layer formation. It is denoted by δ_e.

Refer to Fig. 7.2. Mass of flow per second through the elementary strip = $\rho u dy$

K.E. of this fluid inside the boundary layer

$$= \frac{1}{2} m u^2 = \frac{1}{2} (\rho u dy)\, u^2$$

K.E. of the same mass of fluid before entering the boundary layer

$$= \frac{1}{2} (\rho u dy)\, U^2$$

Loss of K.E. through elementary strip

$$= \frac{1}{2} (\rho u dy)\, U^2 - \frac{1}{2} (\rho u dy)\, u^2 = \frac{1}{2} \rho u\, (U^2 - u^2)\, dy \qquad ...(i)$$

$\therefore$ Total loss of K.E. of fluid = $\displaystyle\int_0^\delta \frac{1}{2} \rho u (U^2 - u^2)\, dy$

Let, δ_e = Distance by which the plate is displaced to compensate for the reduction in K.E.

Then loss of K.E. through δ_e of fluid flowing with velocity U

$$= \frac{1}{2} (\rho U \delta_e) U^2 \qquad ...(ii)$$

Equating eqns. (*i*) and (*ii*), we have

$$\frac{1}{2} (\rho U \delta_e) U^2 = \int_0^\delta \frac{1}{2} \rho u (U^2 - u^2)\, dy$$

or,

$$\delta_e = \frac{1}{U^3} \int_0^\delta u (U^2 - u^2)\, dy$$

$\therefore$

$$\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy \qquad ...(7.3)$$

Problem-1: Velocity distribution : $\frac{u}{U} = \frac{y}{\delta}$

(i) Displacement thickness, δ^* $= \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

$$= \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \delta - \frac{\delta}{2}$$

$$= \delta/2$$

(ii) Momentum thickness, θ $= \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$= \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta}{2} - \frac{\delta}{3}$$

$$= \frac{\delta}{6}$$

(iii) Energy thickness, δ_e $= \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y^2}{\delta^2}\right) dy$$

$$= \frac{\delta}{2} - \frac{\delta}{4} = \frac{\delta}{4}$$

(iv) $\frac{\delta^*}{\theta} = \frac{(\delta/2)}{(\delta/6)} = 3$

(v) $\frac{\delta^*}{\delta_e} = \frac{(\delta/2)}{(\delta/4)} = 2$

Boundary Layer Equation in Laminar Flow Region

[Parallel Flow Over Flat Plate]

The governing equations of motion for the velocity boundary layer region are :

Continuity Equ^n : $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

x-Momentum Equ^n : $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \nu \dfrac{\partial^2 u}{\partial y^2}$

⇨ $\boxed{\dfrac{\delta}{x} \propto \dfrac{1}{\sqrt{Re_x}}}$, where, Local Reynolds Number, $Re_x = \dfrac{V.x}{\nu}$

Blasius Solution For Laminar Boundary Layer :

The Blasius's differential equation obtained from continuity and momentum equ^{n}s is given by

$$2\frac{d^3f}{d\eta^3} + f.\frac{d^2f}{d\eta^2} = 0$$

or, $2f''' + f.f'' = 0$

where, η = Stretching factor $= y\sqrt{\dfrac{V}{\nu . x}}$

This is an ordinary (but non-linear) homogeneous differential equation of order 3. The solution can be obtained by considering following boundary conditions.

$u = 0$ at $y = 0$

$v = 0$ at $y = 0$

$\dfrac{\partial u}{\partial y} = 0$ at $y \to \infty$.

Blasius Exact Solution : $\boxed{\delta = \dfrac{4.94\,x}{\sqrt{Re_x}}}$ or $\boxed{\delta = \dfrac{5x}{\sqrt{Re_x}}}$

and $\boxed{C_{fx} = \dfrac{0.664}{\sqrt{Re_x}}}$, C_{fx} = Local skin friction coefficient.

Von-Karman Momentum Method [Approximate Solution]

For Cubic velocity function, $\dfrac{u}{V} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$,

$\boxed{\delta = \dfrac{4.64x}{\sqrt{Re_x}}}$ and $\boxed{C_{fx} = \dfrac{0.647}{\sqrt{Re_x}}}$

* For Linear velocity profile : $\frac{u}{V} = \frac{y}{\delta}$

$$\boxed{\delta = \frac{3.46x}{\sqrt{Re_x}}}$$

KEY POINTS : ① $\delta \propto x^{1/2}$ and $\delta \propto \frac{1}{\sqrt{V}}$

B.L. Thickness (δ) increases as square root of distance x from the leading edge and inversely proportional as square root of free stream velocity, V.

② Wall Shear Stress (τ)

$$\boxed{\tau = C_{fx} \times \frac{1}{2}\rho V^2}$$

(dynamic head = $\frac{1}{2}\rho V^2$)

C_{fx} = local skin friction co-efficient or, drag co-efficient $= \frac{0.664}{\sqrt{Re_x}}$

$\Rightarrow \tau \propto \frac{1}{x^{1/2}}$ and $\tau \propto V^{3/2}$.

③ Local skin friction co-efficient C_{fx}

$C_{fx} \propto \frac{1}{x^{1/2}}$ and $C_{fx} \propto \frac{1}{V^{1/2}}$

④ Friction Drag Force (F_D) :

$\Rightarrow F_D = C_f \times \frac{1}{2}\rho V^2 \times A$ (on one side/surface of the plate)

$\Rightarrow F_D = 2 \times C_f \times \frac{1}{2}\rho V^2 \times A$ (on both sides/surfaces of the plate)

(A = Surface area of plate over which flow is taking place)

⑤ Mass Flow Rate Through Boundary Layer, $\dot{m}$

$$\boxed{\dot{m} = \frac{5}{8}\rho V \delta}$$

Example 7.1. *The velocity distribution in the boundary layer is given by :* $\frac{u}{U} = \frac{y}{\delta}$*, where u is the velocity at a distance y from the plate and u = U at y = δ, δ being boundary layer thickness. Find :*

(*i*) *The displacement thickness,* (*ii*) *The momentum thickness,*

(*iii*) *The energy thickness, and* (*iv*) *The value of* $\frac{\delta^*}{\theta}$.

Solution. Velocity distribution : $\frac{u}{U} = \frac{y}{\delta}$ ...(Given)

(*i*) **The displacement thickness, δ* :**

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \quad \text{...[Eqn. (7.1)]}$$

$$= \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy \qquad \left(\because \frac{u}{U} = \frac{y}{\delta}\right)$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta$$

$$\delta^* = \left(\delta - \frac{\delta^2}{2\delta}\right) = \delta - \frac{\delta}{2} = \frac{\boldsymbol{\delta}}{\mathbf{2}} \textbf{ (Ans.)}$$

(*ii*) **The momentum thickness, θ :**

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \quad \text{...[Eqn. (7.2)]}$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

or,
$$\theta = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\boldsymbol{\delta}}{\mathbf{6}} \textbf{ (Ans.)}$$

(*iii*) **The energy thickness, δ_e :**

$$\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy \quad \text{...[Eqn. (7.3)]}$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y^2}{\delta^2}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta = \frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3} = \frac{\delta}{2} - \frac{\delta}{4} = \frac{\delta}{4}$$

i.e.,
$$\delta_e = \frac{\boldsymbol{\delta}}{\mathbf{4}} \textbf{ (Ans.)}$$

(*iv*) **The value of $\frac{\delta^*}{\theta}$:**

$$\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = \mathbf{3.0} \textbf{ (Ans.)}$$

Example 7.2. *The velocity distribution in the boundary layer is given by* $\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$, *δ being boundary layer thickness.*

Calculate the following :

(*i*) *The ratio of displacement thickness to boundary layer thickness* $\left(\frac{\delta^*}{\delta}\right)$.

(*ii*) *The ratio of momentum thickness to boundary layer thickness* $\left(\frac{\theta}{\delta}\right)$.

Solution. Velocity distribution : $\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$ ...(Given)

(*i*) δ^*/δ :

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\frac{y^2}{\delta^2}\right) dy$$

$$= \left[y - \frac{3}{2} \times \frac{y^2}{2\delta} + \frac{1}{2} \times \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \left[\delta - \frac{3}{4} \cdot \frac{\delta^2}{\delta} + \frac{1}{2} \times \frac{\delta^3}{3\delta^2}\right] = \left(\delta - \frac{3}{4}\delta + \frac{\delta}{6}\right) = \frac{5}{12}\delta$$

∴
$$\frac{\boldsymbol{\delta}^*}{\boldsymbol{\delta}} = \frac{\mathbf{5}}{\mathbf{12}}$$

(*ii*) θ/δ :

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}\right)\left(1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\frac{y^2}{\delta^2}\right) dy$$

$$= \int_0^\delta \left(\frac{3}{2}\frac{y}{\delta} - \frac{9}{4}\frac{y^2}{\delta^2} + \frac{3}{4} \cdot \frac{y^3}{\delta^3} - \frac{1}{2}\frac{y^2}{\delta^2} + \frac{3}{4}\frac{y^3}{\delta^3} - \frac{1}{4}\frac{y^4}{\delta^4}\right) dy$$

$$= \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \left(\frac{9}{4}\frac{y^2}{\delta^2} + \frac{1}{2}\frac{y^2}{\delta^2}\right) + \left(\frac{3}{4}\frac{y^3}{\delta^3} + \frac{3}{4}\frac{y^3}{\delta^3}\right) - \frac{1}{4}\frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{11}{4}\frac{y^2}{\delta^2} + \frac{3}{2}\frac{y^3}{\delta^3} - \frac{1}{4}\frac{y^4}{\delta^4}\right] dy$$

$$= \left[\frac{3}{2} \times \frac{y^2}{2\delta} - \frac{11}{4} \times \frac{y^3}{3\delta^2} + \frac{3}{2} \times \frac{y^4}{4\delta^3} - \frac{1}{4} \times \frac{y^5}{5\delta^4} \right]_0^\delta$$

$$= \left[\frac{3}{2} \times \frac{\delta^2}{2\delta} - \frac{11}{4} \times \frac{\delta^3}{3\delta^2} + \frac{3}{2} \times \frac{\delta^4}{4\delta^3} - \frac{1}{4} \times \frac{\delta^5}{5\delta^4} \right]_0^\delta$$

$$= \left(\frac{3}{4}\delta - \frac{11}{12}\delta + \frac{3}{8}\delta - \frac{1}{20}\delta \right) = \frac{19}{120}\delta$$

or, $$\frac{\theta}{\delta} = \frac{19}{120} \quad \textbf{(Ans.)}$$

Example 7.5. *Air is flowing over a smooth flat plate with a velocity of 12 m/s. The velocity profile is in the form :*

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

The length of the plate is 1.1 m and width 0.9 m. If laminar boundary layer exists upto a value of $Re = 2 \times 10^5$ and kinematic viscosity of air is 0.15 stokes, find :

(*i*) *The maximum distance from the leading edge upto which laminar boundary layer exists, and*

(*ii*) *The maximum thickness of boundary layer.*

Solution. Velocity distribution : $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Velocity of air, $U = 12$ m/s

Length of plate, $L = 1.1$ m

Width of plate, $B = 0.9$ m

Reynolds number upto which laminar boundary exists, $Re = 2 \times 10^5$

Kinematic viscosity of air, $\nu = 0.15$ stokes $= 0.15 \times 10^{-4}$ m²/s

(*i*) The maximum distance from the leading edge upto which laminar boundary layer exists, x:

$$Re_x = \frac{Ux}{\nu} \quad \text{or} \quad 2 \times 10^5 = \frac{12 \times x}{0.15 \times 10^{-4}}$$

or, $$x = \frac{2 \times 10^5 \times 0.15 \times 10^{-4}}{12} = \mathbf{0.25\,m} \textbf{ (Ans.)}$$

(*ii*) The maximum thickness of boundary layer, δ :

For the given velocity profile, the maximum thickness of boundary layer is given by

$$\delta = \frac{5.48\,x}{\sqrt{Re_x}}$$

$$= \frac{5.48 \times 0.25}{\sqrt{2 \times 10^5}} = 0.00306\,\text{m} \quad \text{or} \quad \mathbf{3.06\,mm} \textbf{ (Ans.)}$$

Example 7.8. *Air is flowing over a flat plate 5 m long and 2.5 m wide with a velocity of 4 m/s at 15°C. If ρ = 1.208 kg/m^3 and ν = 1.47 × 10^{-5} m^2/s, calculate :*

(*i*) *Length of plate over which the boundary layer is laminar, and thickness of the boundary layer (laminar).*

(*ii*) *Shear stress at the location where boundary layer ceases to be laminar, and*

(*iii*) *Total drag force on the both sides on that portion of plate where boundary layer is laminar.*

Solution. Length of the plate, $L = 5$ m

Width of the plate, $B = 2.5$ m

Velocity of air, $U = 4$ m/s

Density of air, $\rho = 1.208$ kg/m^3

Kinematic viscosity of air, $\nu = 1.47 \times 10^{-5}$ m^2/s

(*i*) Length of plate over which the boundary layer is laminar :

Reynolds number $$Re_x = \frac{UL}{\nu} = \frac{4 \times 5}{1.47 \times 10^{-5}} = 1.361 \times 10^6$$

Hence on the front portion, boundary layer is laminar and on the rear, it is turbulent.

$$Re_x = \frac{Ux}{\nu} = 5 \times 10^5$$

$$\therefore \quad \frac{4 \times x}{1.47 \times 10^{-5}} = 5 \times 10^5$$

or, $$x = \frac{5 \times 10^5 \times 1.47 \times 10^{-5}}{4} = 1.837 \text{ m}$$

Hence the boundary layer is **laminar on 1.837 m length of the plate. (Ans.)**

Thickness of the boundary layer (laminar), δ

$$= \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 1.837}{\sqrt{5 \times 10^5}} = 0.01299 \text{ m or } \mathbf{12.99 \text{ mm}} \text{ (Ans.)}$$

(*ii*) Shear stress at the location where boundary layer ceases to be laminar, τ_0 :

Local coefficient of drag, $$C_{fx} = \frac{0.664}{\sqrt{5 \times 10^5}} = 0.000939$$

$$\therefore \quad \tau_0 = C_{fx} \times \frac{1}{2}\rho U^2 = 0.000939 \times \frac{1}{2} \times 1.208 \times 4^2$$

$$= \mathbf{0.00907 \text{ N/m}^2 \text{ (Ans.)}}$$

(*iii*) Total drag force on both sides of plate, F_D :

$$F_D = 2\overline{C}_f \times \frac{1}{2}\rho A U^2$$

where, $\overline{C}_f$ = Average coefficient of drag (or skin friction) = $\dfrac{1.328}{\sqrt{5 \times 10^5}} = 1.878 \times 10^{-3}$

and A = Area of the plate = 1.837 × 2.5 = 4.59 m^2

$$\therefore \quad F_D = 2 \times 1.878 \times 10^{-3} \times \frac{1}{2} \times 1.208 \times 4.59 \times 4^2 = \mathbf{0.167 \text{ N}} \text{ (Ans.)}$$

Dimensionless Numbers & Their Significance

Reynold's Number (Re):

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{VL_c}{\nu} = \frac{\rho V L}{\mu}$$

- Reynolds number is indicative of the relative importance of inertia force and viscous effects in a fluid motion.
- At low Reynolds number, the viscous effects dominate and the fluid motion is Laminar.
- At high Reynolds number, the inertial effects lead to turbulent flow and the associated turbulence level dominates the momentum and energy flux.
- Reynolds number plays an important role in forced Convection heat transfer.

Prandtl Number (Pr):

The relative thickness of the velocity boundary layer (δ) and the thermal boundary layer (δ_{th}) is best described by a dimensionless parameter called as Prandtl number

- $$Pr = \frac{\text{Molecular diffusivity of momentum } (\nu)}{\text{Molecular diffusivity of heat } (\alpha)} = \frac{\nu}{\alpha} = \frac{\mu C_p}{K}$$
- 'Pr' provides a measure of the relative effectiveness of momentum and energy transport by diffusion.
- For air and gases, $Pr = 1$, which indicates that both momentum and heat dissipate through the fluid at about the same rate.
- For liquid metals (such as 'Na', Mercury, Lead, etc.), Prandtl number has a very low value (0.004 - 0.030) due to very high value of thermal conductivity (K) for those fluids. Therefore, liquid metals can be used as coolants in applications where large amount of heat must be removed from a relatively small space as in a Nuclear reactor, Heat Exchangers unit, etc.
-

Fluid	Pr
Liquid Metals	0.004 - 0.030
Air and Gases	0.7 - 1.0
Water	1.7 - 13.7
Light Organic fluids	5 - 50
Oils	50 - 1,00,000
Glycerin	2000 - 1,00,000

Nusselt Number (Nu) :

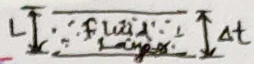
The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection (when there is motion of fluid) to conduction across the same fluid layers.

$$Nu = \frac{\dot{q}_{conv.}}{\dot{q}_{cond.}} = \frac{h.\Delta t}{K.\frac{\Delta t}{L}} = \frac{hL}{K}$$

$$* \; Nu = \frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{\dot{q}_{cond} + \dot{q}_{advection}}{\dot{q}_{cond}}$$

$$= 1 + \frac{\dot{q}_{advection}}{\dot{q}_{conduction}}$$

$$\therefore Nu \geq 1$$



Sometimes, $\boxed{Nu = \frac{hL_c}{K}}$, L_c = Characteristic length.

- The larger the Nusselt numbers, the more effective is the convection heat transfer.
- A Nusselt number, $Nu = 1$ for a fluid layer represents heat transfer across the layer is by 'Pure Conduction'
- The Nusselt number is a convenient measure of the convective heat transfer co.efficient, h in both natural and forced convection.

Stanton Number (St) :

$$\boxed{St = \frac{h}{\rho C_p V} = \frac{Nu}{Re \times Pr}}$$

- The Stanton number, St is a dimensionless numbers that measure the ratio of heat transferred into a fluid to the thermal capacity of the same fluid.
- Its significance to be specific and is used for ease of calculation of convective heat transfer co.efficient (h) in Forced convection heat transfers.

Peclet Number (Pe) :

It is the ratio of heat flow rate by convection to the flow rate by conduction under a unit temperature gradient and through thickness L.

$$\dot{Q}_{conv} = mc_p \Delta t^{\nearrow 1} = (\rho AV).C_p$$

$$\dot{Q}_{cond.} = KA\frac{\Delta t^{\nearrow 1}}{L} = \frac{KA}{L}$$

$$\therefore \; Pe = \frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = \frac{(\rho AV).C_p}{\left(\frac{KA}{L}\right)} = \frac{VL}{\alpha} \qquad \left(\because \alpha = \frac{K}{\rho C_p}\right)$$

$$\boxed{Pe = \frac{VL}{\alpha}}$$

Also, $\boxed{Pe = Re \times Pr}$

Grashof number (Gr) :

The Grashof number is a dimensionless number in fluid dynamics and heat transfer which approximates *the ratio of the buoyancy to viscous force acting on a fluid.*

- It arises in the study of situations involving natural convection and is analogous to the Reynolds number

- Using the Energy equation and the buoyant force combined with Dimensional Analysis provides two different ways to derive the Grashof number.

$$\mathrm{Gr} = \frac{g\beta(T_s - T_0)L_c^3}{\nu^2}.$$

$$Gr = \frac{\textit{buoyant forces}}{\textit{viscous forces}}$$

- In forced convection , the **Reynolds number (Re)** governs the fluid flow.
- But, in natural convection the **Grashof number (Gr)** is the dimensionless parameter that governs the fluid flow.

where:

g is acceleration due to Earth's gravity
β is the coefficient of thermal expansion (equal to approximately $1/T$, for ideal gases)
T_s is the surface temperature
T_∞ is the bulk temperature
L is the vertical length
D is the diameter
ν is the kinematic viscosity.

Rayleigh Number (Ra) :

The Rayleigh number (Ra) is a dimensionless number that characterizes convection problems in heat transfer. Rayleigh Number is the product of Grashof number (Gr) and Prandtl Number (Pr).

Rayleigh Number,Ra = Gr.Pr

Graetz Number

$$G_z = (\text{Peclet No.}) \times \frac{D_w}{L} = P_e \times \left(\frac{D_w}{L}\right) = \frac{\dot{m} C_p}{KL}$$

$$= \frac{\text{HT by convection}}{\text{HT by conduction}} \times \left(\frac{D_w}{L}\right)$$

* This no. is related only for the heat flow to the fluid through circular pipes.

Thermal Boundary Layer

- Consider the flow of a fluid at a temperature of t_∞ (or t_f) over an isothermal flat plate at temperature, t_s. The fluid particles in the layer adjacent to the surface reach thermal equilibrium with the plate and assume the surface temp. t_s. (Here, we have considered, $t_s < t_\infty$). These fluid particles then exchange energy with the particles in the adjoining fluid layer and so on. As a result, a temp. profile develops in the flow field that ranges from t_s (at the surface) to t_∞ (sufficiently far from the surface).

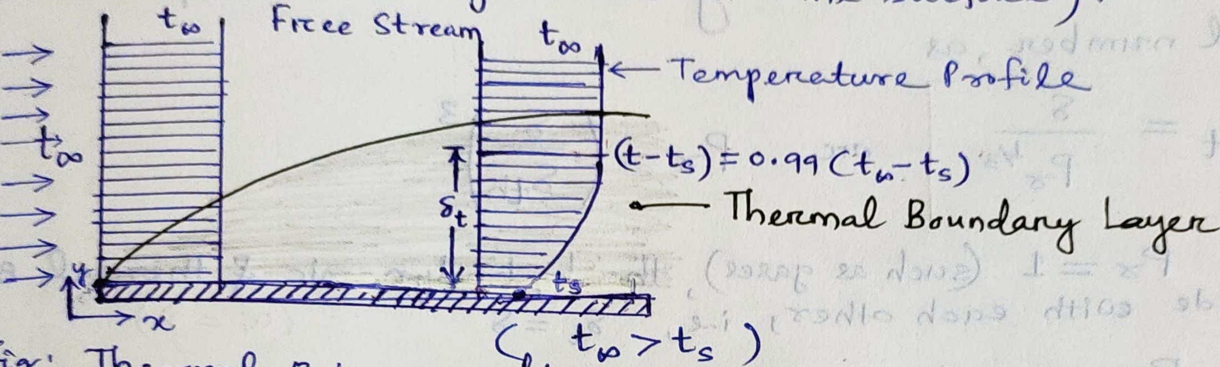


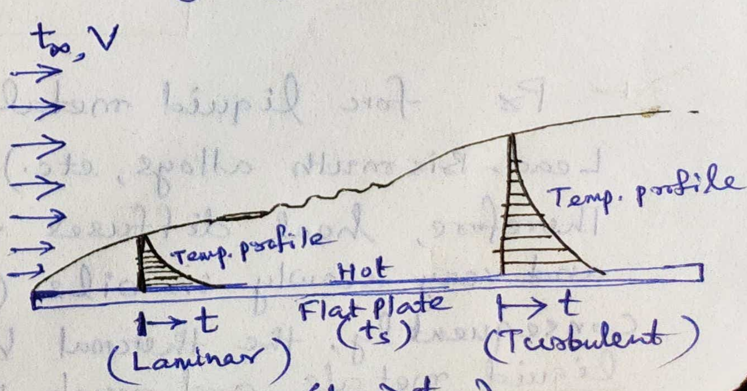
Fig: Thermal B.L. on a flat plate
[the flowing fluid is hotter than the plate surface ($t_\infty > t_s$)]

- The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is called the thermal boundary layer.
- The thickness of the thermal boundary layer, δ_t at any location along the surface is the distance from the surface at which the temperature difference $(t-t_s) = 0.99\,(t_\infty - t_s)$.

  For a special case of $t_s = 0$, we have $t = 0.99\,t_\infty$

- The thickness of the thermal boundary layer increases in the flow direction.

Imp
- At the top of boundary layers, the temperature gradient assumes as ZERO value, because, at that location/region, temp. changes are negligible.



- In flow over a heated or cooled surface, both velocity- and thermal boundary layers develop simultaneously.

Imp

- The fluid velocity has a strong influence on the temp.-profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat surface.

Relationship Between 'δ' and 'δ_t':

The relative thickness of the velocity and thermal boundary layers is best described by the dimensionless parameter Prandtl number, as

$$\boxed{\delta_t = \frac{\delta}{P_r^{1/3}}} \quad \text{or} \quad P_r = \left(\frac{\delta}{\delta_t}\right)^3$$

→ When $P_r = 1$ (such as gases), the hydrodynamic & thermal B.L. coincide with each other, i.e., $\delta_t = \delta$ $(\alpha = \nu)$

→ When $P_r < 1$ (such as liquid metals), then $\delta_t > \delta$:

the thermal boundary layer increases rapidly compared to the hydrodynamic B.L. $(\alpha >> \nu)$

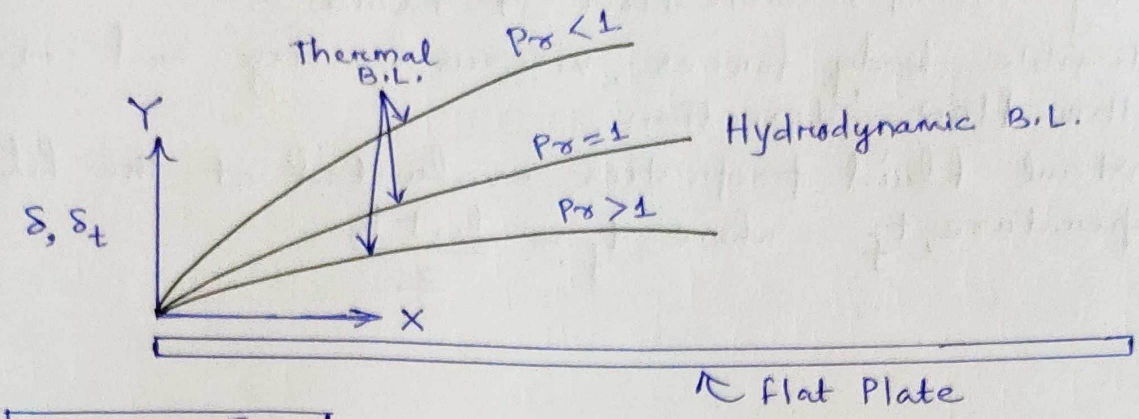
→ When $P_r > 1$ (such as oils), then $\delta_t < \delta$:

the hydrodynamic boundary layer thickness increases rapidly compared to the thermal B.L. $(\alpha << \nu)$

Prandtl Number, P_r = $\dfrac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusity of heat}}$ (i.e., Kinem Viscosi

Or $\boxed{P_r = \dfrac{\nu}{\alpha} = \dfrac{\mu c_p}{K}}$

→ P_r for liquid metals (such as Sodium, Mercury, Lead, Lead-Bismuth alloys, etc.) ranges from 0.004 – 0.030. Therefore, heat diffuses very quickly in liquid metals ($P_r << 1$) and very slowly in oils ($P_r >> 1$) relative to momentum. Consequently, the thermal b.L. layer is much thicker for liquid metals and much thinner for oils relative to the Velocity B.L.



- $$\boxed{\delta_t = \frac{\delta}{P_r^{1/3}}}$$ ← is called Pohlhausen Solution. [or Prandtl's $\frac{1}{3}$rd Law]

- Liquid metals such as Mercury have high thermal-conductivities and are commonly used in applications that requires high heat transfer rates.
However, they have very small Prandtl numbers, and thus the thermal boundary layer develops much faster than the velocity boundary layer.

- $$\delta_t = \frac{\delta}{P_r^{1/3}} = \frac{5x}{Re^{1/2} \cdot P_r^{1/3}}$$
i.e., the thickness of thermal B.L. is proportional to $\sqrt{x}$.

- The concept of thermal B.L. is analogous to that of hydrodynamic or velocity B.L.; the parameters affecting their growth are, however, different.

⇒ The velocity profile of the hydrodynamic B.L. is dependent primarily upon the viscosity of the flowing fluid.

⇒ In a thermal B.L., the temp. profile depends upon the flow velocity, specific heat, viscosity, and thermal conductivity of the flowing fluid.

- The energy equation in the thermal B.L. ~~over~~ for flow past over a flat plate is given by

$$\boxed{u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \cdot \frac{\partial^2 t}{\partial y^2}}$$

The assumptions for the above energy equ^n are

(i) steady incompressible flow of fluid

(ii) negligible body forces, viscous heating and conduction in the flow direction,

(iii) constant fluid properties evaluated at the film-temperature, t_f where $t_f = \dfrac{t_\infty + t_s}{2}$

Bijan Kumar Giri

EXTERNAL FORCED CONVECTION

External forced convection involves flow of liquid and gas ~~th~~ over flat plate, curved surfaces or flow across/over the cylinders & spheres, characterized by the freely growing boundary layers surrounded by a free flow region.

Generally, the most practical cases of external forced convection heat transfer are,

(A) Parallel flow over flat plates

(B) Flow over curved surfaces, eg. turbine blades

(C) Flow across Cylinders and Spheres.

Friction and Pressure Drag:

The force a flowing fluid exerts on a body in the flow direction is called drag or drag force (F_D).

Drag force (F_D) is due to the combined effects of pressure and wall shear forces in the flow direction.

- In case of a thin flat plate aligned parallel to the flow direction, the drag force, F_D depends on the wall shear (skin friction) only and is independent of pressure.
- When the flat plate is normal to the flow direction, the drag force, F_D depends on the pressure only and is independent of the wall shear.
- Drag Co-efficient, $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$
- The total drag co-efficient, $C_D = C_{D,\,friction} + C_{D,\,pressure}$

For Flat Plate, $\boxed{C_D = C_{D,\,friction} = C_f}$, $C_{D,\,pressure} = 0$

* The total drag is ZERO for the case of ideal inviscid fluid flow.

Skin Friction Co.efficient (C_f):

Laminar : $C_{fx} = \frac{0.664}{Re_x^{1/2}}$ and $C_f = \frac{1.328}{Re_L^{1/2}}$

Turbulent : $C_{fx} = \frac{0.059}{Re_x^{1/5}}$ and $C_f = \frac{0.074}{Re_L^{1/5}}$

$$\boxed{\begin{array}{l} C_{fx} \propto Re_x^{-1/2} \text{ and thus, } C_{fx} \propto x^{-1/2} \text{ for Laminar flow} \\ \text{and } C_{fx} \propto Re_x^{-1/5} \text{ and thus, } C_{fx} \propto x^{-1/5} \text{ for Turbulent flow} \end{array}}$$

- C_{fx} is supposedly infinite at the leading edge ($x=0$), decreases by a factor of $x^{-1/2}$ in the flow dir^n for the Laminar flow, and then again reaches its max^m value when flow becomes fully turbulent.
- The local skin friction co.efficients are higher in Turbulent flow than in laminar flow, because of intense mixing that occurs in the turbulent B.L.
- Sometimes, the average skin friction co.efficient, C_f is evaluated over a long plate as

$$C_f = \frac{1}{L}\left[\int_0^{x_{cr}} C_{fx,\text{Laminar}}\,dx + \int_{x_{cr}}^{L} C_{fx,\text{Turbulent}}\,dx\right]$$

- For Laminar flow, the friction co.efficient (C_f) depends on only the Reynolds number (Re) and relative surface-roughness ($\frac{\varepsilon}{L}$) has no effect.
- For Turbulent flow, the friction co.efficient (C_f) depends on only surface roughness and is independent of Re.

Heat Transfer Co.efficient, h

Laminar : $Nu_x = \frac{h_x x}{K} = 0.332\, Re_x^{0.5} \cdot Pr^{\frac{1}{3}}$

$h_x = 0.332\left(\frac{K}{x}\right) \cdot Re_x^{0.5} \cdot Pr^{\frac{1}{3}}$

$\Bigg\}$ $Pr < 0.6$, $Re_x < 5\times10^5$

Turbulent : $Nu_x = \frac{h_x x}{K} = 0.0296\, Re_x^{0.8} \cdot Pr^{1/3}$

$h_x = 0.0296\left(\frac{K}{x}\right) Re_x^{0.8} \cdot Pr^{1/3}$

$\Bigg\}$ $0.6 \le Pr \le 60$, $5\times10^5 \le Re_x \le 10^7$

- h_x is proportional to $Re_x^{0.5}$ or $\sqrt{Re_x}$ for laminar flow, and thus $h_x \propto x^{-0.5}$ in the Laminar region.
- 'h_x' is <u>infinite</u> at the leading edge $(x=0)$ and decreases by a factor of $x^{-0.5}$ in the flow dirn. and finally, h_x reaches its highest values when the flow becomes <u>fully turbulent</u>, and then decreases by a factor of $x^{-0.2}$ in the flow dirn.
- In some cases, 'h' is evaluated over along flat plate as

$$h = \frac{1}{L}\left[\int_0^{x_{cr}} h_{x,\,laminar}\,dx + \int_{x_{cr}}^{L} h_{x,\,turbulent}\,dx\right]$$

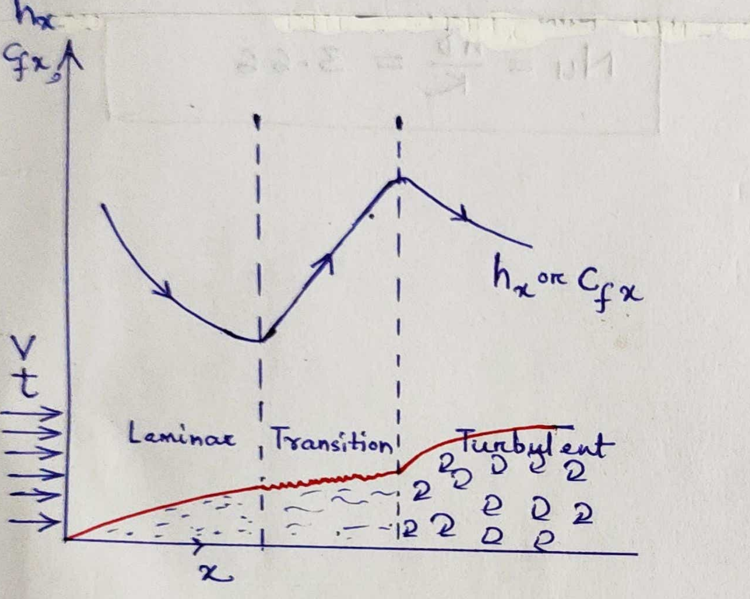


Fig:- The variation of the local friction and heat transfer co-efficients for flow over a flat plate

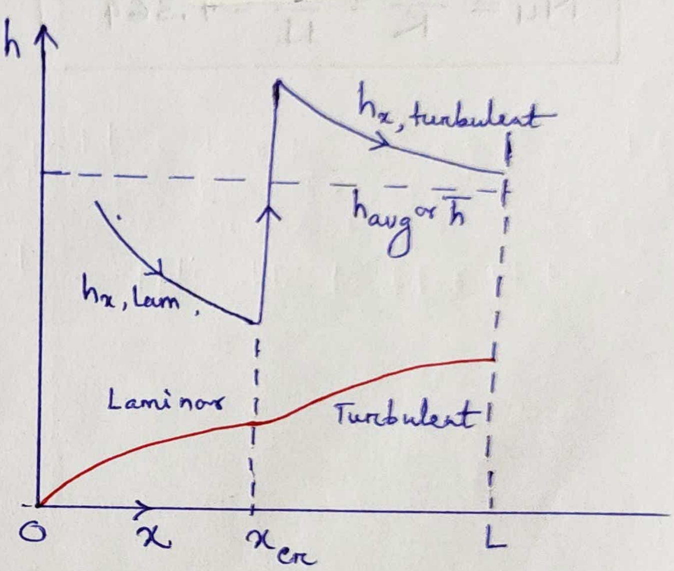


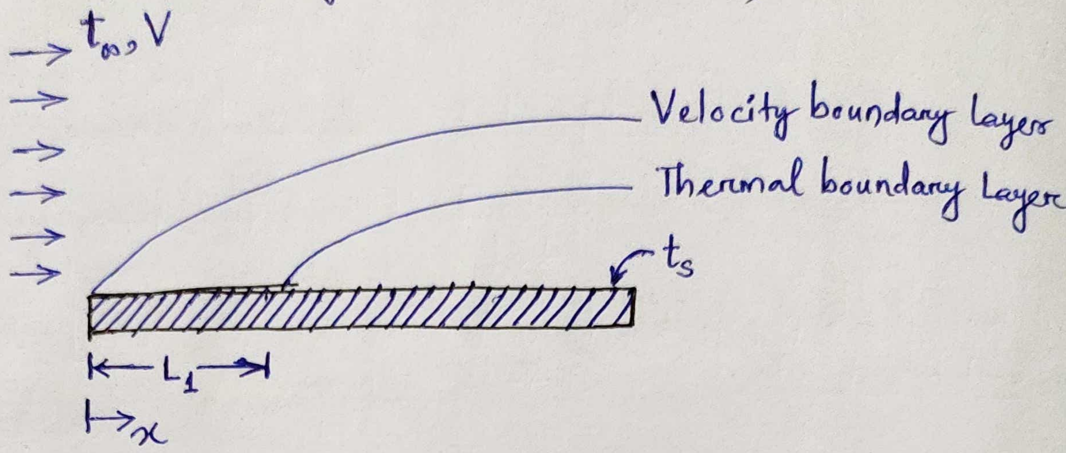
Fig:- Graphical representation of the average heat transfer co-efficient for a flat plate with combined laminar and turbulent flow.

* $C_{fx} \propto \frac{1}{Re_x^{1/2}}$ (or $Re_x^{-1/2}$) and thus to $x^{-1/2}$ for laminar flow, therefore, C_{fx} is supposedly <u>infinity</u> (∞) at the leading edge $(x=0)$) and <u>decreases</u> by a factor of $x^{-1/2}$ in the flow direction.

* The local friction co-efficients are <u>higher</u> in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. C_{fx} reaches its <u>highest</u> values when the flow becomes fully turbulent, and then <u>decreases</u> by a factor of $x^{-1/5}$ in the flow direction.

Flat Plate with Unheated Starting Length

Consider a flat plate of total length L with an unheated starting section of length L_1 and thus, there is no heat transfer for $0 < x < L_1$. In such cases, velocity B.L. starts to develop at the leading edge ($x = 0$), but the thermal B.L. starts to develop where the heating starts ($x = L_1$)



* For Laminar Flow Over a Flat Plate

$C_f = 2 C_{fx}$,

$h = 2 h_x$,

$Nu = 2 Nu_x$,

For Turbulent flow Over a Flat Plate

$C_f \neq 2 C_{fx}$ and $C_f = \frac{5}{4} C_{fx}$

$h \neq 2 h_x$ and $h = \frac{5}{4} h_x$

$Nu \neq 2 Nu$ and $Nu = \frac{5}{4} Nu_x$

Uniform Heat Flux

When a flat plate is subjected to uniform heat flux instead of uniform temperature (i.e., isothermal plate), then

Laminar : $Nu_x = 0.453\, Re_x^{0.5} \cdot Pr^{1/3}$, $Pr > 0.6$, $Re_x < 5 \times 10^5$

Turbulent : $Nu_x = 0.0308\, Re_x^{0.8} \cdot Pr^{1/3}$, $0.6 \leq Pr \leq 60$, $5 \times 10^5 \leq Re_x \leq 10^7$

- These relationships give values that are 36 % higher for laminar flow and 4 % higher for turbulent flow relative to the isothermal plate condition.

Q. For laminar flow of a fluid along a flat plate, one would expect the largest local convective heat transfer co-efficient for the same Reynolds and Prandtl numbers when

a) The same temp. is maintained on the surface
b) The same heat flux is maintained on the surface
c) The plate has an unheated section
d) The plate surface is polished

Ans: (b)

Flow Across Cylinders and Spheres :

Flow across the cylinders and spheres can also be considered as external flow.

Ex: The tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes.

Characteristics length, $L_c = \frac{A_s}{P}$

For Cylinders and Spheres, $\boxed{L_c = D}$ ← External diameter of Cylinder or Sphere

Reynolds Number, $Re = Re_D = \frac{VL_c}{\nu} = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$

- The critical Reynolds number (Re_{cr}) for flow across a circular cylinder or sphere is about, $Re_{cr} = 2\times10^5$ and boundary layer remains Laminar for $Re < 2\times10^5$, boundary Layer becomes Turbulent for $Re \geq 2\times10^5$
- The nature of the flow across a cylinder or sphere strongly affects the total drag co-efficient, C_D. Both the friction drag and pressure drag can be significant in flow-across the cylinder and sphere.
- The drag force (F_D) is primarily
  → due to friction drag at low Reynolds numbers ($Re<10$)
  → due to pressure drag at high Reynolds numbers ($Re>5000$)

and both effects are significants at intermediate Reynolds nos.

Therefore, the average drag co-efficient, C_D for a smooth-single circular cylinder and a sphere is a function of Reynolds number, i.e., $\boxed{C_D = f(Re_D)}$

Nusselt Number (Nu) : $\left[Nu = \frac{h.L}{K}\right]$

⇨ Local Nusselt Number, $Nu_x = 0.332\ Re_x^{1/2} \cdot Pr^{1/3}$

⇨ Average Nusselt Number, $Nu = 0.664\ Re_L^{1/2} \cdot Pr^{1/3}$

Heat Transfer Co-efficient (h) :

⇨ Local heat transfer co-efficient, $h_x = 0.332\left(\frac{K}{x}\right) \cdot Re_x^{1/2} \cdot Pr^{1/3}$

⇨ Average heat transfer co-efficient, $h_L = 0.664\left(\frac{K}{L}\right) \cdot Re_L^{1/2} \cdot Pr^{1/3}$

$$* \quad h \propto Re_x^{1/2} \cdot \frac{1}{x}$$

$$\propto x^{1/2} \cdot x^{-1}$$

$$\Rightarrow \boxed{h \propto x^{-1/2}}$$

Average Heat Transfer Co-efficient (h)

Average value of heat transfer co-efficient is evaluated on the complete or whole length of the plate and is given by

$$h = \frac{1}{L}\int_0^L h_x\, dx = \frac{1}{L}\int_0^L 0.332\left(\frac{K}{x}\right) \cdot \left(\frac{\rho V x}{\mu}\right)^{1/2} \cdot Pr^{1/3} \cdot dx$$

$$= \frac{K}{L} \times 0.332 \times \left(\frac{\rho V}{\mu}\right)^{1/2} \times Pr^{1/3} \int_0^L x^{-1/2}\, dx$$

$$= \frac{K}{L} \times 0.332 \times \left(\frac{\rho V}{\mu}\right)^{1/2} \times Pr^{1/3} \times 2L^{1/2}$$

$$= 0.664\left(\frac{K}{L}\right) Re_L^{1/2}\, Pr^{1/3}$$

⇒ $\boxed{\text{Avg. heat transfer co. eff., } h = 2 \times \text{Local heat transfer co-efficient}}$

Average Skin friction Co-efficient (C_f):

Average value of skin friction co-efficient is evaluated on the full length of the plate and is given by

$$C_f = \frac{1}{L}\int_0^L C_{fx} \cdot dx = \frac{1.328}{\sqrt{Re_L}} \qquad \left[\text{i.e., } C_f = 2\, C_{fx}\right]$$

Turbulent Flow

In turbulent flow, the fluid particles move in a zig-zag way through the fluid. In a turbulent flow, the irregular velocity fluctuations are mainly responsible for heat as well as momentum transfer. Consequently, the turbulent flow, the rates of heat and momentum transfer and the associated friction and heat transfer co.efficients are several times larger than that in laminar.

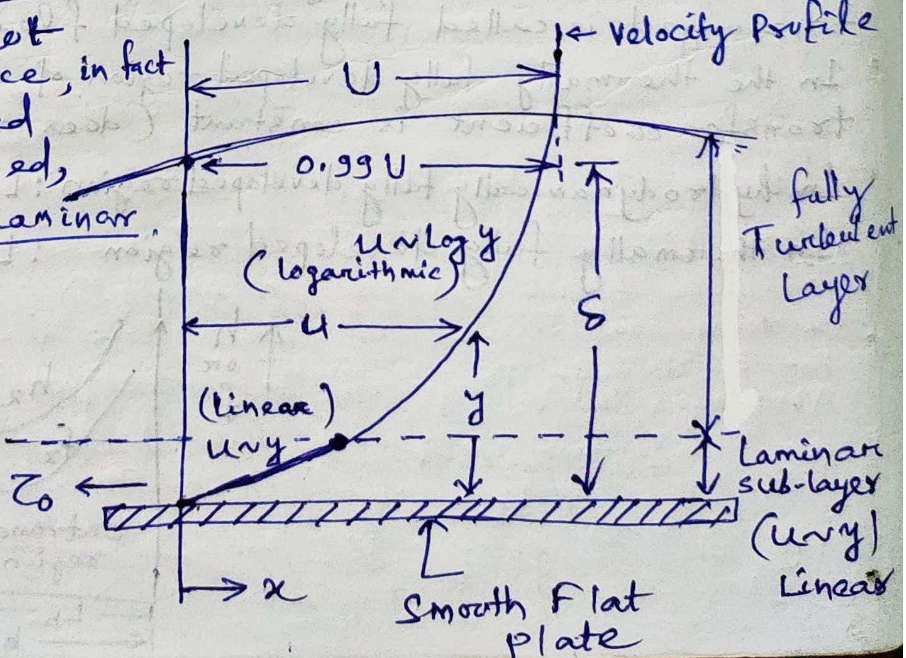
(A) Forced Convection- Flow Over a Flat Plate

- As compared to laminar boundary layers, the turbulent boundary layers are thicker.
- In a turbulent boundary layer, the velocity distribution is ~~more~~ much more uniform, than in a laminar boundary layer, due to intermingling of fluid particles betn different layers of fluid.
- The velocity distribution in a turbulent boundary layer follows a logarithmic law i.e., $u \sim \log y$, which can be represented by a power law of the type,

$$\boxed{\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}} \qquad 5 \times 10^5 < Re < 10^7$$

This is known as 'one-seventh power-law'.

- The turbulent B.L. does not extend to the solid surface, in fact an extremely thin layer, called Laminar sub-layer is formed, wherein the flow is essentially laminar.



Corelations for Flat Plate

Laminar B.L. Thickness, $\delta \propto x^{1/2}$
Turbulent B.L. Thickness, $\delta \propto x^{4/5}$

* $\delta = \dfrac{0.38x}{(Re_x)^{1/5}}$

* Local skin friction co-efficient, $C_{fx} = \dfrac{0.059}{(Re_x)^{1/5}}$

* Shear stress, $\tau_0 = C_{fx} \times \frac{1}{2}\rho U^2$

* Average skin friction coefficient, $\overline{C}_f = \dfrac{0.073}{(Re_L)^{1/5}}$

$$\left(\begin{array}{c} 5\times10^5 \le Re_L \le 10^7 \\ 0.6 \le Pr \le 60 \end{array}\right)$$

* Friction Drag force, $F_D = \overline{C}_f \times \frac{1}{2} \times \rho U^2 \times A$

* $Nu_x = 0.0296\,(Re_x)^{0.8}\,(Pr)^{1/3} = \dfrac{h_x x}{K}$

$$h_x = 0.0296\left(\frac{K}{x}\right)\cdot(Re_x)^{0.8}\cdot(Pr)^{1/3}$$

* $\overline{h} = \dfrac{1}{L}\displaystyle\int_0^L h_x\,dx = \dfrac{1}{L}\int_0^L 0.0296\left(\frac{K}{x}\right)\cdot(Re_x)^{0.8}\cdot(Pr)^{1/3}\,dx$

$$= \frac{K}{L}\times 0.0296\times\left(\frac{\rho U}{\mu}\right)^{0.8}\times Pr^{1/3}\int_0^L x^{0.8-1}\,dx$$

$$= \frac{K}{L}\times 0.0296\times\left(\frac{\rho U}{\mu}\right)^{0.8}\times Pr^{1/3}\int_0^L x^{-0.2}\,dx$$

$$= \frac{K}{L}\times 0.0296\times\left(\frac{\rho U}{\mu}\right)^{0.8}\times Pr^{1/3}\times\left(\frac{5}{4}L^{4/5}\right)$$

$$\Rightarrow \overline{h} = 0.037\times\left(\frac{K}{L}\right)\times(Re_L)^{0.8}\times Pr^{1/3}$$

$\therefore$ $\boxed{\overline{h} = \frac{5}{4}h_x}$ and hence, $\boxed{\overline{Nu} = \frac{5}{4}Nu_x}$

* $\overline{C}_f = \dfrac{1}{L}\displaystyle\int_0^L C_{fx}\,dx$ $\quad\left(\frac{5}{4} = 1.25\right)$

$\therefore$ $\boxed{\overline{C}_f = \frac{5}{4}C_{fx}}$

* **Thermal boundary layer thickness:**

$$(\delta_h)_{turb} \approx (\delta_t)_{turb}$$

Important Notes

① For laminar flow over a flat plate, $\overline{Nu} = 0.664\, Re^{0.5} \cdot Pr^{0.33}$

For turbulent flow over a flat plate, $\overline{Nu} = 0.037\, Re^{0.8}\, Pr^{0.33}$

② For Laminar flow over a flat plate, $h \propto \frac{1}{\sqrt{L}}$ or $h \propto L^{-1/2}$

i.e., $\boxed{\frac{h_2}{h_1} = \left(\frac{L_1}{L_2}\right)^{1/2}}$

For turbulent flow over a flat plate, $h \propto \frac{1}{L^{1/5}}$ or $h \propto L^{-1/5}$

i.e., $\boxed{\frac{h_2}{h_1} = \left(\frac{L_1}{L_2}\right)^{1/5}}$

Reynold's Analogy

This analogy represents the inter-relationship between fluid frict and Newton's law of viscosity.

- This analogy helps to determine the heat transfer coefficient for fluids with $Pr \approx 1$ (like, gases, air, etc.)
- $\boxed{\frac{Nu_x}{Re_x} = \frac{C_{fx}}{2} = \frac{f'}{8}}$ f' = Darcy's friction factor = $4C_f$
- Stanton Number, $St = \frac{Nu_x}{Re_x\, Pr} = \frac{C_{fx}}{2}$ with $Pr = 1$

Modified Reynolds Analogy or Chilton-Colburn Analogy

- This analogy is used to determine the heat transfer co-efficient 'h' from the measurement of fluid friction coefficient (C_f) and is applicable to flat plate [as skin friction is significant in case of a flat plate and drag co-efficient is negligible]
- $\boxed{St_x \cdot Pr^{\frac{2}{3}} = \frac{C_{fx}}{2}}$ (Valid for $0.5 < Pr < 50$)
- $St = \frac{h}{\rho C_p V}$
- For $Pr = 1$, the Reynold's analogy and Colburn analogy are the same.

INTERNAL FORCED CONVECTION

Internal forced convection involves liquid or gas flow through pipes or ducts and is commonly used in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a flow section that is sufficiently long enough to accomplish the desired heat transfer.

- In external flow, the fluid has a free surface and thus the boundary layer over the surface is free to grow indefinitely.

In Internal flow, however, the fluid is completely confined by the inner surfaces of the tube, and thus, there is a limit on how much the boundary layer can grow.

\# Flow through pipes, ducts and conduit are referred to as Internal flow.

- Flow sections of circular c/s are referred to as <u>pipes</u> (especially when the fluid is a liquid).
- Flow sections of non-circular c/s are referred to as <u>ducts</u> (especially when the fluid is a gas).
- Small-diameter pipes are usually referred to as <u>tubes</u>.

Most liquids are transported in circular pipes because pipes with circular c/s can withstand large pressure differences betn the inside and outside without undergoing significant distortion.

Noncircular pipes are usually used in applications such as heating and cooling systems of buildings where the pressure difference is relatively small.

\# The primary consequence of friction in fluid flow is <u>pressure drop</u> because temp. rise due to frictional heating is usually too small and thus can be disregarded. Therefore, any significant temp. change in the fluid is due to heat transfer.

But frictional heating must be considered for flows that involves highly viscous fluids with large velocity gradients.

The fluid velocity in a pipe changes from zero at the wall (because of the no-slip condition) to a max^m at the pipe centre. In fluid flow, the average velocity, V_{avg} remains constant in incompressible flow when the cross-sectional area of the pipe is constant.

For flow in a circular tube,

Reynolds number, $Re = \dfrac{V_{avg} D}{\nu} = \dfrac{\rho \cdot V_{avg} D}{\mu}$

$Re < 2300$, Flow in a circular tube is Laminar

$2300 \leq Re \leq 10000$, Flow is in transition

$Re > 10000$, Flow is turbulent

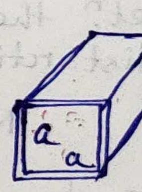
For flow through non-circular tubes (such as ducts), Reynolds number (Re), Nusselt number (Nu) and the friction-factor (C_f) are based on the hydraulic diameter, D_h.

$$D_h = \frac{4 \times \text{Cross-sectional Area}}{\text{Wetted Perimeter}} = \frac{4A_c}{P}$$

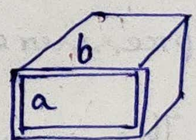
→ The hydraulic diameter, D_h is defined as such that it reduces to ordinary diameter, D for circular tubes.

Circular Tubes: $D_h = \dfrac{4A_c}{P} = \dfrac{4 \times \left(\frac{\pi D^2}{4}\right)}{\pi D} = D$

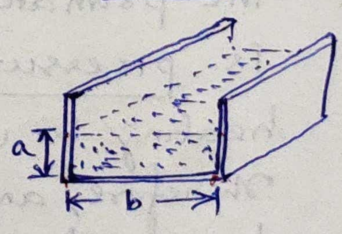
Square Ducts: $D_h = \dfrac{4A_c}{P} = \dfrac{4 \times a^2}{4 \times a} = a$



Rectangular Ducts: $D_h = \dfrac{4ab}{2(a+b)} = \dfrac{2ab}{a+b}$



Open Rectangular Channel: $D_h = \dfrac{4ab}{2a+b}$



Hydrodynamic and Thermal Entrance Regions

- Consider a fluid entering a circular pipe at a uniform velocity. Because of the no-slip condition, the fluid particles in the layer in contact with the wall of the pipe come to a complete stop. This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction. This initiates a velocity gradient along the pipe and the development of a velocity boundary layer or just a boundary layer (B.L.).

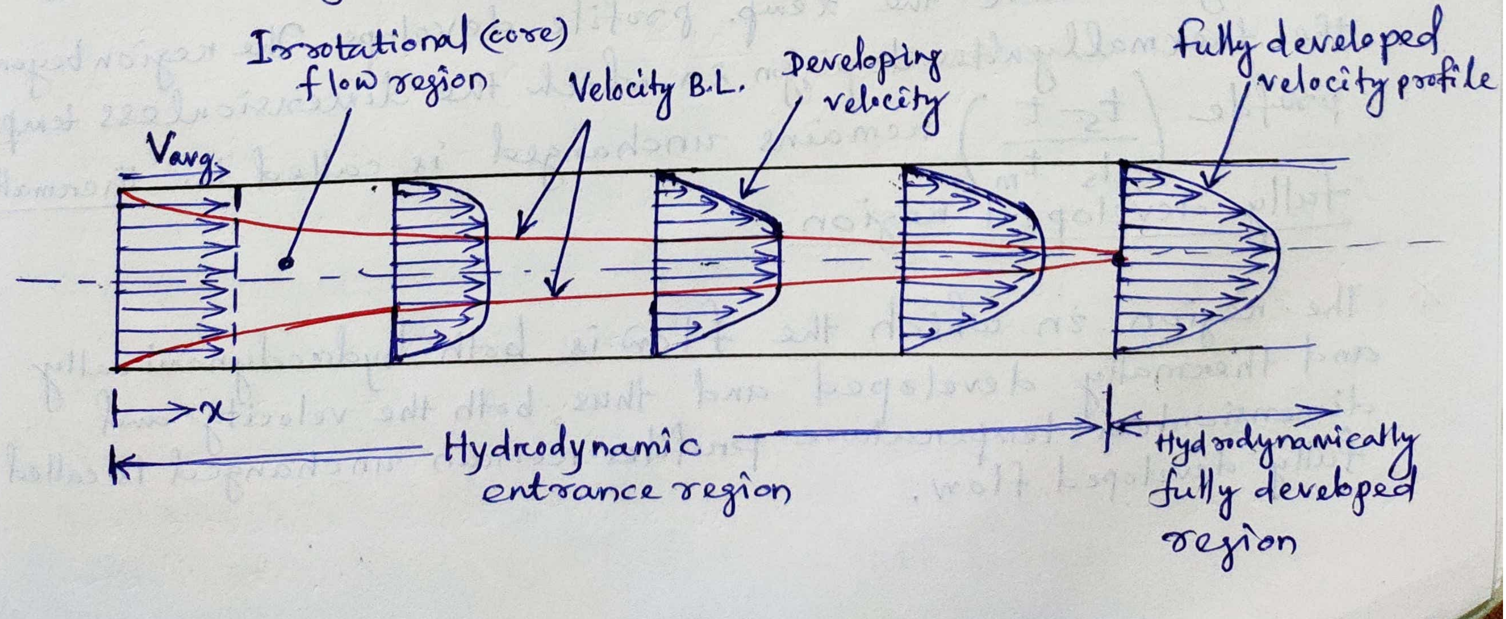
- The thickness of this boundary layer increases ... flow direction until the B.L. reaches the pipe centre and thus, fills the entire pipe.

- The region from the pipe inlet to the point at which the velocity profile is fully developed is called the hydrodynamic entrance region and the length of this region is called the hydrodynamic entry length, L_h. Flow in the entrance region is called hydrodynamic developing flow since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged, is called the hydrodynamically fully developed region.

The velocity profile in the fully developed region

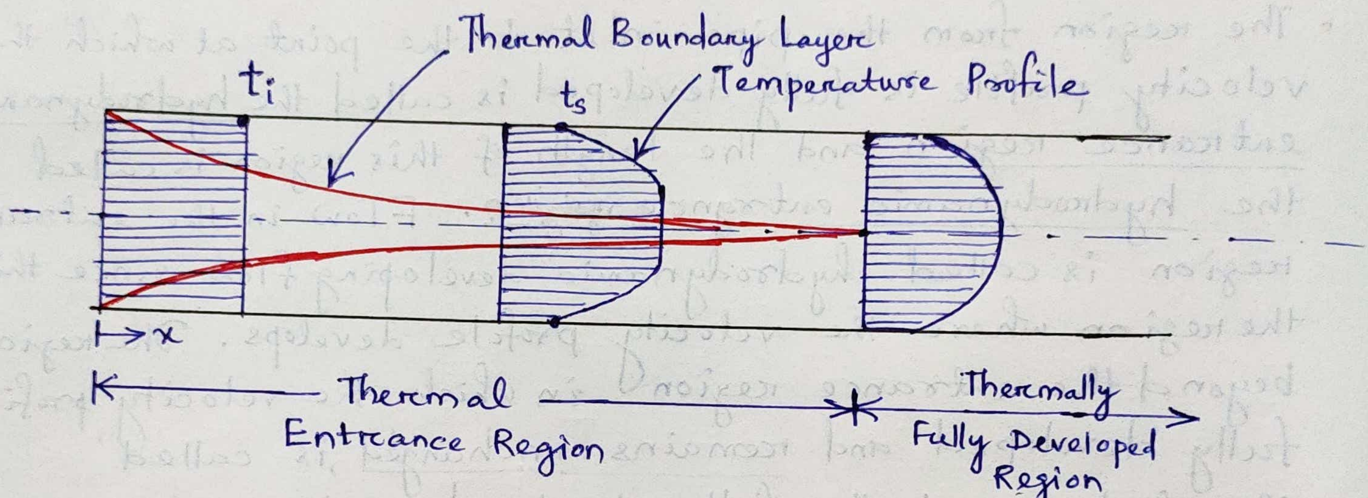
→ is parabolic in laminar flow region, and

→ is somewhat flatter or fuller in turbulent flow due to eddy motion and more vigorous mixing in radial dir^n



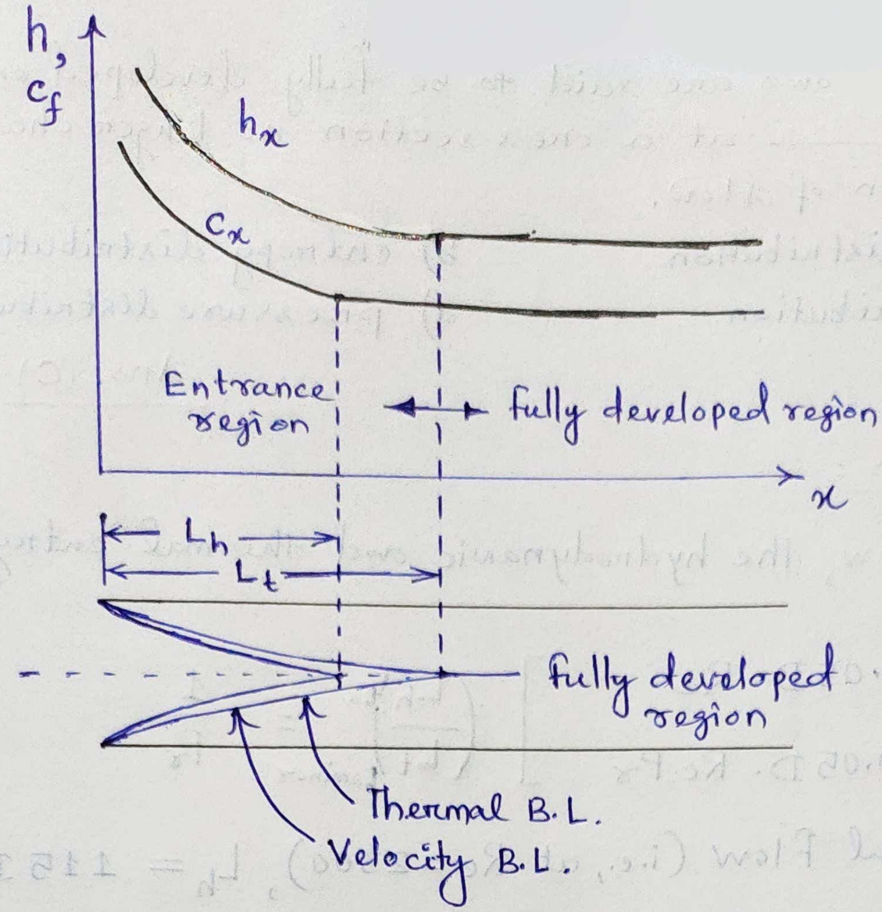
Consider a fluid at a uniform temp. entering a circular tube whose surface is maintained at a different temp.. (i.e. isothermal pipe surface but its temp. is somewhat different from that of fluid temp fluid inside it.). The fluid particles in the layer in contact with the surface of the tube assume the pipe surface temp.. This initiates convection heat transfer in the tube and the development of a thermal boundary layer along the tube.

- The thickness of this thermal B.L. also increases in the flow direction until the thermal B.L. reaches the tube centre and thus, fills the entire tube.



- The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entrance region and the length of this region is called the thermal entry length, L_t. Flow in the thermal entrance region is called thermally developing flow, since this is the region where the temp. profile develops. The region beyond the thermal entrance region in which the dimensionless temp. profile $\left(\frac{t_s - t}{t_s - t_m}\right)$ remains unchanged is called the thermally fully developed region.

* The region in which the flow is both hydrodynamically and thermally developed and thus, both the velocity and dimensionless temperature profiles remain unchanged is called fully developed flow.



* <u>In the entrance region</u> [where the thickness of B.L. is smallest]
The <u>skin friction co.efficient, c_{fx}</u> (and thus, the pressure drop), the <u>convective heat transfer co.efficient, h_x</u> (and, the heat flux) the <u>Nusselt number, Nu_x</u> are much higher in the entrance regions of a tube. This enhancement can be significant for short tubes but negligible for long tubes.

* <u>In the fully developed region</u> [where the thickness of B.L. is maximum]
The skin friction co.efficient, c_{fx} and the convective heat transfer co.efficient, h_x <u>remain constant</u> in the hydrodynamically and thermally fully developed regions respectively.

* The <u>velocity profile</u> no longer changes in the direction of flow in the fully developed flow.

* The <u>temperature profile</u> in the thermally fully developed region may vary with 'x' in the flow dir^n, because the temp. profile can be different at different cross-sections of the tube in the developed region. However, the <u>dimensionless temp. profile remains unchanged</u> in the thermally developed region, when the temp. or heat flux at the tube surface remains constant.

Q. Internal force flows are said to be fully developed, once the ________________ at a cross-section no longer changes in the direction of flow.

a) temperature distribution
b) entropy distribution.
c) velocity distribution
d) pressure distribution.

Ans: (C)

Entry Lengths :

→ In Laminar Flow, the hydrodynamic and thermal entry lengths are given as

$$\left.\begin{aligned} L_{h,\,Laminar} &\approx 0.05 D \cdot Re \\ L_{t,\,Laminar} &\approx 0.05 D \cdot Re\, Pr \end{aligned}\right] \quad \left(\frac{L_h}{L_t}\right)_{Laminar} = \frac{1}{Pr}$$

→ In Transitional Flow (i.e., at $Re = 2300$), $L_h = 115 D$

→ In Turbulent Flow, the hydrodynamic and thermal entry-lengths are about the same size and independent of the Prandtl number.

$$L_{h,\,Laminar} \approx L_{t,\,turbulent} \approx 10 D$$

- The entry length is much shorter in turbulent flow and its dependence on the Raynolds no. is weaker.

Thermal Boundary Conditions :

The thermal boundary conditions at the surface can be of

- constant surface temperature ($t_s = \text{constant}$)
- constant surface heat flux ($\dot{q}_s = \text{constant}$)

→ The constant surface temp. condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube.

→ The constant surface heat flux condition is realized when the tube is subjected to radiation or electric-resistance heating uniformly from all directions.

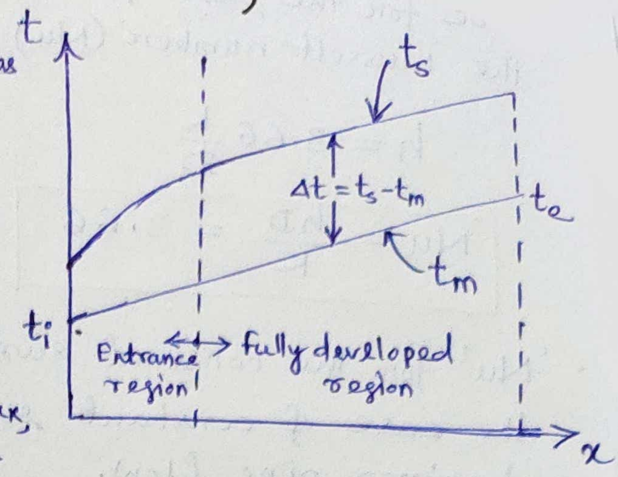
→ When $h_x = h = \text{constant}$, the surface temp., t_s must change when $\dot{q}_s = \text{constant}$, and the surface heat flux, $\dot{q}_s$ must change when $t_s = \text{constant}$. Thus, we may have either $t_s = \text{constant}$ or $\dot{q}_s = \text{constant}$ at the surface of a tube, but not both.

(i) Constant Surface Heat flux ($\dot{q}_s$ = constant)

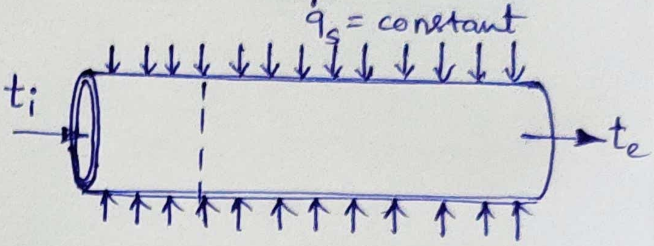
The rate of heat transfer is given as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (t_e - t_i)$$

Also, $\dot{q}_s = h(t_s - t_m)$ Bulk mean fluid temp. $= \frac{t_i + t_e}{2}$



In fully developed flow in a tube subjected to constant surface heat flux, the temp. gradient $\left(\frac{dt}{dx}\right)$ is independent of 'x' and thus, the shape of the temp. profile does not change along the tube.



Convective heat transfer co-efficient,

$$h = \frac{24}{11}\frac{K}{R} = \frac{48}{11}\frac{K}{D} = 4.36\frac{K}{D}$$

$$\Rightarrow \boxed{Nu = \frac{hD}{K} = 4.36 \text{ or } \frac{48}{11}}$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt-number (Nu) is a constant.

There is no dependance on the Reynolds or the Prandtl numbers.

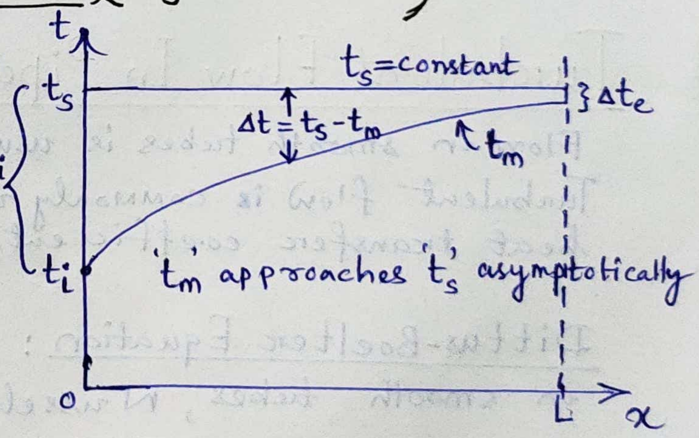
(ii) Constant Surface Temperature (t_s = constant)

The heat transfer rate is given by

$$\dot{Q} = \dot{m} c_p (t_e - t_i) = h A_s \cdot \Delta t_{LMTD}$$

where, Δt_{LMTD} = Logarithmic Mean Temp. Difference

$$= \frac{\Delta t_e - \Delta t_i}{\ln\left(\frac{\Delta t_e}{\Delta t_i}\right)}$$



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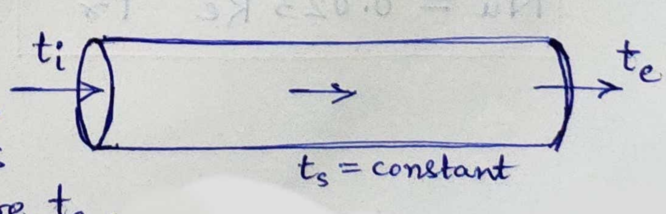
Therefore, we should always use the log. mean temperature difference when determining the convection heat-transfer in a tube whose surface is maintained at a constant temperature, t_s.



- For fully developed laminar flow in a circular in a circular tube for the case of constant surface temperature, t_s, the Nusselt number (Nu) is obtained as

$$h = 3.66 \frac{K}{D}$$

$$\boxed{Nu = \frac{hD}{K} = 3.66}$$

- 'Nu' for the constant surface heat flux is 16% higher than the case of constant surface temperature for the fully developed Laminar pipe flow.

Turbulent Flow In Pipes

Flow in smooth tubes is usually fully turbulent for $Re > 10{,}000$. Turbulent flow is commonly utilised in practice because of the higher heat transfer co-efficients associated with it.

Dittus-Boelter Equation: For fully developed turbulent flow in smooth tubes, Nusselt number (Nu) can be obtained as

$$\boxed{Nu = 0.023\, Re^{0.8}\, Pr^{n}}$$

where $n = 0.4$ for heating of the fluid and $n = 0.3$ for cooling of the fluid flowing through the tube.

HIGHLIGHTS

Important Formulae

A. Laminar Flow

I. ***Flow over flat plate :***

If $\frac{Ux}{\nu} < 5 \times 10^5$ ...boundary layer is *laminar* (velocity distribution is *parabolic*)

If $\frac{Ux}{\nu} > 5 \times 10^5$ ...boundary layer is *turbulent* on that portion (velocity distribution follows *log law or, a power law*)

(*i*) Displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

(*ii*) Momentum thickness, $\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

(*iii*) Energy thickenss, $\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$

(*iv*) $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ (Blasius)

(*v*) $\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$ (Von-Karman)

(*vi*) $\frac{\delta_{th}}{x} = \frac{5}{\sqrt{Re_x}} = \frac{\delta}{x}$ for $Pr = 1$

(*vii*) $\frac{\delta_{th}}{\delta} = \frac{1}{(Pr)^{1/3}}$ (Pohlhausen)

(*viii*) $C_{fx} = \frac{0.664}{\sqrt{Re_x}}$ (Blasius)

(*ix*) $C_{fx} = \frac{0.646}{\sqrt{Re_x}}$ (Von-Karman)

(*x*) $\bar{C}_f = \frac{1.328}{\sqrt{Re_L}}$ (Blasius)

(*xi*) $h_x = 0.332 \frac{k}{x} (Re_x)^{1/2} (Pr)^{1/3}$

(*xii*) $Nu_x = \frac{h_x x}{k} = 0.332 (Re_x)^{1/2} (Pr)^{1/3}$

(*xiii*) $\bar{h} = 2h_x$

(*xiv*) $\overline{Nu} = \frac{\bar{h}L}{k} = 0.664 (Re_L)^{1/2} (Pr)^{1/3}$

II. ***Laminar tube flow :***

(*i*) $u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$ ... Most commonly used equation for the velocity distribution for laminar flow through pipes.

(*ii*) $h = \frac{48k}{11D}$

(*iii*) $Nu = 4.364$

(*iv*) $Nu = 3.65$ ... For constant wall temperature.

B. Turbulent Flow

I. *For flat plate :*

(*i*) $\dfrac{\delta}{x} = \dfrac{0.371}{(Re_x)^{1/5}}$

(*ii*) $\tau_o = \dfrac{\rho U^2}{2} \times \dfrac{0.0576}{(Re_x)^{1/5}} \left[= \dfrac{0.0288\, \rho U^2}{(Re_x)^{1/5}} \right]$

(*iii*) $C_{fx} = \dfrac{0.0576}{(Re_x)^{1/5}}$

(*iv*) $\bar{C}_f = \dfrac{0.072}{(Re_L)^{1/5}}$ Valid for $5 \times 10^5 < Re_L < 10^7$

$\bar{C}_f = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}}$ ...Relation suggested by Prandtl and Schlichting, for Re between, 10^7 and 10^9, when the boundary layer is turbulent from the leading edge onwards.

$\bar{C}_f = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}} - \dfrac{1670}{Re_L}$ ...for laminar and turbulent flow at $Re_c = 5 \times 10^5$

(*v*) $Nu_x = 0.0288\, (Re_x)^{0.8}\, (Pr)^{1/3}$

$h_x = 0.0288 \left(\dfrac{k}{x}\right) (Re_x)^{0.8}\, (Pr)^{1/3}$

(*vi*) $\overline{Nu} = 0.036\, (Re_L)^{0.8}\, (Pr)^{1/3}$

$\bar{h} = 0.036 \left(\dfrac{k}{L}\right) (Re_L)^{0.8}\, (Pr)^{1/3}$

(*vii*) $\overline{Nu} = (Pr)^{1/3}\, [0.036\, (Re_L)^{0.8} - 836]$ ...when $Re_c = 5 \times 10^5$

II. *For tubes :*

$\overline{Nu} = 0.023\, (Re)^{0.8}\, (Pr)^{1/3}$

$\bar{h} = 0.023\, \dfrac{k}{D}\, (Re)^{0.8}\, (Pr)^{1/3}$

The above expressions are valid for

$1 \times 10^4 < Re < 1 \times 10^5;\ 0.5 < Pr < 100;\ \dfrac{L}{D} > 60.$

III. *Turbulent flow over cylinders:*

(*i*) The following *emprical correlation* is widely used for turbulent flow over cylinders:

$\overline{Nu} = \dfrac{\bar{h}D}{k} = C\, (Re)^n\, (Pr)^{1/3}$

where C and n are constants and have the values as givn in the table below:

Table: Constants for eqn. (7.175) for flow across cylinder (Hilpert, 1933; Knudsen, 1958)

S.No	Re	C	n
1.	0.4 to 4	0.989	0.330
2.	4 to 40	0.911	0.385
3.	40 to 4×10^3	0.683	0.466
4.	4×10^3 to 4×10^4	0.193	0.618
5.	4×10^4 to 4×10^5	0.026	0.805

(*ii*) *Churchill and Bernstein* have suggested the following comprehensive *empirical correlation* which covers the entire range of Re and wide range of Pr:

$$\overline{Nu} = 0.3 + \frac{0.62 (Re)^{0.5} (Pr)^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{0.25}} \left[1 + \left(\frac{Re}{28200} \right)^{5/8} \right]^{0.8}$$

This is *valid* for $100 < Re < 10^7$, and $Re.Pr > 0.2$ and correlates very well all available data.

(*iii*) The following equation may be used in the *mid-range* of Reynolds numbers, i.e. $20{,}000 < Re < 400{,}000$:

$$\overline{Nu} = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62 (Re)^{0.5} (Pr)^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{0.25}} \left[1 + \left(\frac{Re}{28200} \right)^{0.5} \right]$$

for $20{,}000 < Re < 400{,}000$, and $Re.Pr > 0.2$

(*iv*) The following relation is recommended by Nakai and Okazaki for $Pe\ (= Re.Pr) < 0.2$:

$\overline{Nu} = [0.8237 - \ln(Pe)^{0.5}]^{-1}$

Example 7.22. *Castor oil at 25°C flows at a velocity of 0.1 m/s past a flat plate, in a certain process. If the plate is 4.5 m long and is maintained at a uniform temperature of 95°C, calculate the following using exact solution :*

(*i*) *The hydrodynamic and thermal boundary layer thicknesses on one side of the plate,*

(*ii*) *The total drag force per unit width on one side of the plate,*

(*iii*) *The local heat transfer coefficient at the trailing edge, and*

(*iv*) *The heat transfer rate.*

The thermo-physical properties of oil at mean film temperature of (95 + 25)/2 = 60°C are :

$\rho = 956.8$ kg/m^3; $\alpha = 7.2 \times 10^{-8}$ m^2/s; $k = 0.213$ W/m°C; $\nu = 0.65 \times 10^{-4}$ m^2/s.

Solution. *Given* : $t_\infty = 25°C$, $t_s = 95°C$, $L = 4.5$m, $U = 0.1$ m/s.

(*i*) **The hydrodynamic and thermal boundary layer thicknesses, δ, δ_{th} :**

Reynolds number at the end of the plate,

$$Re_L = \frac{UL}{\nu} = \frac{0.1 \times 4.5}{0.65 \times 10^{-4}} = 6923$$

Since Reynolds number is less than 5×10^5, hence the flow is *laminar* in nature.

The hydrodynamic boundary layer thickness,

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times L}{\sqrt{Re_L}} = \frac{5 \times 4.5}{\sqrt{6923}}$$

$$= 0.2704 \text{ m or } \mathbf{270.4 \text{ mm}} \quad \textbf{(Ans.)}$$

The thermal boundary layer thickness, according to Pohlhausen, is given by :

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}}$$

where, $\quad Pr$ (Prandtl number) $\dfrac{\nu}{\alpha} = \dfrac{(0.65 \times 10^{-4})}{7.2 \times 10^{-8}} = 902.77$

$\therefore \quad \delta_{th} = \dfrac{0.2704}{(902.77)^{1/3}} = 0.02798$ m or **27.98 mm** **(Ans.)**

(*ii*) **The total drag force per unit width on one side of the plate, F_D :**

The average skin friction coefficient is given by,

$$\bar{C}_f = \frac{1.328}{\sqrt{Re_L}}$$

or, $\quad \bar{C}_f = \dfrac{1.328}{\sqrt{6923}} = 0.01596$

The drag force, $\quad F_D = \bar{C}_f \times \dfrac{1}{2} \rho U^2 \times$ area of plate (for one side)

or, $\quad F_D = 0.01596 \times \dfrac{1}{2} \times 956.8 \times 0.1^2 \times (4.5 \times 1) =$ **0.3436 N per meter width. (Ans.)**

(*iii*) **The local heat transfer coefficient at the trailing edge, h_x (at $x = L$) :**

$$Nu_x = \frac{h_x x}{k} = 0.332\,(Re_x)^{1/2}\,(Pr)^{1/3}$$

$$= 0.332 \times (6923)^{1/2}\,(902.77)^{1/3} = 266.98$$

or, $\quad h_x = \dfrac{266.98 \times k}{x} = \dfrac{266.98 \times 0.213}{4.5} =$ **12.64 W/m² °C** **(Ans.)**

(*iv*) **The heat transfer rate, Q :**

$$Q = \bar{h}\, A_s (t_s - t_\infty)$$

where, $\quad \bar{h} = 2h_x = 2 \times 12.64 = 25.28$W/m^2°C

$\therefore \quad Q = 25.28 \times (4.5 \times 1)\,(95 - 25) =$ **7963.2 W** **(Ans.)**

Example 7.23. *Air at 20°C and at atmospheric pressure flows at a velocity of 4.5 m/s past a flat plate with a sharp leading edge. The entire plate surface is maintained at a temperature of 60°C. Assuming that the transition occurs at a critical Reynolds number of 5×10^5, find the distance from the leading edge at which the flow in the boundary layer changes from laminar to turbulent. At the location, calculate the following :*

(*i*) *Thickness of hydrodynamic layer,*

(*ii*) *Thickness of therml boundary layer,*

(*iii*) *Local and average convective heat transfer coefficients,*

(*iv*) *Heat transfer rate from both sides for, unit width of the plate,*

(*v*) *Mass entrainment in the boundary layer, and*

(*vi*) *The skin friction coefficient.*

Assume cubic velocity profile and approximate method.

The thermo-physical properties of air at mean film temperature (60 + 20)/2 = 40°C are :
$\rho = 1.128$ *kg/m³*, $\nu = 16.96 \times 10^{-6}$ *m²/s*, $k = 0.02755$ *W/m°C*, $Pr = 0.699$.

Solution. *Given*: $t_\infty = 20$ °C, $t_s = 60$°C, $U = 4.5$ m/s.

At the transition point, $Re_c = (Re)_{trans.} = \dfrac{Ux_c}{\nu}$

or,
$$x_c = \frac{Re_c \times \nu}{U} = \frac{5 \times 10^5 \times 16.96 \times 10^{-6}}{4.5} = \mathbf{1.88\ m} \quad \textbf{(Ans.)}$$

(where, x_c = Distance from the leading edge at which the flow in the boundary layer changes *from laminar to turbulent*).

(*i*) **Thickness of hydrodynamic layer, δ :**

The thickness of hydrodynamic layer for, cubic velocity profile is given by,

$$\delta = \frac{4.64\, x_c}{\sqrt{Re_c}}$$

$$= \frac{4.64 \times 1.88}{\sqrt{5 \times 10^5}} = 0.01234 \text{ m or, } \mathbf{12.34\ mm} \quad \textbf{(Ans.)}$$

(*ii*) **Thickness of thermal boundary layer, δ_{th} :**

The thermal boundary layer is given by,

$$\delta_{th} = \frac{0.975\,\delta}{(Pr)^{1/3}}$$

or,
$$\delta_{th} = \frac{0.975 \times 0.01234}{(0.669)^{1/3}} = 0.01355 \text{ m or, } \mathbf{13.55\ mm}$$

(*iii*) **Local and average convective heat transfer coefficients:**

The Nusselt number at $x = x_c$ is given by

$$Nu_c = 0.332\,(Re_c)^{1/2}\,(Pr)^{1/3}$$
$$= 0.332\,(5 \times 10^5)^{1/2}\,(0.699)^{1/3} = 208.34$$

Put
$$Nu_x = \frac{h_x \times x_c}{k} \quad \text{or,} \quad h_c = \frac{Nu_c \times k}{x_c}$$

(where, h_c = Local heat transfer coefficient at $x = x_c$)

or,
$$h_c = \frac{208.34 \times 0.02755}{1.88} = \mathbf{3.05\ W/m^2\,°C} \quad \textbf{(Ans.)}$$

Average heat transfer coefficient,

$$\bar{h} = \frac{1}{x_c}\int_0^{x_c} h_x\,dx$$

$$\bar{h} = 2h_c = 2 \times 3.05 = \mathbf{6.1\ W/m^2°C} \quad \textbf{(Ans.)}$$

(*iv*) **Heat transfer rate from both sides for, unit width of the plate; Q :**

$$Q = \bar{h}\,(2A_s)\,\Delta t = 6.1\,(2 \times 1.88 \times 1)\,(60 - 20) = \mathbf{917.44\ W} \quad \textbf{(Ans.)}$$

(*v*) **Mass entrainment in the boundary layer, m :**

$$m = \frac{5}{8}\,\rho\,U\,(\delta_2 - \delta_1)$$

Here, $\delta_1 = 0$ at $x = 0$ and $\delta_2 = 0.01234$ *m* at $x = x_c = 1.88$ m

$$\therefore\ m = \frac{5}{8} \times 1.128 \times 4.5\,(0.01234 - 0) = \mathbf{0.039\ kg/s\ or,\ 140.4\ kg/h} \quad \textbf{(Ans.)}$$

(*vi*) **The skin friction coefficient, C_{fx} :**

$$C_{fx} = \frac{0.646}{\sqrt{Re_x}}$$

or,
$$C_{fx} = \frac{0.646}{\sqrt{5 \times 10^5}} = \mathbf{9.136 \times 10^{-4}} \quad \textbf{(Ans.)}$$

Example 7.33. *Ambient air at 20°C flows at a velocity of 10 m/s parallel to a wall 5 m wide and 3 m high. Calculate the heat transfer rate if the wall is maintained at 40°C. The critical Reynolds number is equal to 5×10^5. The properties of air at the mean film temperature may be taken as : $k = 0.0263$ W/m K, $\nu = 15.89 \times 10^{-6}$ m²/s and $Pr = 0.707$.*

If the entire boundary layer is assumed turbulent, what will be the percentage error in the computation of heat transfer rate ? Comment on the comparable values of the two results.

Appropriate correlation from the following may be used :

$$\overline{Nu} = 0.664\, Re_L^{0.5}\, Pr^{1/3}$$

$$\overline{Nu} = 0.0375\, Re_L^{0.8}\, Pr^{1/3}$$

$$\overline{Nu} = 0.0375\, [Re_L^{0.8} - 23200]\, Pr^{1/3} \quad \textbf{(AMIE, Summer, 1999)}$$

Solution. *Given :* $t_\infty = 20°C;\ U = 10$ m/s; $L = 5$ m; $B = 3$ m; $t_s = 40°C$; $(Re)_{cr} = 5 \times 10^5$

$k = 0.0263$ W/m K; $\nu = 15.89 \times 10^{-6}$ m²/s; $Pr = 0.707$.

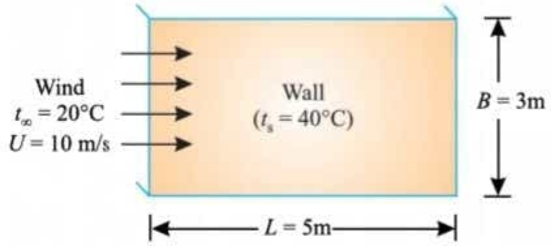


Fig. 7.20.

Percentage error in computation of heat transfer rate :

$$Re_L = \frac{UL}{\nu} = \frac{10 \times 5}{15.89 \times 10^{-6}} = 3.1466 \times 10^6$$

i.e., $> (Re)_{cr}$

Using the following equation, we get

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.0375\left[Re_L^{0.8} - 23200\right] Pr^{1/3}$$

...(*Combination of laminar and turbulent flow*)

$$\therefore \quad \bar{h} = \frac{k}{L} \times 0.0375\left[Re_L^{0.8} - 23200\right] Pr^{1/3}$$

$$= \frac{0.0263}{5} \times 0.0375\, [(3.1466 \times 10^6)^{0.8} - 23200] \times (0.707)^{1/3}$$

$$= 0.00019725\,(157860.4 - 23200) \times 0.8908 = 23.66 \text{ W/m}^2\,°C$$

Heat transfer rate,

$$Q = \bar{h}A\,(t_s - t_\infty) = 23.66 \times (5 \times 3) \times (40 - 20) = 7098 \text{ W}$$

If *entire boundary is assumed turbulent,*

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.0375\, Re_L^{0.8}\, Pr^{1/3}$$

$$\therefore \quad \bar{h} = \frac{k}{L} \times 0.0375\, Re_L^{0.8}\, Pr^{1/3}$$

$$= \frac{0.0263}{5} \times 0.0375 \times (3.1466 \times 10^6)^{0.8} \times (0.707)^{1/3}$$

$$= 0.00019725 \times 157860.4 \times 0.8908 = 27.74 \text{ W/m}^2\,°C$$

$$Q = 27.74 \times (5 \times 3) \times 20 = 8322 \text{ W}$$

$$\therefore \quad \textbf{Percentage error} = \frac{8322 - 7098}{7098} \times 100 = \mathbf{17.24\%} \quad \textbf{(Ans.)}$$

Comments. The difference between the two values is nearly 17%. Further the correlation for turbulent heat transfer are empirical, which may have a variation of ± 25%. Hence, the rate of heat transfer in such cases *may be calculated for all purposes, by assuming the boundary layer to be entirely turbulent.*

Example 7.34. *Air flows over a heated plate at a velocity of 50 m/s. The local skin friction co-efficient at a point on a plate is 0.004. Estimate the local heat transfer coefficient at this point. The following property data for air are given :*

Density = 0.88 kg/m³; viscosity = 2.286 × 10⁻⁵ kg m/s;

specific heat, c_p = 1.001 kJ/kg K; conductivity = 0.035 W/m K.

Use $St\ Pr^{1/3} = \dfrac{C_{fx}}{2}$ **(U.P.S.C., 1993)**

Solution. *Given :* $U = 50$ m/s; $C_{fx} = 0.004$; $\rho = 0.88$ kg/m³; $\mu = 2.286 \times 10^{-5}$ kg m/s;

$c_p = 1.001$ kJ/kg K; $k = 0.035$ W/m K.

Local heat transfer coefficient, h_x :

Prandtl number, $Pr = \dfrac{\mu \cdot c_p}{k} = \dfrac{2.286 \times 10^{-5} \times (1.001 \times 1000)}{0.035} = 0.654$

Stanton number, $St = \dfrac{h_x}{\rho \cdot c_p \cdot U} = \dfrac{h_x}{0.88 \times (1.001 \times 1000) \times 50} = \dfrac{h_x}{44044}$

Now, $St \cdot (Pr)^{2/3} = \dfrac{C_{fx}}{2}$ ...(Given)

$$\frac{h_x}{44044}(0.654)^{2/3} = \frac{0.004}{2}$$

or, $h_x = \dfrac{0.004}{2} \times \dfrac{44044}{(0.654)^{2/3}}$ **= 116.9 W/m² K** **(Ans.)**

Example 7.35. *The crankcase of an I.C. engine measuring 80 cm × 20 cm may be idealised as a flat plate. The engine runs at 90 km/h and the crankcase is cooled by the air flowing past it at the same speed. Calculate the heat loss from the crank surface maintained at 85°C, to the ambient air at 15°C. Due to road induced vibration, the boundary layer becomes turbulent from the leading edge itself.*

Solution. *Given :* $U = 90$ km/h $= \dfrac{90 \times 1000}{3600} = 25$ m/s; $t_s = 85$°C; $t_\infty = 15$°C; $L = 80$ cm $= 0.8$ m; $B = 20$ cm $= 0.2$m.

The properties of air at $t_f = \dfrac{85 + 15}{2} = 50$°C are :

$k = 0.02824$ W/m°C, $\nu = 17.95 \times 10^{-6}$ m²/s, $Pr = 0.698$ ... (From tables)

Heat loss from the crankcase, Q :

The Reynolds number, $Re_L = \dfrac{UL}{\nu} = \dfrac{25 \times 0.8}{17.95 \times 10^{-6}} = 1.114 \times 10^6$

Since $Re_L > 5 \times 10^5$, the nature of flow is *turbulent*.

For turbulent boundary layer,

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.036\,(Re_L)^{0.8}\,(Pr)^{0.333} = 0.036\,(1.114 \times 10^6)^{0.8}\,(0.698)^{0.333} = 2196.92$$

or, $\bar{h} = \dfrac{k}{L} \times 2196.92 = \dfrac{0.02824}{0.8} \times 2196.92 = 77.55$ W/m²°C

$\therefore$ $Q = \bar{h}A\,(t_s - t_\infty) = 77.55 \times (0.8 \times 0.2)\,(85 - 15)$ **= 868.56 W** **(Ans.)**

Example 7.36. *Air at 20°C and 1.013 bar flows over a flat plate at 40 m/s. The plate is 1m long and is maintained at 60°C. Assuming unit depth, calculate the heat transfer from the plate. Use the following correlation :* **(M.U.)**

$$Nu_L = (Pr)^{0.33}\,[0.037\,(Re_L)^{0.8} - 850]$$

Solution. *Given :* $t_\infty = 20$°C; $U = 40$ m/s; $L = 1$m; $B = 1$ m; $t_s = 60$°C.

Properties of air at (60 + 20)/2 = 40°C, from the tables:

$\rho = 1.128$ kg/m³; $c_p = 1.005$ kJ/kg°C; $k = 0.0275$ W/m°C; $\nu = 16.96 \times 10^{-6}$ m²/s; $Pr = 0.699$.

Heat transfer from the plate, Q :

Reynolds number, $Re_L = \dfrac{UL}{\nu} = \dfrac{40 \times 1}{16.96 \times 10^{-6}} = 2.36 \times 10^6$

$\therefore$ $Nu_L = \dfrac{\bar{h}L}{k} = (0.699)^{0.33}\,[0.037\,(2.36 \times 10^6)^{0.8} - 850] = 3365.6$

or, $\bar{h} = \dfrac{0.0275 \times 3365.6}{1} = 92.55$ W/m²°C

$\therefore$ $Q = \bar{h}\,A_s\,(t_s - t_\infty) = 92.55 \times (1 \times 1)\,(60 - 20)$

$= 3702$ W or, **3.702 kW** **(Ans.)**

Example 7.41. *A square plate maintained at 95°C experiences a force of 10.5 N when forced air at 25°C flows over it at a velocity of 30 m/s. Assuming the flow to be turbulent and using Colburn analogy calculate :*

(*i*) *The heat transfer coefficient;*

(*ii*) *The heat loss from the plate surface.*

Properties of air are :

$\rho = 1.06\ kg/m^3$, $c_p = 1.005\ kJ/kg\ K$, $\nu = 18.97 \times 10^{-6}\ m^2/s$, $P_r = 0.696$.

Solution. *Given :* $F_D = 10.5$ N, $t_s = 95$°C, $t_\infty = 25$°C, U = 30 m/s

(*i*) The heat transfer coefficient, $\bar{h}$:

For turbulent flow, the drag force is given by

$$F = \bar{C}_f \times \frac{1}{2} \rho A U^2$$

or,
$$10.5 = \frac{0.072}{(Re_L)^{0.2}} \times \frac{1}{2} \times 1.06 \times (L \times L) \times (30)^2$$

$$= 0.072 \left(\frac{\nu}{UL}\right)^{0.2} \times \frac{1}{2} \times 1.06 \times L^2 \times 900$$

$$= 0.072 \left(\frac{18.97 \times 10^{-6}}{25 \times L}\right)^{0.2} \times \frac{1}{2} \times 1.06 \times L^2 \times 900$$

$$= 1.424 \times \frac{L^2}{(L)^{0.2}} = 2.05\ (L)^{1.8}$$

or,
$$L = \left(\frac{10.5}{2.05}\right)^{1/1.8} = 2.478 \text{ m}$$

The Reynolds number at the end of the plate,

$$Re_L = \frac{UL}{\nu} = \frac{30 \times 2.478}{18.97 \times 10^{-6}} = 3.919 \times 10^6$$

Average skin friction coefficient; $\bar{C}_f = \dfrac{0.072}{(Re_L)^{0.2}}$

$$= \frac{0.072}{(3.919 \times 10^6)^{0.2}} = 3.457 \times 10^{-3}$$

From Colburn analogy, we have

$$\bar{St}\ (Pr)^{2/3} = \frac{\bar{Cf}}{2}$$

or,
$$\frac{\bar{h}}{\rho\, c_p U} (Pr)^{2/3} = \frac{\bar{C}_f}{2}$$

or,
$$\bar{h} = \frac{\rho\, c_p U}{(Pr)^{2/3}} \times \frac{\bar{C}_f}{2}$$

or,
$$\bar{h} = \frac{1.06 \times (1.005 \times 10^3) \times 30}{(0.696)^{0.666}} \times \left(\frac{3.457 \times 10^{-3}}{2}\right)$$

$= 70.32$ **W/m²°C (Ans.)**

(*ii*) Heat loss from the plate surface, Q :

$$Q = \bar{h}\, A\, \Delta t = 70.32 \times (2.478 \times 2.478) \times (95 - 25) = 30226 \text{ W}$$

$= 30.226$ **kW (Ans.)**

Chapter - 2

Natural (or Free) Convection Heat Transfer

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Free or Natural Convection

Free or Natural convection is the process of heat transfer which occurs due to movement of the fluid particles by density changes/differences associated with temperature differential in a fluid.

When a surface is maintained in still fluid at a temp. higher or lower than that of fluid, a layer of fluid adjacent to the surface gets heated or cooled. A density difference is created between this layer and the still fluid surrounding it. The density difference introduces a buoyant force causing flow of fluid near the surface. Heat transfer under such conditions is known as 'free or natural convection'.

Examples :-

(i) The cooling of transmission lines, electric transformers, and rectifiers.

(ii) The heating of rooms by use of radiators

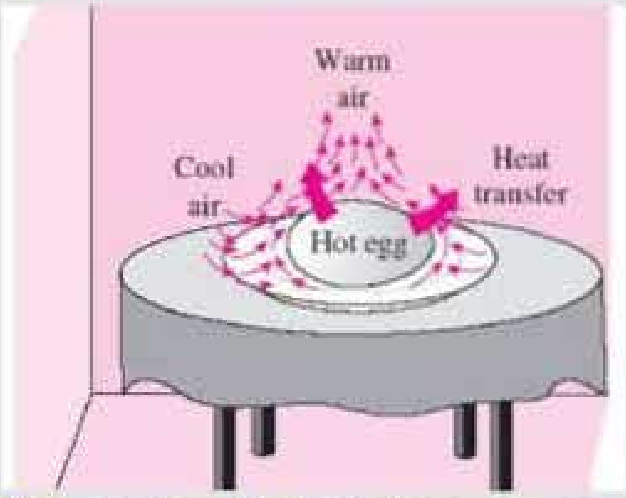
(iii) The heat transfer from hot pipes and ovens surrounded by cooler air.

PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Examples?

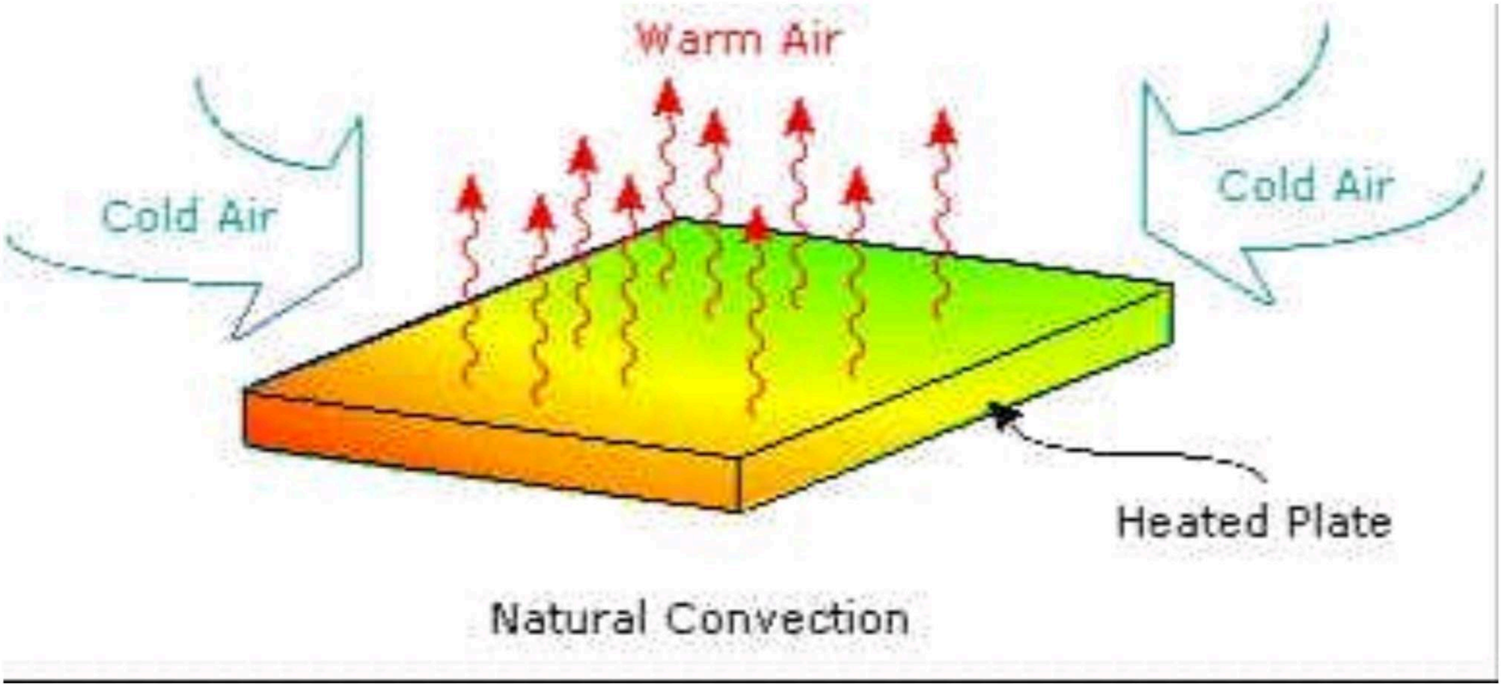
Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

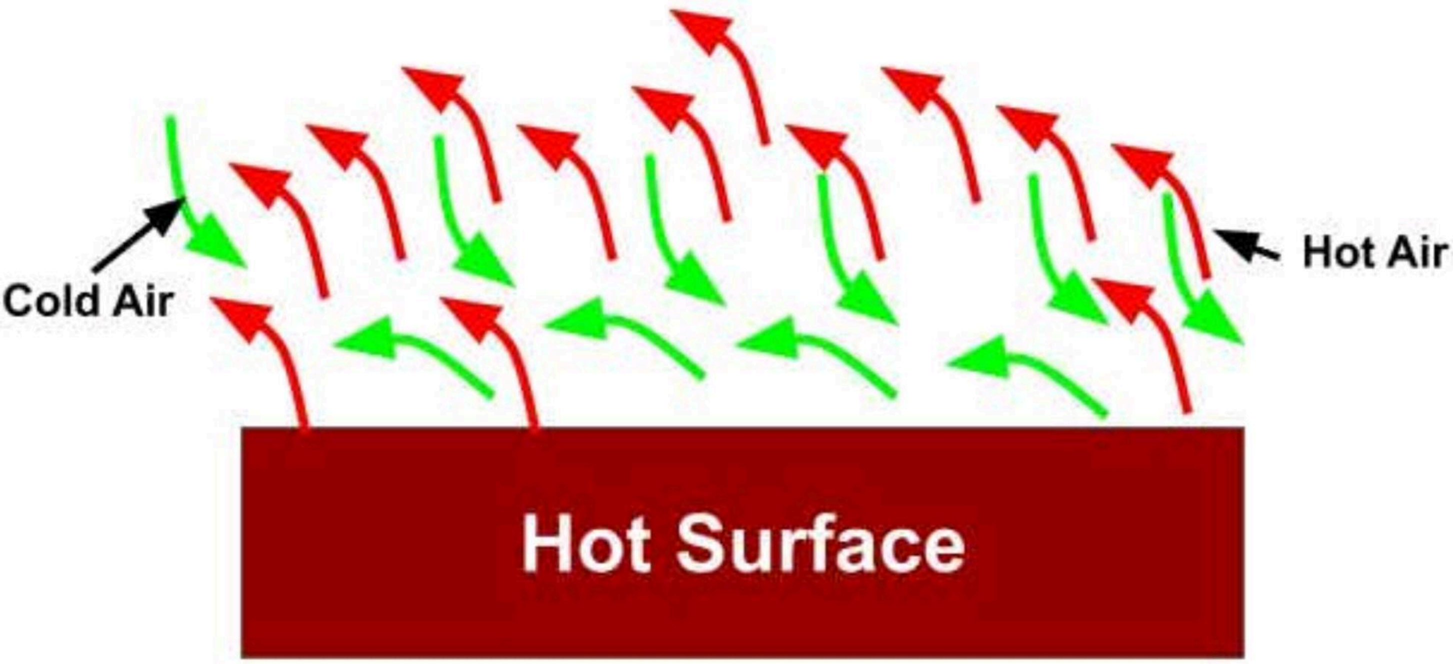
The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current, and the heat transfer that is enhanced as a result of this current is called natural convection heat transfer.



The cooling of a boiled egg in a cooler environment by natural convection.

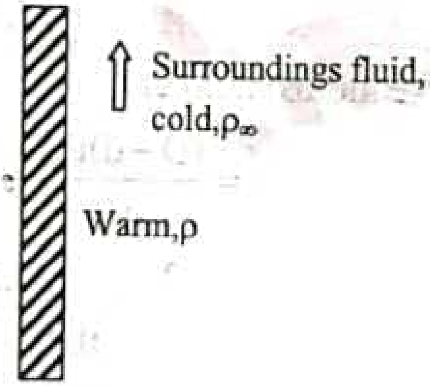
The warming up of a cold drink in a warmer environment by natural convection.


Warm Air
Cold Air
Cold Air
Heated Plate
Natural Convection


Cold Air
Hot Air
Hot Surface

A free convection flow field is a self-sustained flow driven by the presence of a temperature gradient. (As opposed to a forced convection flow where external means are used to provide the flow). As a result of the temperature difference, the density field is not uniform also. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer. It is therefore important only when there is no external flow exists.

Buoyancy effects:



Net force = $(\rho_\infty - \rho)gV$

The density difference is due to the temperature difference and it can be characterized by their volumetric thermal expansion coefficient, β :

$$\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{1}{\rho}\frac{(\rho_\infty - \rho)}{(T_\infty - T)} = \frac{1}{\rho}\left(\frac{\Delta\rho}{\Delta T}\right)$$

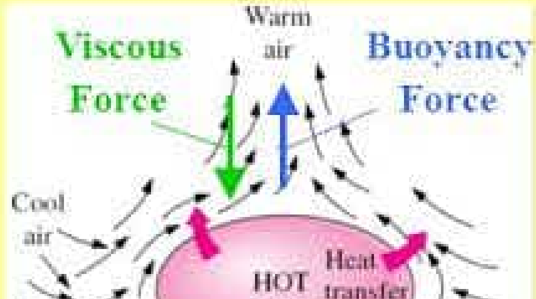
$$\Delta\rho \approx \beta\Delta T$$

For Ideal Gas,

$$\beta = 1 / T$$

- Buoyancy forces are responsible for the fluid motion in natural convection.
- Viscous forces appose the fluid motion.
- Buoyancy forces are expressed in terms of fluid temperature differences through the volume expansion coefficient

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)\bigg|_P = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)\bigg|_P \qquad (1/\mathrm{K}) \qquad \textbf{(9-3)}$$



Characteristic Parameters in Free Convection

A property that comes into play in free or natural convection is the 'coefficient of volume expansion' of the fluid and is defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \rho\left(\frac{\partial v}{\partial T}\right)_P \quad \begin{cases} v \rightarrow \text{Specific volume} \\ P \rightarrow \text{pressure} \end{cases}$$

for an ideal gas, $v = \frac{RT}{P}$, $\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$

$$\therefore \boxed{\beta = \frac{1}{T}} \qquad (\because T = 273 + t)$$

From the thermodynamic point of view,

* Volume expansivity (β) is defined as the ratio of change in volume with temp at constant 'P' to the volume.

$$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}$$

For a perfect gas, $PV = RT$

$$\therefore \beta = \frac{P}{RT} \times \frac{\partial}{\partial T}\left(\frac{RT}{P}\right)_P = \frac{P}{RT} \times \frac{R}{P} = \frac{1}{T}$$

$$\Rightarrow \boxed{\beta = \frac{1}{T}} \quad K^{-1} \text{ or } {}^{1}/{}_{^\circ C}$$

Apparently, the volume expansivity of a perfect gas varies inversely with absolute temp and is independent of both pressure and volume.

<u>Grashof number</u> (Gr) :

The Grashof number is a dimensionless number in fluid dynamics and heat transfer which approximates *the ratio of the buoyancy to viscous force acting on a fluid.*

- It arises in the study of situations involving natural convection and is analogous to the Reynolds number

- Using the Energy equation and the buoyant force combined with Dimensional Analysis provides two different ways to derive the Grashof number.

$$\mathrm{Gr} = \frac{g\beta(T_s - T_0)L_c^3}{\nu^2}.$$

$$Gr = \frac{\text{buoyant forces}}{\text{viscous forces}}$$

- In forced convection , the **Reynolds number (Re)** governs the fluid flow.
- But, in natural convection the **Grashof number (Gr)** is the dimensionless parameter that governs the fluid flow.

where:

g is acceleration due to Earth's gravity
β is the coefficient of thermal expansion (equal to approximately $1/T$, for ideal gases)
T_s is the surface temperature
T_∞ is the bulk temperature
L is the vertical length
D is the diameter
ν is the kinematic viscosity.

<u>Rayleigh Number (Ra)</u> :

The Rayleigh number (Ra) is a dimensionless number that characterizes convection problems in heat transfer. Rayleigh Number is the product of Grashof number (Gr) and Prandtl Number (Pr).

Rayleigh Number,Ra = Gr.Pr

The ratio of the Grashof number to the square of the Reynolds number (this characteristic ratio is called *<u>Richardson number , Ri</u>*)may be used to determine whether forced or free convection may be neglected for a system, or if there's a combination of the two as follows :

$\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2} \gg 1$ forced convection may be ignored

$\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2} \approx 1$ combined forced and free convection

$\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2} \ll 1$ free convection may be neglected

Grashof number (Gr):

It is ratio between Buoyancy force and Viscous force

$$Gr = \frac{g\beta\Delta T L_c^{\ 3}}{\nu^2} = \frac{\rho^2 L_c^{\ 3}\beta g\Delta T}{\mu^2} = \left(\rho L_c^{\ 3}\beta g\Delta T\right)\frac{\rho}{\mu^2}$$

$$= \left(\rho L_c^{\ 3}\beta g\Delta T\right)\frac{\rho V^2 L_c^{\ 2}}{(\mu V L_c)^2}$$

$$\text{Or},\quad Gr = \text{Buoyancy force} \times \frac{\text{Inertia force}}{(\text{Viscous force})^2}$$

$$Gr = \frac{\text{Buoyancy force}}{\text{Viscous force}} \times Re$$

L_c = characteristic length

β = fluid coefficient of thermal expansion

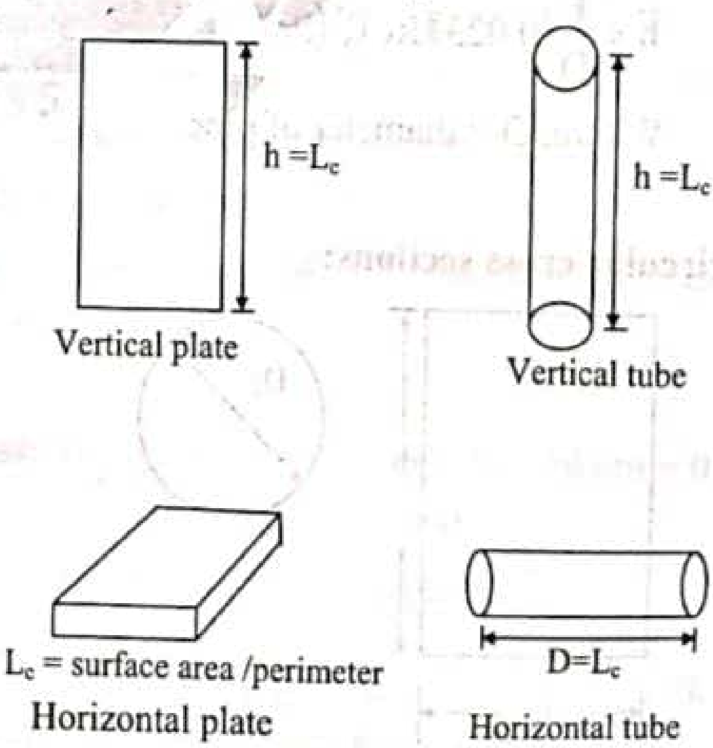
g = acceleration due to gravity

ΔT = temp difference $(T_w - T_\infty)$

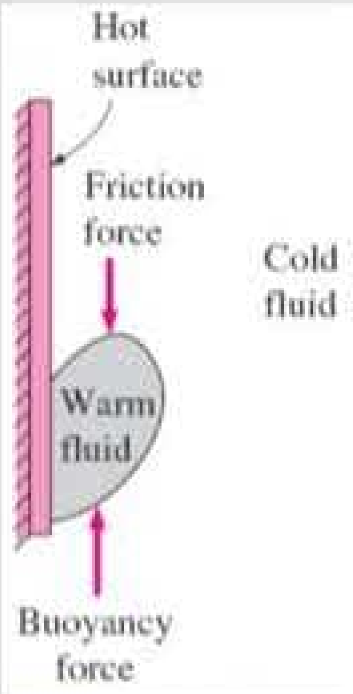
ρ = density

μ = dynamic viscosity

ν = kinematic viscosity



- The Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.
- For vertical plates, the critical Grashof number is observed to be about 10^9.



The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

When a surface is subjected to external flow, the problem involves both natural and forced convection.

The relative importance of each mode of heat transfer is determined by the value of the coefficient Gr/Re^2:

- Natural convection effects are negligible if $Gr/Re^2 << 1$.
- Free convection dominates and the forced convection effects are negligible if $Gr/Re^2 >> 1$.
- Both effects are significant and must be considered if $Gr/Re^2 \approx 1$ (mixed convection).

* For Convection mode of heat transfer, $\boxed{Nu = f(Re, Gr, Pr)}$
* In Natural Convection,

Nusselt Number, $\boxed{Nu = f(Gr, Pr)}$ or $\boxed{Nu = f(R_a)}$

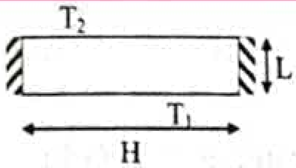
* Rayleigh Number, $\boxed{R_a = Gr \cdot Pr}$

*
$10^4 < Gr \cdot Pr < 10^9 \Rightarrow$ for Laminar flow

$Gr \cdot Pr > 10^9 \Rightarrow$ for Turbulent flow

↳ The values of Gr and Pr are evaluated at the mean film temp, $\left(t_f = \frac{t_s + t_\infty}{2}\right)$.

Natural convection in rectangular cavities:



In case of horizontal rectangular cavity of height L and width H as shown above figure, it is assumed that aspect ratio (H/L) is large. The bottom surface is heated and maintained at a temperature T_1, while the top surface is at a temperature T_2 with $T_1 > T_2$. The remaining two surfaces are insulated.

For small values of temperature difference ($T_1 - T_2$), the buoyancy forces are not strong enough to overcome the viscous forces in the fluid. As a result, the fluid is stationary and the heat transfer takes place by pure conduction in the upward direction.

As a temperature difference is increased, natural convection flow begins at a certain value of ($T_1 - T_2$). This observation is expressed in terms of a Rayleigh number defined by

$$Ra_L = Gr_L \Pr = \left(\frac{g\beta(T_1 - T_2)L^3}{\nu^2}\right)\Pr$$

External Flows:

The local Nusselt number Nu_x for mixed convection on vertical plates is given by

$$Nu_x = 0.332(\mathrm{Re}_x)^{1/2}(\Pr)^{1/3} \; if \left(\frac{Gr_x}{R_{e_x}^2}\right) \le A$$

$$Nu_x = 0.508(\Pr)^{1/2}(0.952 + \Pr)^{-1/4}(Gr_x)^{1/4} \; if \left(\frac{Gr_x}{R_{e_x}^2}\right) > A$$

Where, A = 0.6 for $\Pr \le 10$

A = 1.0 for $\Pr = 100$

For horizontal plates when $\left(\frac{Gr_x}{R_e^{2.5}}\right) \le 0.083$ the following equation for forced convection may be used.

$$Nu_x = 0.332\, R_{e_x}^{1/2} \Pr^{1/3}$$

Internal flows:

(i) For mixed convection in laminar flow, Brown and Gauvin recommended a correlation of the form as:

$$\overline{Nu} = 1.75\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\left[Gz + 0.012\left(Gz.Gr^{1/3}\right)^{4/3}\right]^{1/3}$$

Where, Gz = Graetz number = Re Pr(D/L)

μ_b, μ_s = Viscosities of the fluid at the bulk mean temperature and surface temperature respectively.

(ii) For mixed convection with turbulent flow in horizontal tubes, Metais and Eckert suggest

$$\overline{Nu} = 4.69(Re)^{0.27}(Pr)^{0.21}(Gr)^{0.07}(D/L)^{0.36}$$

Example 09 :

In laminar natural convection a vertical wall is maintained at 150°C having average Nusselt number 45 with surrounding air at 30°C. If the wall temperature becomes 45°C, all other parameters remaining same, what would be the average Nusselt number?

(a) 24.56 (b) 23.67 (c) 26.75 (d) 30

Sol: $T_1 = 150°C$, $T_\infty = 30°C$, $T_2 = 45°C$

$\Delta T_1 = 150 - 30 = 120°C$

$Nu_1 = 45$

$\Delta T_2 = 45 - 30 = 15°C$

In laminar flow natural convection

$$Nu = (Ra)^{1/4} = (Gr \times Pr)^{1/4}$$

$$Nu \propto (Gr)^{1/4} \Rightarrow Nu \propto (\Delta T)^{1/4}$$

$$\left(\frac{\Delta T_1}{\Delta T_2}\right)^{1/4} = \frac{Nu_1}{Nu_2}$$

$$Nu_2 = 45 \times \left(\frac{15}{120}\right)^{1/4} = 26.75$$

Example 10 :

Air at 35°C flows across a cylinder of 40mm diameter at a velocity of 50m/s. The cylinder surface is maintained at 180°C. Find the heat lost per unit length. Properties at mean temperature of 50°C are; $\rho = 1\ kg/m^3$, $\mu = 50\times10^{-5}$ kg/ms, k = 0.08 W/m°C, c_p = 1.0 kJ/kg°C.

Use relation: $(Nu)_D = 0.023\,(Re)^{0.8}\,(Pr)^{1/3}$..

Sol: Given, D = 40mm = 0.05m, $t_\infty = 35°C$,

$t_s = 145°C$, V = 50m/s

Heat lost per unit length, Q/L:

Reynolds number, $Re = \dfrac{\rho VD}{\mu}$

$$= \frac{1\times50\times0.04}{50\times10^{-6}} = 40000$$

Prandtl number, $Pr = \dfrac{\mu C_p}{k}$

$$= \frac{50\times10^{-6}\times1000}{0.08} = 0.625$$

∴ Nusselt number is given by

$$(Nu)_D = 0.023\,Re^{0.8}\,Pr^{1/3}$$

$$= 0.023\times(40000)^{0.8}\times(0.625)^{1/3} = 95$$

$$Nu_D = \frac{\bar{h}D}{k} = 95$$

Heat transfer coefficient,

$$\bar{h} = \frac{295.16\times0.0312}{0.05}$$

$$= 190\ W/m^2°C$$

∴ Heat transferred, $Q = \bar{h}\,A_s\,(t_s - t_\infty)$

$$= 190\times(\pi\times0.04\times L)\times(180-50)$$

Heat transferred per unit length,

$$\frac{Q}{L} = 3103.89\ W$$

Grashof number has a role in free convection similar to that played by Reynolds number in forced convection.

In free convection, buoyancy driven flow sometimes dominates the flow inertia, therefore, the Nusselt number is a function of the Grashof number and the Prandtle number alone.

$\therefore Nu = f(Gr, Pr)$

Reynolds number will be important if there is an external flow. (combined forced and free convection).

In many instances, it is better to combine the Grashof number and the Prandtle number to define a new parameter, the Rayleigh number, Ra = Gr. Pr. The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow.

$Ra \le 10^9$ (laminar flow over flat plate)

$10^9 < Ra < 10^{11}$ (Transition flow over flat plate)

$Ra > 10^{11}$ (turbulent flow over flat plate)

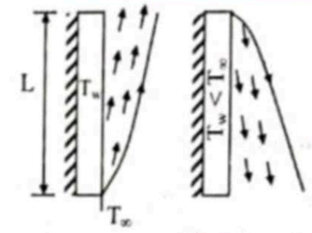
Empirical Correlations for free convection:

(i) Nusselt number $Nu = \dfrac{hL}{k}$

(ii) Grashoff number $Gr = \dfrac{L^3 \beta g \Delta t}{\upsilon^2}$

(iii) Prandtl number $Pr = \dfrac{\mu c_p}{k}$

1. Flow over vertical plates & cylinders (natural convection):



Heated vertical plate $T_w > T_\infty$ Cooled vertical plate $T_w < T_\infty$

$Ra < 10^9 \Rightarrow$ Laminar flow

$\overline{Nu}_L = 0.59\ Ra^{0.25}$

$R_a \ge 10^9 \Rightarrow$ Turbulent flow

$\overline{Nu}_L = 0.1 Ra^{0.33}$

L = characteristic Length

All the fluid properties are evaluated at the mean film temperature $\left(T_f = \dfrac{T_w + T_\infty}{2}\right)$

For laminar flow:

$(Nu)_x = 0.59(Gr.Pr)^{0.25}$

$$\frac{hx}{k} = 0.59\left[\frac{g\beta \Delta T x^3}{\upsilon^2} \times \frac{\mu c_p}{k}\right]^{\frac{3}{4}}$$

$\therefore h \propto x^{-\frac{1}{4}}$

$$\frac{h_2}{h_1} = \left(\frac{x_2}{x_1}\right)^{-\frac{1}{4}}$$

Average heat transfer coefficient, , $\bar{h} = \dfrac{1}{L}\displaystyle\int_0^L h\,dx$

$$\bar{h} = \frac{1}{L}\int_0^L x^{\frac{-1}{4}}\,dx = \frac{1}{L}\left[\frac{x^{3/4}}{3/4}\right]_0^L = \left(\frac{4}{3}\right)(L)^{-1/4}$$

$$\bar{h} = 1.33(h_L)$$

$$\overline{Nu} = 1.33(Nu)_L$$

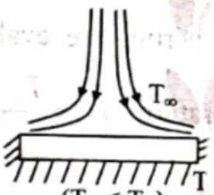
$$\overline{Nu} = 1.33 \times 0.59(Ra)_L^{0.25}$$

$$\overline{Nu} = 0.78 \times (Ra)_L^{0.25}$$

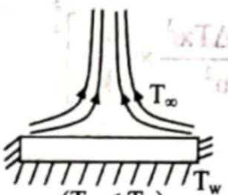
Flow over Horizontal Plate:

Incase of an irregular plate, the characteristic length is defined as the surface area divided by the perimeter of the plate.

a) Upper surface cooled

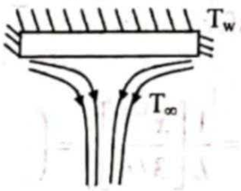


b) Upper surface Heated



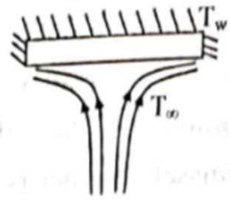
c) Lower surface cooled

(Tw < T∞)



d) Lower surface heated

(Tw < T∞)



For the figure b & c (upper surface heated or the lower surface cooled):

$Ra < 10^7 \Rightarrow$ Laminar flow

$\overline{Nu}_L = 0.54\,Ra^{0.25}$ for $10^4 < Ra_L < 10^7$

$Ra \geq 10^7 \Rightarrow$ Turbulent flow

$\overline{Nu}_L = 0.15\ Ra^{0.33}$ for $10^7 < Ra_L < 10^{10}$

For the figure a & d (the lower surface heated or upper surface cooled):

$Ra < 7\times10^8 \Rightarrow$ Laminar flow

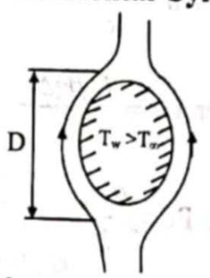
$\overline{Nu}_L = 0.27\ Ra^{0.25}$ for $3\times10^5 < Ra_L < 3\times10^{10}$

$Ra \geq 7\times10^8 \Rightarrow$ Turbulent flow

$\overline{Nu}_L = 0.107\,Ra^{0.33}$

For the above case the characteristic dimension L is to be taken as the length of the side for a square plate, the mean of the two sides for a rectangular plate and 0.9 times the diameter for a circular plate.

Flow over Horizontal Cylinders:



$Ra < 10^9 \Rightarrow$ Laminar flow

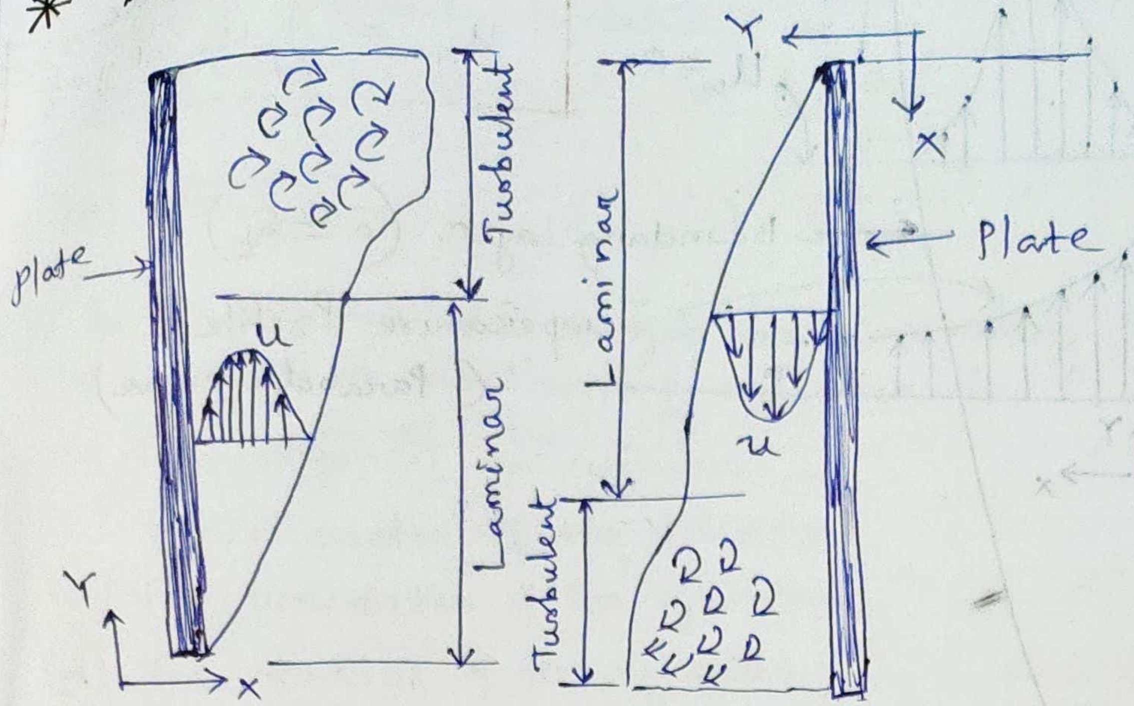
$\overline{Nu}_L = 0.53\,Ra^{0.25}$

$Ra \geq 10^9 \Rightarrow$ Turbulent flow

$\overline{Nu}_L = 0.13\,Ra^{0.33}$

All the fluid properties are evaluated at the mean film temperature $\left(T_f = \dfrac{T_w + T_\infty}{2}\right)$

* Velocity Boundary Layer for Free/Natural Convection



Hot Wall
$T_W > T_\infty$

Cold Wall
$T_W < T_\infty$

* Empirical Correlation for Natural Convection

$$Nu = \frac{hL_c}{K}, \quad Gr = \frac{L_c^3 \cdot \beta \cdot g \cdot \Delta t}{\nu^2}, \quad Pr = \frac{\mu \cdot C_p}{K}$$

1. Vertical Plates & Cylinders

Laminar flow : $\overline{Nu_L} = 0.59\,(Gr \cdot Pr)^{1/4} \leftarrow 10^4 < Gr \cdot Pr < 10^9$

Turbulent flow: $\overline{Nu_L} = 0.13\,(Gr \cdot Pr)^{1/3} \leftarrow 10^9 < Gr \cdot Pr < 10^{12}$

2. Horizontal Plates

Laminar flow : $\overline{Nu_L} = 0.13\,(Gr \cdot Pr)^{1/3} \leftarrow 10^5 < Gr \cdot Pr < 2 \times 10^8$

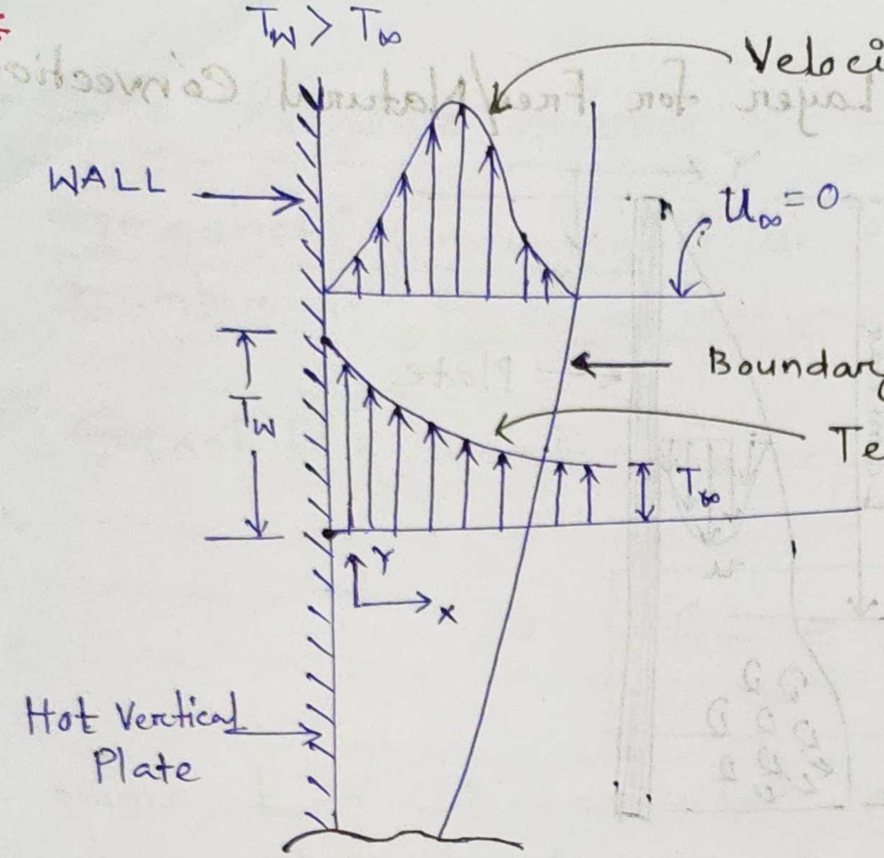
Turbulent flow: $\overline{Nu_L} = 0.16\,(Gr \cdot Pr)^{1/3} \leftarrow 2 \times 10^8 < Gr \cdot Pr < 3 \times 10^{11}$

* Combined Free & Forced Convection

$\frac{Gr}{Re^2} \geq 1 \longrightarrow$ Pure free convection becomes dominate

$\frac{Gr}{Re^2} = 1 \longrightarrow$ Mixed Convection (free + force), $\frac{Gr}{Re^2} \leq 1 \longrightarrow$ Pure forced Convection

*



$$\boxed{u_{max} = \frac{4}{27} u_o, \text{ at } y = \frac{\delta}{3}}$$

(Temperature and Velocity Profiles for Natural Convection on a hot Vertical Plate)

* Velocity, $u = 0$ at $y = \delta$

$u = $ Maximum at $y = \frac{\delta}{3}$

and $u_{max} = \frac{4}{27} u_o$

\# For Natural convection over a Vertical Flat plate or Vertical cylinder in the Turbulent Region ($Gr_L > 10^9$),

$$\boxed{\overline{Nu}_L = 0.13 (Gr_L \cdot Pr)^{1/3}}$$

$\rightarrow Nu \propto Gr^{1/3}$

$\Rightarrow \frac{h \cdot L}{K} \propto (L^3)^{1/3}$

$\Rightarrow \frac{h \cdot L}{K} \propto L$

$\Rightarrow \boxed{h \propto \text{Constant}}$

and 'h' is independent of diameter.

\# for Laminar Flow, over a vertical plate or tube $\boxed{h \propto \frac{1}{D^{1/4}}}$

\# The simple empirical correlations for the average Nusselt number in natural convection, $Nu = \frac{hL_c}{K} = c(Gr_L \cdot Pr)^n = c(Ra_L)^n$

The values of constants 'C' and 'n' depend on the geometry of the surface and the flow regime. The value of the constant $C < 1$ and $n = 1/4$ for laminar flow, $n = 1/3$ for turbulent flow.

Important Notes

* **Hydraulic Diameter, D_h** : In order to find Reynolds number, friction factor etc, we use a parameter called hydraulic diameter which is given by below equation :

$$\text{Hydraulic Diameter } (D_h) = \frac{4A_c}{P}$$

A_c= area of cross-section
P= wetted Perimeter (perimeter of cross section in contact with fluid)

So D_h relates the ***non-circular section to circular section*** and thus we can use this D_h in finding friction factor(f' or C_{fx}) using Moody's chart.

Also, For a fully filled duct or pipe whose cross section is a regular polygon, the hydraulic diameter is equivalent to the diameter of a circle inscribed within the wetted perimeter. It can be calculated by $\frac{4A_c}{P}$

* **Equivalent Diameter D_e**: It is the diameter of a circular duct or pipe that gives the same pressure loss as an equivalent rectangular duct or pipe for particular flow rate.

In real life, we use equivalent diameter in duct sizing where we know the flow rate(Q) and we need to find the size of the rectangular duct which gives same pressure drop as circular duct.

It is very easy to get confused and it seems Dh and De are the same thing. But flow rate plays an important role since to find De flow rate should same. Now without confusing you much let's see all these mathematically:

* **Characteristic Length, L_c :**

(1) In conduction mode of heat transfer, Characteristic Length, L_c is defined as the volume of one body divided by its surface area (i.e., $L_c = \frac{V}{A_s}$). It is used mainly in unsteady state of heat conduction across solids and when Biot's number (**B_i**) & Fourier's number (**F_o**) are being calculated.

(2) In convection mode of heat transfer, Characteristic Length, L_c is defined as the surface area of one body divided by its perimeter (i.e., $L_c = \frac{A_s}{P}$). It is used mainly in natural convection heat transfer across flat plates,cylinders, ducts etc surrounded by bulk fluid and when Grashoff's number (**Gr**) & Nusselt Number (**Nu**) are being calculated.

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Characteristic Length of Geometry (L_c)

→ In natural / free convection, the characteristic length, L_c used in Nusselt and Grashoff numbers is

= Length of a side of a square

= Mean of the two dimensions of a ~~horizontal~~ rectangular surface with negligible thickness $\left(\text{i.e., } \frac{a+b}{2}\right)$

= $\boxed{\frac{1}{L_c} = \frac{1}{L_h} + \frac{1}{L_v}}$, for rectangular solids

L_h = avg. of the horizontal dimension of the body

L_v = height or length of vertical dimension

= $0.9D$, D = diameter of a circular disk

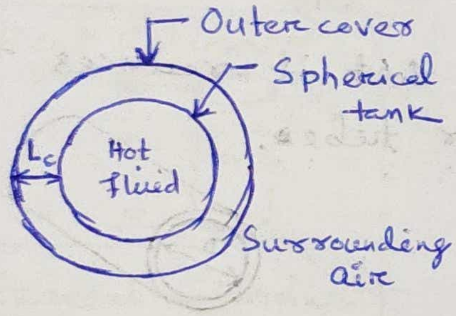
→ According to Cengel book,

The characteristic length for a horizontal surface is given by

$$\boxed{L_c = \frac{A_s}{P}}$$, A_s = surface area, P = perimeter

- $L_c = \frac{a}{4}$, for a horizontal square of side 'a'
- $L_c = \frac{D}{4}$, for a horizontal circular surface of diameter D

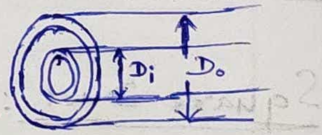
→

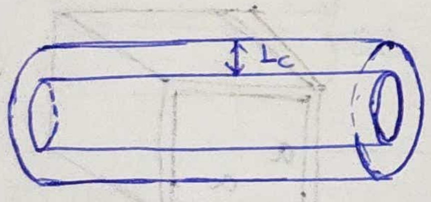


$$\boxed{L_c = \frac{D_o - D_i}{2}}$$

But $D_h = \frac{4A_c}{P} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)}$

$= D_o - D_i$

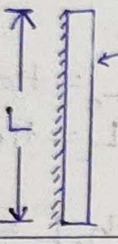
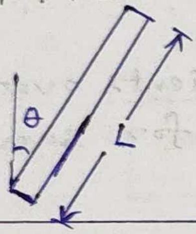
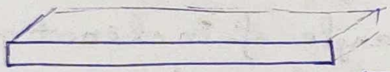
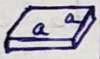
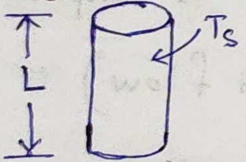
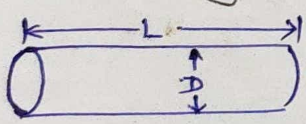
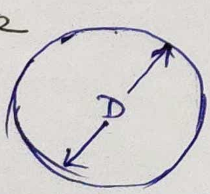




$$\boxed{L_c = \frac{D_o - D_i}{2}}$$

Characteristic Length (L_c) of Geometry

$$\text{Characteristic Length, } L_c = \frac{\text{Surface area}}{\text{Perimeter}} = \frac{A_s}{P}.$$

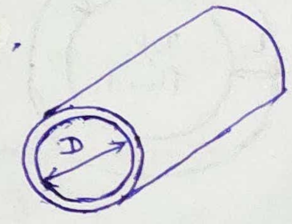
Geometry	Characteristic Length (L_c)	
Vertical Plate 	L	
Inclined Plate 	L	
Horizontal Plate 	$\frac{A_s \leftarrow \text{Surface area}}{P \leftarrow \text{Perimeter}}$	
* Horizontal Square Plate 	$L_c = \frac{A_s}{P} = \frac{2 \times a^2}{2 \times (4a)} = \frac{a}{4}$	
* Horizontal Circular plate 	$L_c = \frac{A_s}{P} = \frac{2 \times \left(\frac{\pi D^2}{4}\right)}{2 \times (\pi D)} = \frac{D}{4}$	
Vertical Cylinder 	L	
Horizontal Cylinder 	D	
Sphere 	D	

Hydraulic Diameter (D_h):

For the flow through non-circular tubes, the Reynolds number, Nusselt number and the friction factor are based on the hydraulic diameter.

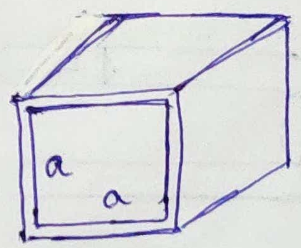
$$\boxed{D_h = \frac{4A_c}{P_w}} = \frac{4 \times \text{Cross sectional area of flow}}{\text{Wetted perimeter}}$$

- The hydraulic diameter is such that it reduces to ordinary diameter for flow in a circular tube.



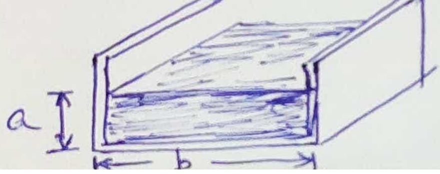
Circular tube: $D_h = \dfrac{4\left(\pi D^2/4\right)}{\pi D} = D$

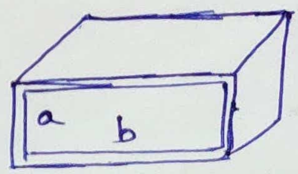
Square Duct: $D_h = \dfrac{4a^2}{4a} = a$



Rectangular Duct: $D_h = \dfrac{4ab}{2(a+b)} = \dfrac{2ab}{a+b}$

Channel (open-channel flow): $D_h = \dfrac{4a}{2a+b}$





* $\delta \propto x^{1/4}$ (As 'x' increases, δ also increases)

* $h \propto \frac{1}{x^{1/4}}$ (Laminar region) (As 'x' increases, 'h' decreases)

In the Laminar region, over vertical plates or tubes,

* Average Heat transfer co-efficient, $\bar{h} = \frac{1}{L}\int_0^L h_x \, dx$

$$= \frac{1}{L}\int_0^L C \cdot x^{-1/4} \, dx$$

$$= \left[C_1 \cdot x^{3/4}\right]_{x=L} \times \frac{4}{3}$$

$$\Rightarrow \boxed{\bar{h} = \frac{4}{3} h_x}$$

Hence,

$$\boxed{\overline{Nu} = \frac{4}{3} Nu_x}$$

Forced Convection

Laminar flow over flat plate

$\bar{h} = 2h_x$

and $\overline{Nu} = 2Nu_x$

Objective Type Questions

Q.1: Grashof Number is defined as

(a) $\dfrac{g \cdot \beta \cdot \theta \cdot L}{\nu}$ (b) $\dfrac{g \cdot \beta \cdot \theta \cdot L^2}{\nu^2}$

(c) $\dfrac{g \cdot \beta \cdot \theta \cdot L^3}{\nu^2}$ (d) $\dfrac{g \cdot \beta \cdot \theta \cdot L^3}{\nu^3}$

Ans: (c)

Q.2: The characteristic length for computing Grashof number in case of a horizontal cylinder is

(a) the length of the cylinder
(b) the diameter of the cylinder
(c) the perimeter of the cylinder
(d) the radius of the cylinder

Ans: (b)

Q.3: In Natural convection heat transfer, the Nusselt Number is a function of

(a) Re and Pr (b) Re and Gr
(c) Gr and Pr (d) Gr and Bi

Ans: (c)

Q.4: The maximum velocity in the laminar boundary layer in natural convection heat transfer is equal to

(a) 0 (b) $\delta/3$
(c) $\delta/2$ (d) δ

Ans: (b)

Q.5: Natural Convection dominates if

(a) $Gr/Re^2 << 1$ (b) $Gr/Re^2 >> 1$

(c) $Gr/Re^2 = 1$ (d) $\dfrac{Pr \cdot Gr}{Re^2} >> 1$

Ans: (b)

Q.6: forced convection dominates if ________ Ans: (a)

Q.7: In natural convection heat transfer under uniform heat flux, the modified Grashof number, Gr_x^* is defined as

$$\boxed{Gr_x^* = Nu \times Gr}$$

Q.8: In natural convection heat transfer from a horizontal tube of 3 cm diameter is given by the relation

$Nu \propto Gr^{1/4}$ and the convective heat transfer co-efficient is 100 $W/m^2\text{-}K$. If the diameter of the tube is 12 cm, the value of 'h' would be, with other parameters remaining the same,

(a) 100 $W/m^2\text{-}K$

(b) 90 $W/m^2\text{-}K$

(c) 80 $W/m^2\text{-}K$

(d) 70.71 $W/m^2\text{-}K$

Solution:

$D_1 = 3\text{ cm}$, $h_1 = 100\ W/m^2\text{-}K$

$D_2 = 12\text{ cm}$, $h_2 = ?$

We know, $Gr = \frac{L_c^3 g . \beta . \Delta t}{\nu^2}$

$\Rightarrow Gr \propto L_c^3$ ($\because L_c$ = characteristic length)

$\Rightarrow \boxed{Gr \propto D^3}$ $L_c = D$, for a horizontal cylinder

$\therefore Nu \propto Gr^{1/4}$

$\Rightarrow \frac{h.L}{K} \propto (D^3)^{1/4}$

$\Rightarrow h \propto K . \frac{D^{3/4}}{L}$ ($\because L = D$)

$\Rightarrow h \propto D^{3/4 - 1}$

$\Rightarrow h \propto \frac{1}{D^{1/4}}$

$\frac{h_2}{h_1} = \left(\frac{D_1}{D_2}\right)^{1/4}$

$\Rightarrow h_2 = h_1 \times \left(\frac{D_1}{D_2}\right)^{1/4}$

$= 100 \times \left(\frac{3}{12}\right)^{1/4}$

$= 50\sqrt{2}\ W/m^2\text{-}K$

$\Rightarrow h_2 = 70.71\ W/m^2\text{-}K$

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Ans

Q.9: Assertion (A): In natural convection turbulent flow over heated vertical plate, 'h' is independent of the characteristic length.

Reasoning (R): In turbulent flow natural convection heat transfer over a heated vertical plate, $Nu = C(Pr)^{1/3}$

Code:
(a) Both A and R are false
(b) Both A and R are true
(c) A is true, R is false
(d) A is false, R is true

Ans: (b)

* For a vertical plate, characteristic length, L_c = Height of the plate
For a vertical cylinder, characteristic length, L_c = Height of the cylinder
For a horizontal cylinder, characteristic length, L_c = Diameter of the cylinder

* Natural convection heat transfer co-efficients over surface of a vertical pipe and vertical flat plate for same height and fluid are EQUAL.

* For free convection over inclined plates, Grashoff number is multiplied by cos θ, where θ is the angle of inclination from the vertical and use vertical plate constants.

* For free convection on vertical planes or cylinders, the convection heat transfer co-efficient is empirically given by

$$h = 1.42\left(\frac{\Delta t}{L}\right)^{0.25} \quad \text{(for Laminar flow)}$$

$$h = 1.31\left(\Delta t\right)^{0.33} \quad \text{(for turbulent flow)}$$

Problem : Two horizontal steam pipes having diameters 100 mm and 300 mm are so laid in a boiler house that the mutual heat transfer may be neglected. The surface temp. of each of the steam pipes is 475°C. If the temp. of the ambient air is 35°C, calculate the ratio of heat transfer co-efficients and heat losses per metre length of the pipes.

Solution : Given $D_1 = 100\,MM = 0.1\,m$, $t_s = 475°C$

$D_2 = 300\,MM = 0.3\,m$, $t_a = 35°C$

The steam pipes are located in the boiler house where the ambient air is stationary, thus this is a case of free convection for which

$$Nu = C \cdot (Gr \cdot Pr)^{1/4} \quad \text{and} \quad Gr \propto D^3$$

So, $\boxed{h \propto \frac{1}{D^{1/4}}}$

Ratio of heat transfer co-efficients.

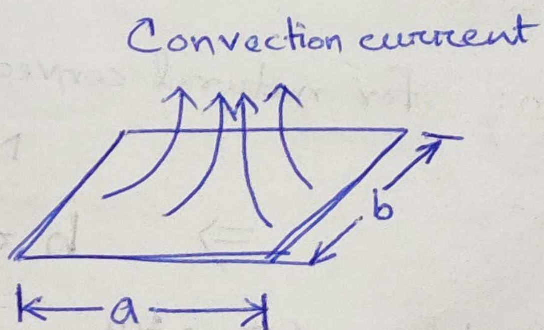
$$\frac{h_1}{h_2} = \left(\frac{D_2}{D_1}\right)^{1/4} = \left(\frac{0.3}{0.1}\right)^{1/4} = 1.316 \quad \text{Ans}$$

Ratio of Heat Losses per unit length :

$$\frac{Q_1}{Q_2} = \frac{h_1 D_1}{h_2 \cdot D_2} = 1.316 \times \frac{0.1}{0.3} = 0.438 \quad \text{Ans}$$

$(A = \pi D L)$

Q. Natural convection occurs over horizontal flat plate as shown in figure. What will be the characteristic length for this case ?



(A) $\frac{2ab}{a+b}$ (B) $\frac{ab}{2(a+b)}$

(C) $\frac{a+b}{2}$ (D) a

Ans : (C)

But according to some other books, both (B) & (C) are correct.

* Option A (i.e. $\frac{2ab}{a+b}$) represents the hydraulic mean dia of rectangular conduit.

Q. The convective heat transfer co-efficient from a hot cylindrical surface exposed to still air varies in accordance with

(a) $(\Delta T)^{0.25}$ (b) $(\Delta T)^{0.5}$

(c) $(\Delta T)^{0.75}$ (d) $(\Delta T)^{1.25}$

Ans : (a)

Q. In respect of free convection over a vertical flat plate the Nusselt numbers for laminar and turbulent flows varies respectively with Grashoff number (Gr) as

(a) Gr and $Gr^{1/4}$ (b) $Gr^{1/2}$ and $Gr^{1/3}$

(c) $Gr^{1/4}$ and $Gr^{1/3}$ (d) $Gr^{1/3}$ and $Gr^{1/4}$

Ans: (c)

Q*. The Nusselt number is related to Reynolds number in laminar and turbulent flows respectively as

(a) $Re^{-1/2}$ and $Re^{0.8}$ (b) $Re^{1/2}$ and $Re^{0.8}$

(c) $Re^{-1/2}$ and $Re^{-0.8}$ (d) $Re^{1/2}$ and $Re^{-0.8}$

Ans : (b)

Q. The Nusselt number of convective heat transfer between a horizontal tube and water surrounding, it is prescribed by the relation

$$Nu = 0.52\,(Gr.Pr)^{0.25}$$

For a 4 cm diameter tube, the heat transfer co-efficient is stated to be 5910 KJ/m²-hr-deg. Subsequently, the tube is replaced by one with 16 cm diameter tube. If temperature and surface of the fluid remain same, the heat transfer co-efficient will change to

(a) 2955 (b) 4180 (c) 11820 (d) 23645 KJ/m²-hr-deg

Ans : (b)

Solution: For natural convection,

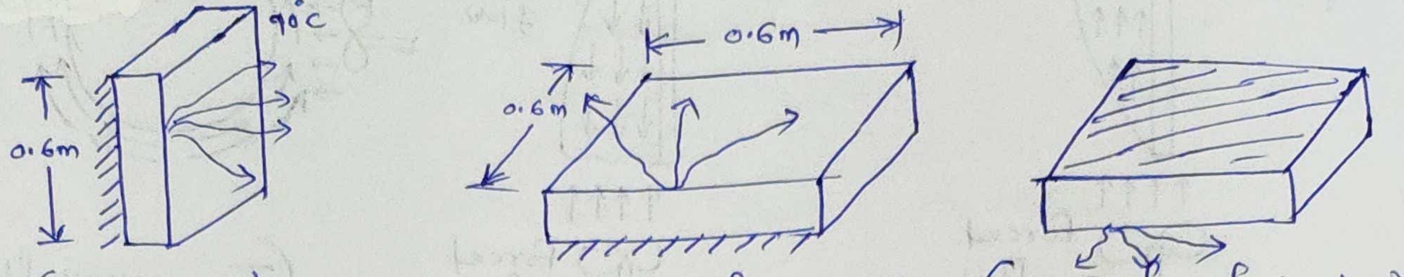
$$Nu = 0.52\,(Gr.Pr)^{0.25}$$

$$\Rightarrow \quad h \propto \frac{1}{D^{1/4}}$$

$$\therefore \frac{h_2}{h_1} = \left(\frac{D_1}{D_2}\right)^{1/4} = \left(\frac{4\times10^{-2}}{16\times10^{-2}}\right)^{1/4} = \sqrt{2}$$

$$\Rightarrow \quad h_2 = \sqrt{2}\,h_1 = \sqrt{2}\times 5190 = 4180 \text{ KJ/m}^2\text{-hr-deg.}$$

Q. Consider a 0.6m × 0.6m thin square plate in a room at 30°c. One side of the plate is maintained at a temp. of 90°c, while the other side is insulated. In which orientation of the plate, the heat transfer, by natural convection from the square plate is minimum.



(Vertical) (Hot surface facing up) (Hot surface facing down)

Vertical	Hot surface facing up	Hot surface facing down
$L_c = L = 0.6\,m$	$L_c = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = 0.15\,m$	$L_c = \frac{A_s}{P} = \frac{L}{4} = 0.15\,m$
$h = \frac{K}{L} Nu = 5.302$	$h = \frac{K}{L} Nu = 5.944\ W/m^2K$	$h = \frac{K}{L} Nu = 2.971\ W/m^2$
$Q = 115\,W$	$Q = 128\,W$	$Q = 64.2\,W$ (Minimum)

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and can not get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer

Q. In natural convection heat transfer, the Grashof number is a measure of the relative magnitudes of .

(a) the buoyancy force only (b) the opposing viscous force only
(c) the buoyancy force and the opposing viscous force acting on the fluid
(d) none of the above

Ans: (c)

Q. The primary driving force for natural convection is
(a) shear stress forces (b) buoyancy forces (c) pressure forces
(d) surface tension forces (e) none of them

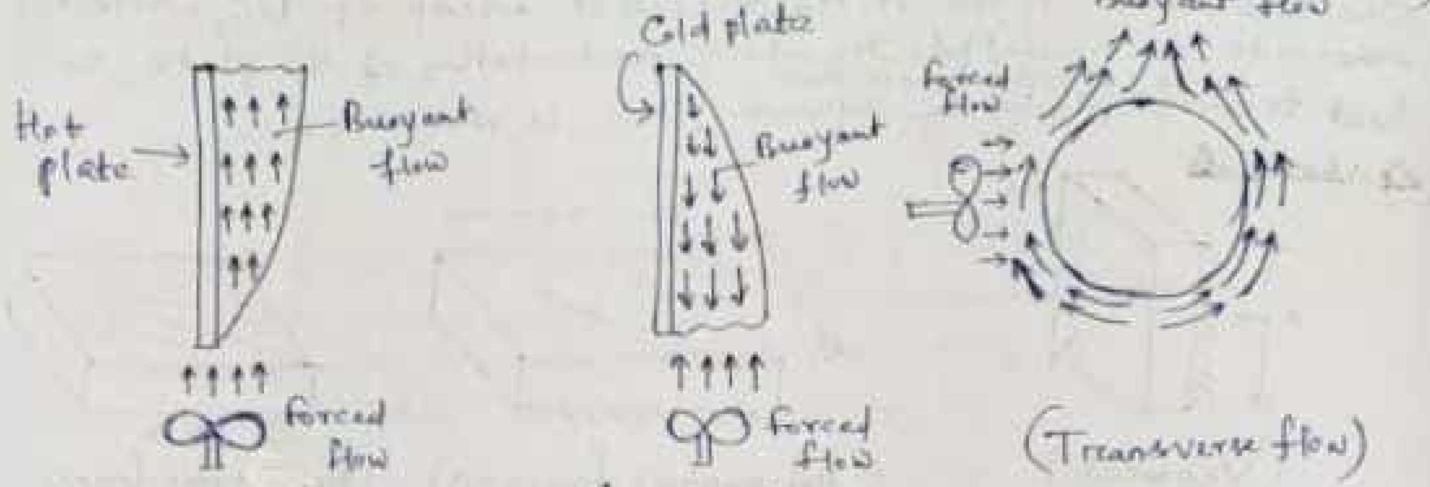
Ans: (b)

Q. Assertion (A): Heat sinks with closely spaced fins are not suitable for natural convection cooling.

Reason (R): The friction force increases as more and more solid surfaces are introduced, seriously disrupturing the fluid flow and heat transfer.

Ans: Both 'A' and 'R' are correct and 'R' is the explanation of 'A'.

* Mixed Convection (Combined Natural and Forced Convection)



(Assisting flow) (Opposing flow) (Transverse flow)

For forced convection, $Nu = f(Re_L, Pr)$

For Natural convection, $Nu = f(Gr_L, Pr)$

For combined natural and forced convection, $Nu = f(Re_L, Gr_L, Pr)$

- In assisting flow, the buoyant motion is in the same direction as the forced motion. Therefore, natural convection assists forced convection and enhances heat transfer.
  Ex:- upward forced flow over a hot surface

- In opposing flow, the buoyant motion is in the opposite dirn. to the forced motion. Therefore, natural convection resists forced convection and decreases heat transfer.
  Ex:- Upward forced flow over a cold surface.

- In transverse flow, the buoyant motion is perpendicular to the forced motion. Transverse flow enhances fluid mixing and thus enhances heat transfer.
  Ex:- horizontal forced flow over a hot or cold cylinder or sphere.

$$Nu_{combined} = Nu^n_{forced} + Nu^n_{free} \quad \text{(Assisting and Transverse flow)}$$

$$Nu_{combined} = Nu^n_{forced} - Nu^n_{free} \quad \text{(Opposing flow)}$$

Natural Convection Heat Transfer

MCQs with Answers

1. Generally, natural convection occurs due to

a. change in velocity of a fluid
b. change in density of a fluid
c. change in molecular structure of a fluid
d. none of the above

ANSWER: b. change in density of a fluid

2. The buoyancy forces which give rise to the natural convection are called as

a. convection forces
b. fluid forces
c. body forces
d. none of the above

ANSWER: c. body forces

3. The intensity of mixing of fluid in natural convection is

a. more than the intensity of mixing of fluid in forced convection
b. less than the intensity of mixing of fluid in forced convection
c. equal to the intensity of mixing of fluid in forced convection
d. unpredictable

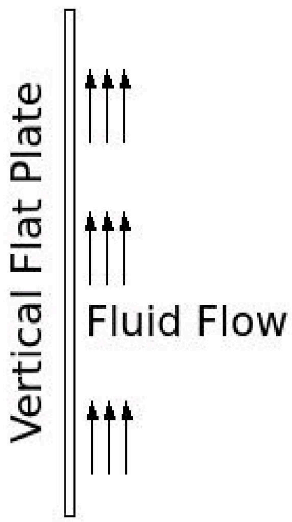
ANSWER: b. less than the intensity of mixing of fluid in forced convection

4. What is the relation between convection heat transfer coefficients of natural convection and forced convection?

a. convection heat transfer coefficient of natural convection is lower than the convection heat transfer coefficient of forced convection
b. convection heat transfer coefficient of natural convection is more than the convection heat transfer coefficient of forced convection
c. convection heat transfer coefficients in both natural and forced convection are the same for same system
d. unpredictable

ANSWER: a. convection heat transfer coefficient of natural convection is lower than the convection heat transfer coefficient of forced convection

5. Below figure shows a natural convection heat transfer on a vertical flat plate surrounded by a fluid.



Natural convection heat transfer on a vertical flat plate surrounded by a fluid

What is the relation between the upward velocity of the fluid and the distance from the bottom of the plate, when plate is hotter that fluid?

a. as the distance from the bottom of the plate increases, the upward velocity of the fluid near the plate surface decreases
b. as the distance from the bottom of the plate increases, the upward velocity of the fluid near the plate surface increases
c. the upward velocity of the fluid near the plate surface is same all over the plate
d. unpredictable

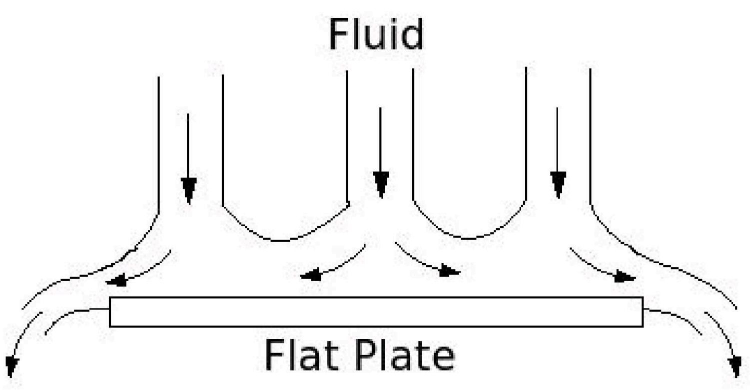
ANSWER: b. as the distance from the bottom of the plate increases, the upward velocity of the fluid near the plate surface increases

6. Assume a natural convection heat transfer on a vertical flat plate surrounded by a fluid. Where will be the fully developed turbulent layer of fluid established, if the plate is hotter than the fluid?

a. At the bottom of the plate
b. At the middle of the plate
c. At the top of the plate
d. Nowhere

ANSWER: c. At the top of the plate

7. Below figure shows a natural convection heat transfer on a horizontal flat plate and fluid is above it. Which condition satisfies the figure below?



Natural Convection on Horizontal Flat Plate

a. Plate temperature is lower than the fluid temperature
b. Plate temperature is higher than the fluid temperature
c. Plate temperature is equal to the fluid temperature
d. unpredictable

ANSWER: b. Plate temperature is higher than the fluid temperature

8. In heat transfer, the ratio of the buoyancy force to the viscous force acting on a fluid called?

a. Prandtl number (Pr)
b. Reynolds number (Re)
c. Nusselt number (Nu)
d. Grashof number (Gr)

ANSWER: d. Grashof number (Gr)

9. In natural convection, the Nusselt number (Nu) depends on

a. Pr and Re
b. Gr and Re
c. Gr and Pr
d. none of the above

ANSWER: c. Gr and Pr

10. Which of the following condition is correct for natural convection?

a. $(Gr / Re^2) = 1$
b. $(Gr / Re^2) << 1$
c. $(Gr / Re^2) >> 1$
d. none of the above

ANSWER: c. $(Gr / Re^2) >> 1$

11. If there are no externally induced flow velocities, then the Nusselt number (Nu) does not depend upon

a. Prandtl number (Pr)
b. Reynolds number (Re)
c. Grashof number (Gr)
d. none of the above

ANSWER: b. Reynolds number (Re)

12. The Grashof number in natural convection plays same role as

a. Prandtl number (Pr) in forced convection
b. Reynolds number (Re) in forced convection
c. Nusselt number (Nu) in forced convection
d. none of the above

ANSWER: b. Reynolds number (Re) in forced convection

13.
In free convection heat transfer transition from laminar to turbulent flow is governed by the critical value of the

A. Reynold's number

B. Grashoff's number

C. Reynold's number, Grashoff's number

D. Prandtl number, Grashoff's number

Ans : D

Bijan Kumar Giri

Example 8.1. *A vertical cylinder 1.5 m high and 180 mm in diameter is maintained at 100°C in an atmosphere environment of 20°C. Calculate heat loss by free convection from the surface of the cylinder. Assume properties of air at mean temperature as,* $\rho = 1.06$ *kg/m³*, $\nu = 18.97 \times 10^{-6}$ *m²/s*, $c_p = 1.004$ *kJ/kg°C and* $k = 0.1042$ *kJ/mh°C.* **(AMIE Summer, 2000)**

Solution. *Given* : $L = 1.5$ m; $D = 180$ mm $= 0.18$ m, $t_s = 100$°C;

$t_\infty = 20$°C; $\rho = 1.06$ kg/m³; $\nu = 18.97 \times 10^{-6}$ m²/s;

$c_p = 1.004$ kJ/kg°C; $k = 0.1042$ kJ/m h°C.

Heat loss by free convection, Q :

$$\mu = \rho\nu = 1.06 \times (18.97 \times 10^{-6} \times 3600) = 0.07239 \text{ kg/mh}$$

$$\beta = \frac{1}{T} = \frac{1}{273 + t_f} = \frac{1}{273 + \left(\frac{100 + 20}{2}\right)} = 0.003\,\text{K}^{-1}$$

$$Gr = \frac{L^3 g \beta \Delta t}{\nu^2}$$

$$= \frac{(1.5)^3 \times 9.81 \times 0.003 \times (100 - 20)}{(18.97 \times 10^{-6})^2} = 2.208 \times 10^{10}$$

$$Pr = \frac{\mu c_p}{k} = \frac{0.07239 \times 1.004}{0.1042} = 0.6975$$

$$Gr\,Pr = 2.208 \times 10^{10} \times 0.6975 = 1.54 \times 10^{10}$$

For this value of $Gr\,Pr$ (tubulent range),

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.10\,(Gr . Pr)^{1/3} \quad (\text{for } 10^9 < Gr.Pr < 10^{12})$$

$$= 0.10\,(1.54 \times 10^{10})^{1/3} = 248.79$$

$$\therefore \quad \bar{h} = \frac{k}{L} \times 248.79 = \frac{0.1042}{1.5} \times 248.79 = 17.283 \text{ kJ/h m}^2\text{°C}$$

$\therefore$ Rate of heat loss, $Q = \bar{h}A\,(t_s - t_\infty)$

$= 17.283 \times (\pi \times 0.18 \times 1.5) \times (100 - 20)$

$=$ **1172.8 kJ/h** **(Ans.)**

Bijan Kumar Giri

Example 8.2. *A cylindrical body of 300 mm diameter and 1.6 m height is maintained at a constant temperature of 36.5°C. The surrounding temperature is 13.5°C. Find out the amount of heat to be generated by the body per hour if* $\rho = 1.025$ *kg/m*^{3}; $c_p = 0.96$ *kJ/kg°C*; $v = 15.06 \times 10^{-6}$ *m*^{2}*/s*; $k = 0.0892$ *kJ/m-h-°C and* $\beta = \frac{1}{298} K^{-1}$. *Assume* $Nu = 0.12\,(Gr\,.\,Pr)^{1/3}$ *(the symbols have their usual meanings).* **(AMIE Winter, 1997)**

Solution. *Given* : $D = 300$ mm $= 0.3$ m; $L = 1.6$ m; $t_s = 36.5$°C; $t_\infty = 13.5$°C;

$\rho = 1.025$ kg/m^3; $c_p = 0.96$ kJ/kg°C; $v = 15.06 \times 10^{-6}$ m^2/s;

$k = 0.0892$ kJ/m-h-°C; $\beta = \frac{1}{298} K^{-1}$; $Nu = 0.12\,(Gr\,.\,Pr)^{1/3}$

The amount of heat to be generated :

Grashoff number,
$$Gr = \frac{L^3 g\beta\,(t_s - t_\infty)}{v^2}$$
$$= \frac{(1.6)^3 \times 9.81 \times \left(\frac{1}{298}\right) \times (36.5 - 13.5)}{(15.06 \times 10^{-6})^2} = 1.3674 \times 10^{10}$$

Prandtl number,
$$Pr = \frac{\mu c_p}{k} = \frac{\rho v c_p}{k} = \frac{1.025 \times (15.06 \times 10^{-6} \times 3600) \times 0.96}{0.0892} \qquad \left(\because v = \frac{\mu}{\rho}\right)$$

Nusselt number,
$$Nu = \frac{hL}{k} = 0.12\,(Gr\,.\,Pr)^{1/3}$$
$$= 0.12\,(1.3674 \times 10^{10} \times 0.598)^{1/3} = 241.75$$

$\therefore$
$$h = \frac{k}{L} \times 241.75 = \frac{0.0892}{1.6} \times 241.75 = 13.478 \text{ kJ/h m}^2\text{°C}$$

Heat lost from the surface by natural convection,
$$Q = hA\,(t_s - t_\infty)$$
$$= 13.478 \times (\pi \times 0.3 \times 1.6)(36.5 - 13.5) = \textbf{*467.5 kJ/h} \qquad \textbf{(Ans.)}$$

* This is the amount of heat to be generated.

Example 8.6. *Air flows through long rectangular heating duct of width and height of 0.75 m and 0.3 m respectively. The outer surface temperature of the duct is maintained at 45°C. If the duct is exposed to air at 15°C in a cramp-space beneath a home, what is the heat loss from the duct per metre length ?*

Use the following correlations :

(*i*) *Top surface :*

Take $L_c = \dfrac{B}{2} = 0.375\ m.$

$$\overline{Nu_L} = 0.54\ (Ra_L)^{0.25} \text{ if } 10^4 \le Ra_L \le 10^7$$
$$= 0.15\ (Ra_L)^{0.33} \text{ if } 10^7 \le Ra_L \le 10^{11}$$

(*ii*) *Bottom surface :*

Take $L_c = \dfrac{B}{2} = 0.375\ m$

$$\overline{Nu_L} = 0.27\ (Ra_L)^{0.25},\ 10^5 \le Ra_L \le 10^{10}$$

(*iii*) *Sides of the duct :*

$$\overline{Nu_L} = 0.68 + \frac{0.67\ (Ra_L)^{0.25}}{\left[1 + \left(\dfrac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

Use the following properties of air :

$$\nu = 16.2 \times 10^{-6}\ m^2/s,\ \alpha = 22.9 \times 10^{-6}\ m^2/s$$
$$k = 0.0265\ W/m\ K,\ \beta = 0.0033\ K^{-1} \text{ and } Pr = 0.71.$$

(N.M.U. Winter, 1995)

Solution. The configuration of the duct is shown in Fig. 8.5.

$$Ra_L = \frac{g\beta\ (t_s - t_a)\ L_c^3}{\alpha\nu}$$

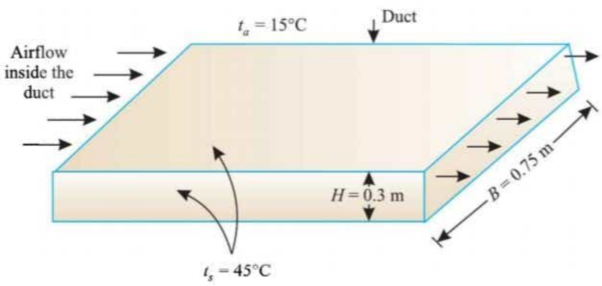


Fig. 8.5

$$= \frac{9.81 \times 0.0033 \times (45 - 15)\ L_c^3}{(22.9 \times 10^{-6}) \times (16.2 \times 10^{-6})} = 2.62 \times 10^9\ (L_c^3)$$

For two sides :

$$L_c = H = 0.3\text{ m}$$

$\therefore$ $$Ra_L = 2.62 \times 10^9\ (0.3)^3 = 7.07 \times 10^7$$

$\therefore$ $$Nu_L = 0.68 + \frac{0.67\ (7.07 \times 10^7)^{0.25}}{\left[1 + \left(\dfrac{0.492}{0.71}\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{61.44}{1.3} = 47.94$$

$\therefore$ $$\frac{h_s\ L_c}{k} = 47.94$$

or, $$h_s = \frac{k}{L_c} \times 47.94 = \frac{0.0265 \times 47.94}{0.3} = 4.23\text{ W/m}^2\text{°C}$$

For top surface :

$$\overline{Nu_L} = \frac{h_t\ L_c}{k} = 0.15\ (Ra_L)^{0.33} = 0.15\ (7.07 \times 10^7)^{0.33} = 58.4$$

$\therefore$ $$h_t = \frac{k}{L_c} \times 58.4 = \frac{0.0265 \times 58.4}{0.375} = 4.127\text{ W/m}^2\text{°C}$$

$$\left(\because L_c = \frac{B}{2} = \frac{0.75}{2} = 0.375\text{ m}\right)$$

For bottom surface :

$$\overline{Nu_L} = \frac{h_b\ L_c}{k} = 0.27\ (Ra_L)^{0.25} = 0.27\ (7.07 \times 10^7)^{0.25} = 24.76$$

$\therefore$ $$h_b = \frac{k}{L_c} \times 24.76 = \frac{0.0265}{0.375} \times 24.76 = 1.75\text{ W/m}^2\text{°C}$$

The rate of heat loss per unit length of the duct is given by,

$$Q = Q_s + Q_t + Q_b$$
$$= [2 \times h_s \times (0.3 \times 1) + h_t \times (0.75 \times 1) + h_b \times (0.75 \times 1)]\ (t_s - t_a)$$
$$= [2 \times 4.23 \times 0.3 + 4.127 \times 0.75 \times 1 + 1.75 \times 0.75 \times 1]\ (45 - 15)$$
$$= \mathbf{208.4\ W/m} \quad \textbf{(Ans.)}$$

Example 8.11. *A nuclear reactor with its core constructed of parallel vertical plates 2.2 m high and 1.4 m wide has been designed on free convection heating of liquid bismuth. The maximum temperature of the plate surfaces is limited to 960°C while the lowest allowable temperature of bismuth is 340°C. Calculate the maximum possible heat dissipation from both sides of each plate.*

For the convection coefficient, the appropriate correlation is

$$Nu = 0.13\ (Gr.Pr)^{0.333}$$

where different parameters are evaluated at the mean film temperature.

Solution. The mean film temperature, $t_f = \dfrac{960 + 340}{2} = 650°\text{C}$

The thermo-physical properties of bismuth are:

$\rho = 10^4\ \text{kg/m}^3$; $\mu = 3.12$ kg/m-h; $c_p = 150.7$ J/kg°C; $k = 13.02$ W/m°C

$$\beta = \frac{1}{650 + 273} = 1.08 \times 10^{-3}\ \text{K}^{-1}$$

$$\therefore \quad Pr = \frac{\mu c_p}{k} = \frac{(3.12/3600) \times 150.7}{13.02} = 0.01$$

$$Gr = \frac{L^3 \rho^2 g \beta \Delta t}{\mu^2} = \frac{(2.2)^3 \times (10^4)^2\ 9.81 \times 1.08 \times 10^{-3} \times (960 - 340)}{(3.12/3600)^2}$$

$$= 9.312 \times 10^{15}$$

$$Gr.Pr = 9.312 \times 10^{15} \times 0.01 = 93.12 \times 10^{12}$$

Using the given correlation, we get

$$Nu = \frac{hL}{k} = 0.13\ (Gr.Pr)^{0.333}$$

or,

$$h = \frac{k}{L} \times 0.13\ (Gr.Pr)^{0.333}$$

$$= \frac{13.02}{2.2} \times 0.13\ (93.12 \times 10^{12})^{0.33} = 34500\ \text{W/m}^2\text{°C}$$

$\therefore$ Heat dissipation from both sides of each plate,

$$Q = 2\, h\, A_s\, \Delta t$$

$$= 2 \times 34500 \times (2.2 \times 1.45) \times (960 - 340)$$

$= 136.47 \times 10^6$ W = **136.47 MW** **(Ans.)**

Example 8.16. *A steam pipe 7.5 cm in diameter is covered with 2.5 cm thick layer of insulation which has a surface emissivity of 0.9. The surface temperature of the insulation is 80°C and the pipe is placed in atmospheric air at 20°C. Considering heat loss both by radiation and natural convection, calculate*

(*i*) *The heat loss from 6 m length of the pipe;*

(*ii*) *The overall heat transfer coefficient and the heat transfer coefficient due to radiation alone.*

Solution. *Given* : $D = 7.5 + 2 \times 2.5 = 12.5$ cm $= 0.125$ m, $\varepsilon = 0.9$, $t_s = 80°C$, $t_\infty = 20°C$

The mean film temperature, $t_f = \dfrac{80 + 20}{2} = 50°C.$

The thermo-physical properties of air at 50°C are:

$\rho = 1.092$ kg/m³; $c_p = 1007$ J/kg°C; $\mu = 19.57 \times 10^{-6}$ kg/ms; $k = 27.81 \times 10^{-3}$ W/m°C

$$\beta = \frac{1}{50 + 273} = 3.096 \times 10^{-3}\ \text{K}^{-1}$$

$$Pr = \frac{\mu c_p}{k} = \frac{19.57 \times 10^{-6} \times 1007}{27.81 \times 10^{-3}} = 0.708$$

$$Gr = \frac{D^3 \rho^2 g \beta \Delta t}{\mu^2} = \frac{(0.125)^3 \times (1.092)^2 \times 9.81 \times 3.096 \times 10^{-3} \times (80 - 20)}{(19.57 \times 10^{-6})^2}$$
$$= 11.08 \times 10^6$$

Using the eqn. (8.45), we get

$$\overline{Nu} = \frac{\bar{h} D}{k} = 0.53\ (Gr.Pr)^{1/4}$$

$$\bar{h} = h_{conv.} = \frac{k}{D} \times 0.53\ (Gr.Pr)^{1/4}$$
$$= \frac{27.81 \times 10^{-3}}{0.125} \times 0.53\ (11.08 \times 10^6 \times 0.708)^{0.25} = 6.24\ \text{W/m}^2\text{°C}$$

(*i*) **Heat loss from 6m length of pipe:**

Heat lost by *convection,*

$$Q_{conv.} = \bar{h}\ A_s\ \Delta t = 6.24 \times (\pi \times 0.125 \times 6) \times (80 - 20) = 882.16\ \text{W}$$

Heat lost by *radiation,*

$$Q_{rad.} = \varepsilon\ \sigma A\ (T_1^4 - T_2^4)$$
$$= 0.9 \times (5.67 \times 10^{-8}) \times (\pi \times 0.125 \times 6)\ [(80 + 273)^4 - (20 + 273)^4]$$
$$= 980.81\ \text{W}$$

Total heat loss, $Q_t = Q_{conv.} + Q_{rad.} = 882.16 + 980.81 =$ **1862.97 W (Ans.)**

(*ii*) **Heat transfer coefficients, overall (h_t) and due to radiation alone ($h_{rad.}$) :**

$$Q_t = h_t\ .\ A\ \Delta t$$

or, $$h_t = \frac{Q_1}{A.\Delta t} = \frac{1862.97}{(\pi \times 0.125 \times 6) \times (80 - 20)} = 13.17\ \text{W/m}^2\text{°C}$$

∴ $h_{rad.} = h_t - h_{conv.} = 13.17 - 6.24 =$ **6.93 W/m²°C (Ans.)**

Example 8.17. *Calculate the heat transfer from a 60W incandescent bulb at 115°C to ambient air at 25°C. Assume the bulb as a sphere of 50 mm diameter. Also, find the percentage of power lost by free convection.*

The correlation is given by: Nu = 0.60 (Gr.Pr)$^{1/4}$

Solution. *Given* : $t_s = 110°C$, $t_\infty = 25°C$, $D = 50$ mm $= 0.05$ m.

The film temperature, $t_f = \dfrac{t_s + t_\infty}{2} = \dfrac{115 + 25}{2} = 70°C$

The thermo-physical properties of air at 70°C are:

$k = 2.964 \times 10^{-2}$ W/m°C, $\nu = 20.02 \times 10^{-6}$, $Pr = 0.694$,

$$\beta = \frac{1}{t_f + 273} = \frac{1}{70 + 273} = 2.915 \times 10^{-3}$$

$$Gr = \frac{D^2 g \beta \Delta t}{\nu^2} = \frac{(0.05)^3 \times 9.81 \times 2.915 \times 10^{-3} \times (115 - 25)}{(20.20 \times 10^{-6})^2}$$

($\because$ The characteristic length $L = D$ in this case)

Using given correlation, we get

$$\overline{Nu} = \frac{\bar{h} L}{k} = 0.6\ (Gr.Pr)^{1/4}$$

$$h = \frac{k}{D} \times 0.6\ (Gr.Pr)^{1/4}$$

$$= \frac{2.964 \times 10^{-2}}{0.05} \times 0.6\ (8.026 \times 10^5 \times 0.694)^{1/4} = 9.72\ \text{W/m}^2\text{°C}$$

Heat transfer $= \bar{h}\ A_s\ \Delta t = 9.72 \times (\pi \times 0.05^2) \times (115 - 25) =$ **6.87 W** **(Ans.)**

%age of power lost by free convection $= \frac{6.87}{60} \times 100 =$ **11.45 % (Ans.)**

Example 8.18. *A horizontal high pressure steam pipe of 10 cm outside diameter passes through a large room whose walls and air are at 23°C. The pipe outside surface temperature is 165°C and it emissivity is 0.85. Estimate the heat loss from the pipe per unit length. Use the following correlation for the calculation of film coefficient*

$$\overline{Nu} = \left[0.6 + \frac{0.387\,(Ra)^{1/6}}{\left\{1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}} \right]^2$$

where Ra is known as Rayleigh Number and is given by

$$Ra = Gr.Pr = \frac{\beta g\ (\Delta T)\ (L_c)^3}{\nu\alpha}$$

Take the following properties of air :

$k = 0.0313$ *W/m K;* $\nu = 22.8 \times 10^{-6}$ *m²/s*

$\alpha = 32.8 \times 10^{-6}$ *m²/s,* $Pr = 0.697$, $\beta = 2.725 \times 10^{-3}\ K^{-1}$.

(N.M.U. Winter, 1999)

Solution. *Given :* $D = 10$ cm $= 0.1$ m; $t_a = 23$°C; $t_s = 165$°C; $\varepsilon = 0.85$

Properties of air : $k = 0.0313$ W/m K; $\nu = 22.8 \times 10^{-6}$ m²/s;

$\alpha = 32.8 \times 10^{-6}$ m²/s, $Pr = 0.697$, $\beta = 2.725 \times 10^{-3}$ K^{-1}

Heat loss from the pipe per unit length, Q :

Heat lost by convection and radiation from the sphere is given by :

$$Q = Q_{conv.} + Q_{rad.}$$

$$Q_{rad.} = \varepsilon A \sigma\ (T_s^4 - T_a^4) = \varepsilon\ (\pi D L)\ \sigma\ (T_s^4 - T_a^4)$$

$$= 0.85\ (\pi \times 0.1 \times 1) \times 5.67 \left[\left(\frac{165 + 273}{100}\right)^4 - \left(\frac{23 + 273}{100}\right)^4 \right]$$

...(where $L = 1$ m)

$$= 1.514\ [368.04 + 76.76] = 441\ \text{W/m}$$

$$Q_{conv.} = h \times (\pi D L)\ (t_s - t_a)$$

where h is calculated by using the given relation.

$$Ra = Gr.Pr = \frac{\beta g\ (\Delta t)\ (L_c)^3}{\nu\alpha} = \frac{2.725 \times 10^{-3} \times 9.81 \times (165 - 23) \times (0.1)^3}{22.8 \times 10^{-6} \times 32.8 \times 10^{-6}} = 5 \times 10^6$$

$$Nu = \frac{hD}{k} = \left[0.6 + \frac{0.387\ (5 \times 10^6)^{1/6}}{\left\{1 + \left(\frac{0.559}{0.698}\right)^{9/16}\right\}^{8/27}} \right]^2 = \left(0.6 + \frac{5.06}{1.206}\right)^2 = 23$$

$\therefore$ $$h = \frac{k}{D} \times 23 = \frac{0.0313}{0.1} \times 23 = 7.2\ \text{W/m}^2\text{°C}$$

$\therefore$ $Q_{conv.} = 7.2 \times (\pi \times 0.1 \times 1)\ (165 - 23) = 321.2$ W, when $L = 1$ m

Hence, $Q = 441 + 321.2$

$=$ **762.2 W/m** **(Ans.)**