Revision Notes

Heat Transfer (HT)

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Books To Be Referred :

[1] Heat Transfer by P.K. Nag, TMH

[2] Heat and Mass Transfer by Dr D.S. Kumar - S.K.Kataria and Sons publication

[3] Heat Transfer R.K.Rajput, Laxmi Publications

[4] Heat and Mass Transfer: A Practical Approach, Y.A.Cengel, Tata Mcgraw Hill Education Private Limited

[5] Heat Transfer : J.P.Holman, TMH Publications

[6] Fundamentals of Engineering Heat and Mass Transfer: R.C.Sachdeva, New Age International Publishers, 4th Edition

MODULE - I : Conduction Heat Transfer

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CHAPTER - 1

Introduction To Heat Transfer

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Heat Transfer :

• Heat transfer refers to the transmission of energy from one region to another in a sane medium or between two medium , as a result of temperature gradient.

• The temperature difference between two points in the same medium, or between two mediums which are in thermal contact represents the driving potential as applied to heat transfer problems.

THERMODYNAMICS VERSUS HEAT TRANSFER

• Thermodynamics is concerned with the **equilibrium states of matter**, and precludes the existence of a temperature gradient. For heat transfer or exchange, temperature gradient must exist and as such heat transfer is inherently a **non-equilibrium process** .

• Thermodynamics helps to determine the quantity of work and heat interactions when a system changes from one equilibrium state to another. The analysis, however, does not provide any information on the nature of interactions and the time rate at which interactions occur. It simply describes how much heat is to be exchanged during a process without caring to explain how that could be achieved.But Analysis of Heat Transfer overcomes these limitations of Thermodynamics .

• Heat transfer helps to predict he temperature distribution, which may be the function of both space coordinates and time within regions of matter.

Heat transfer also helps to determine the rate at which energy is transferred across a surface of a specimen or body due to temperature gradients at the surface, and temperature difference between the different surfaces.

For Example : The difference between thermodynamics and heat transfer can be well appreciated by considering the cooling of a hot steel bar which is placed in a water bath.

Thermodynamic analysis would help to predict the final equilibrium temperature of the composite system.comprised by steel bar-water combination Analysis, herenver fails to predict how long it will take to reach the equilibrium condition or what would be the temperature of the bar after a certain length of time before the equilibrium condition is attained.

Heat transfer analysis does help to predict the temperature of both the bar and theatre as a function of time. That is, the temperature at all paints of interest within the bar or temperature at any specific point such as at the centre of the bar where it is the highest) at any time can be predicted. Also, the instantaneous heat transfer rate can be predicted from all or from any part of the surface of the bar at any time.

Other Importance Of Study Of Heat Transfer :

The design of the heat exchange equip ment such as boilers, heaters, refrigerators, and heat exchangers depends not only on the amount of heat to be transmitted but rather on the rate at which heat is to be transferred under given conditions. The discipline of heat transfer seeks to quantify the rate at which heat transfer occurs in term of the degree of non-equilibrium. This is accomplished through the rate equations for the different modes of heat transfer(Conduction , Convection and Radiation) .

Introduction To Heat Transfer

Heat Transfer Mechanisms Or Modes Of Heat Transfer

Heat as the form of energy, can be transferred from one system to another as a result of temperature difference. Heat can be transferred in three different modes:

1. Conduction
2. Convection
3. Radiation

→ All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.

Conduction :- Thermal conduction is a mechanism of heat propagation from a region of higher temperature to a region of low temperature within a medium (Solid, Liquid or Gaseous) or betⁿ different mediums in direct physical contact.

→ Conduction can take place in Solids, Liquids or Gases.

→ Conduction does not involve any movement of macroscopic portions of matter relative to one another.

Mechanisms Of Thermal Conduction :

In Solids
- (i) Migration of free electrons/valence electrons
- (ii) Lattice Vibrations

In Liquids & Gases
- (iii) Collision of molecules
- (iv) Diffusion of molecules

⇒ In solids, the thermal conduction heat transfer is due to the combination of vibrations of the molecules in a lattice (i.e., lattice vibrations) and the energy transport by free electrons (i.e., Migration of free electrons)

⇒ In Gases and Liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.

* Since conduction is essentially due to random molecular motion, the concept is termed as Microform of heat transfer and is usually referred to as diffusion of energy.

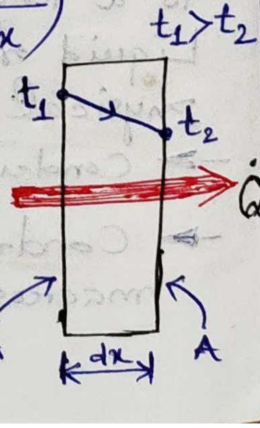
* The rate equation for one-dimensional steady flow of heat by conduction is described/governed by the Fourier's law.

Fourier's Law Of Heat Conduction : It states that the rate of heat conduction through a plane layer is proportional to the temperature difference (Δt or dt) across the surfaces of the layer and the heat transfer area (A) normal to the dirn. of heat flow and is inversely proportional to the thickness of the layer (Δx or dx)

Mathematically, $\dot{Q} \propto \frac{A \cdot dt}{dx}$

$$\Rightarrow \quad \dot{Q} = -KA\frac{dt}{dx} = -KA\left(\frac{t_2 - t_1}{dx}\right)$$

$$\Rightarrow \quad \dot{Q} = KA\left(\frac{t_1 - t_2}{dx}\right) \text{ watt.}$$



where 'K' is the proportionality constant and is known as 'thermal conductivity of material', which is a measure of the ability of a material to conduct heat.

Unit of 'K' : $W/m\cdot°C$ or $W/m\cdot°K$

Dimension of K: $\frac{W}{m\cdot °C} = \frac{N\cdot m}{s} \times \frac{1}{m\cdot °C} = \frac{[MLT^{-2}][L]}{[T] \times [L][\theta]}$

$= [MLT^{-2}\theta^{-1}]$

A material is considered to be comprised of
(a) Free electrons, and (b) atoms which are bound in a periodic arrangement called lattice.

Accordingly, thermal conductivity (K) of a material is the outcome of migration of free electrons and lattice vibrational waves.

In metals, the moleculars are closely packed; molecular activity is rather small and so thermal conductivity is substantially due to the flow of free electrons.

In fluids, the free electron movement is negligibly small and therefore, the thermal conductivity (K) results primarily from the frequency of interactions betn. the lattice atoms.

KEY POINTS OF 'K'

- $K_{solid} > K_{Liquid} > K_{Gas}$ & $K_{purest\ form\ of\ metal} > K_{alloy}$
- A high value of th. conductivity (K) indicates that the material is good conductor of heat and a low value of K indicates that material is a poor heat conductor or Insulator

$K_{Diamond} > K_{Silver} > K_{Copper} > K_{Gold} > K_{Al} > K_{Iron}$

$K_{Mercury} > K_{liquid} > K_{insulating\ material} > K_{air}$

→ Copper and Silver : Good electric conductor as well as good heat conductor.

→ Mica, Diamond : Good conductor of heat but an electric insulator
[no free electron but conducts heat by vibrations in atoms]
But Graphite can conduct electricity because of the free electrons in its structure. So Graphite is a good conductor of electricity and a bad conductor of heat.

→ Vanadium dioxide and some gases and some semiconductors : are good conductor of electricity but not of heat.

- K of solids and liquids (→ Water is being exceptation) decreases with increase in temperature but K of gases increases with increase in temperature.
- K of gases is independent of pressure and K of Liquids is also insensitive to pressure except at the thermodynamic critical point.
- Heat transfer by conduction in gases occurs through transport of the K.E. of molecular motion resulting from the random movement and collisions of the molecules
- Gases with higher molecular weight have small value of thermal conductivity (K) than those with lower mol. weight.

$K_{H_2} = 0.190\ W/m\text{-}^\circ C$ (Mol. Wt. of $H_2 = 2$) $K_{O_2} = 0.0272\ W/m\text{-}^\circ C$ (Mol. wt. of $O_2 = 32$)

- The dependance of K for most materials on temp., is almost linear. $K = K_0(1 + \beta t)$
- Thermal conductivity of damp material is considerably higher than the thermal conductivity of the dry material.
- Thermal conductivity increases with density growth.

$K_{ice\ or\ snow} > K_{liquid}$

- Materials having a crystalline structure have a high value of K than the substances in amorphous form.
- Thermal conductivity of most types of wood is large in the dirⁿ parallel to the grain compared to that in a dirⁿ across the grain.

JMP

Q. Thermal conductivity of water

a) First increases with temp. then decreases with temperature.
b) Increases steadily with temperature
c) Decreases with temperature
d) Does not depend on temperature

A

Ans: (a) Water and Mercury are being exception and they do not follow general trend for liquids (i.e, $K_{Liquid} \propto \frac{1}{T}$). Thermal-conductivity, K of water and Mercury first increases and then decreases

Example 1.1. *Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C and the other face at 50°C. Take thermal conductivity of copper as 370 W/m°C.*

Solution. Temperature difference, $dt \,(= t_2 - t_1) = (50 - 350)$

Thickness of copper plate, $L = 45$ mm $= 0.045$ m

Thermal conductivity of copper, $k = 370$ W/m°C

Rate of heat-transfer per unit area, q :

From Fourier's law

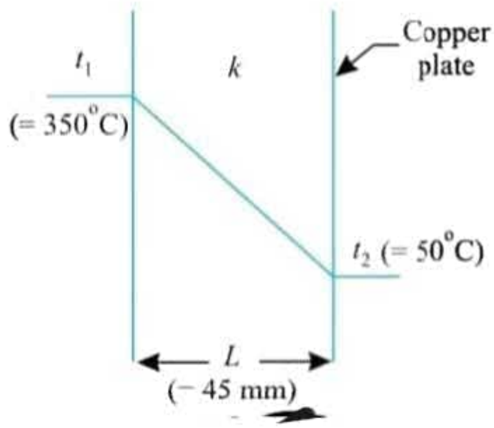
$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L} \quad \text{...(Eqn. 1.1)}$$

or,
$$q = \frac{Q}{A} = -k\frac{dt}{dx}$$

$$= -370 \times \frac{(50 - 350)}{0.045}$$

$$= 2.466 \times 10^6 \text{ W/m}^2 \text{ or}$$

2.466 MW/m² (Ans.)



Example 1.2. *A plane wall is 150 mm thick and its wall area is 4.5 m². If its conductivity is 9.35 W/m°C and surface temperatures are steady at 150°C and 45°C, determine :*

(*i*) *Heat flow across the plane wall;*

(*ii*) *Temperature gradient in the flow direction.*

Solution. Thickness of the plane wall,

$$L = 150 \text{ mm} = 0.15 \text{ m}$$

Area of the wall, $A = 4.5$ m²

Temperature difference, $dt = t_2 - t_1 = 45 - 150 = -105$°C

Thermal conductivity of wall material,

$$k = 9.35 \text{ W/m°C}$$

(*i*) **Heat flow across the plane wall, Q :**

As per Fourier's law,

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15} = \mathbf{29452.5\ W}$$

(*ii*) **Temperature gradient,** $\frac{dt}{dx}$ **:**

From Fourier's law, we have

$$\frac{dt}{dx} = -\frac{Q}{kA} = \frac{29452.5}{9.35 \times 4.5} = \mathbf{-700°C/m}$$

CONVECTION :-

Thermal convection is the mode of heat transfer bet? a solid surface and the adjacent liquid or gas that is in motion or by the circulation/mixing of fluid medium (gas, liquid or a powdery substance)

- The faster the motion/circulation/mixing of fluid, the greater the convection heat transfer.
- Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.
- Convection constitutes the macroform of the heat transfer.
- Heat conduction in the presence of fluid motion → Convection.

There are TWO types of convections are distinguished:

(i) Forced Convection

(ii) Free or Natural Convection

→ Convection is called forced convection, if the fluid is forced to flow over the surface by external means such as a fan, pump, blower or the wind.

→ Convection is called natural (or free) convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of the temp. in the fluid. [as the warmer air adjacent to the hot body being lighter goes up and thus fall of the cooler air (as being heavier) to fill its place.]

* Convection heat transfer is also seen in phase change processes such as Boiling & Condensation, as these processes involve change of phase of a fluid accompanied by the fluid motion induced during the process, such as rise of vapour bubbles during boiling or the fall of the liquid droplets during condensation.

- Convection heat transfer is governed by Newton's law of cooling, i.e.,

$$\dot{Q} = hA_s(t_s - t_f) \text{ watt.}$$

where A_s = surface area through which convection heat transfer takes place, m^2

t_s = surface temperature, °c

t_f = temp. of the fluid sufficiently far from the surface, °c

h = convective heat transfer co.efficient or heat transfer coeff.

Unit of 'h' : $W/m^2\text{-}°C$ or $W/m^2\text{-}°K$

Dimension of 'h' : $[MT^{-2}\Theta^{-1}]$

KEY POINT ABOUT 'h' : The convection heat transfer-coefficient, h is not a property of the fluid.

- It is an experimentally determined parameter.
- Its value depends on all the variables influencing convection such as surface geometry, the nature of fluid motion, the properties of the fluid and the bulk fluid velocity

Types of Convection	h, $W/m^2.K$
Free convection of gases =	2 - 25
Free convection of liquids =	10 - 1000
forced convection of gases =	25 - 250
Forced convection of liquids =	50 - 20,000
Boiling and Condensation =	2500 - 100,000 (very large)

Heat Transfer Coefficient (h)

The units of *h* are,

$$h = \frac{Q}{A\,(t_s - t_f)} = \frac{W}{m^2\,°C} \quad \text{or} \quad W/m^2\,°C$$

or, W/m^2K

The coefficient of convective heat transfer '*h*' (also known as *film heat transfer coefficient*) may be defined as "*the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time.*"

The value of '*h*' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.* viscosity, density, specific heat etc.).

(*ii*) Nature of fluid flow.

(*iii*) Geometry of the surface.

(*iv*) Prevailing thermal conditions.

Since '*h*' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however, by dimensional analysis an equation for the purpose can be obtained.

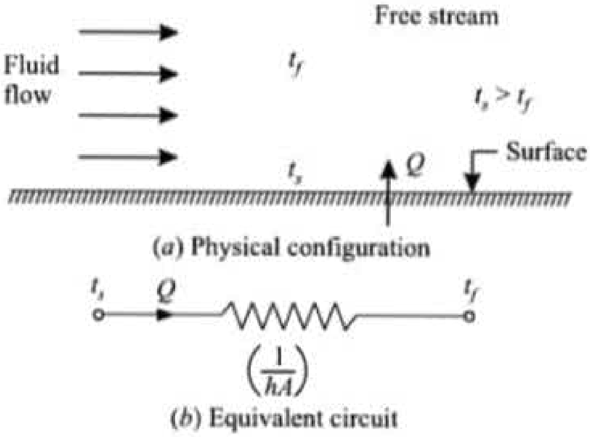


Fig. 1.9. Convective heat-transfer

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation*. Refer Fig. 1.9 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \text{...Eqn (1.6)}\right]$ is called **convection thermal resistance** $[(R_{th})_{conv}]$ to heat flow.

Example 1.4. *A hot plate 1m × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5 \text{ m}^2$

Plate surface temperature, t_s = 300°C

Temperature of air (fluid), t_f = 20°C

Connvective heat-transfer coefficient, h = 20 W/m² °C

Rate of heat transfer, Q :

From Newton's law of cooling,

$$Q = hA\,(t_s - t_f)$$

$$= 20 \times 1.5\,(300 - 20) = 8400 \text{ W or } \mathbf{8.4\ kW}$$

Example 1.5. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, d = 1.5 mm = 0.0015 m

Length of the wire, L = 150 mm = 0.15 m

∴ Surface area of the wire (exposed to heat transfer),

$$A = \pi\, d\, L = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4} \text{ m}^2$$

Wire surface temperature, t_s = 120°C

Water temperature, t_f = 100°C

Convective heat transfer coefficient, h = 4500 W/m² °C

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

∴ $Q = hA\,(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\,(120 - 100) = \mathbf{63.6\ W}$

RADIATION: Thermal radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a results of the changes in the electronic configurations of the atoms or molecules.

- Unlike conduction and convection, the heat transfer by radiation does not necessarily require the presence of intervening medium.
- Thermal radiation heat exchange occurs most effectively in vacuum.
- Heat transfer by radiation is fastest among all three modes of heat transfer.
- The radiation heat exchange rate is based on "Stefan-Boltzman Law".

Energy radiated per unit time, $E_b = \sigma \cdot A \cdot T^4$ ← Ideal emitter (eg-Black body)

or $E = \varepsilon \sigma A T^4$ ← Real surface

Laws of Radiation :

1. **Wien's law.** It states that *the wavelength* λ_m *corresponding to the maximum energy is inversely proportional to the absolute temperature T of the hot body.*

$$\text{i.e.,} \quad \lambda_m \propto \frac{1}{T} \quad \text{or,} \quad \lambda_m T = \text{constant} \qquad ...(1.7)$$

2. **Kirchhoff's law.** It states that the *emissivity of the body at a particular temperature is numerically equal to its absorptivity for radiant energy from body at the same temperature.*

3. **The Stefan-Boltzmann law.** The law states that *the emissive power of a black body is directly proportional to fourth power of its absolute temperature.*

$$\text{i.e.,} \quad Q \propto T^4 \qquad ...(1.8)$$

Refer Fig. 1.10 (*a*)

$$Q = F \sigma A (T_1^4 - T_2^4) \qquad ...(1.9)$$

where, F = A factor depending on geometry and surface properties,

σ = Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ W/m^{2}K^4,

A = Area, m^2, and

T_1, T_2 = Temperatures, degrees kelvin (K).

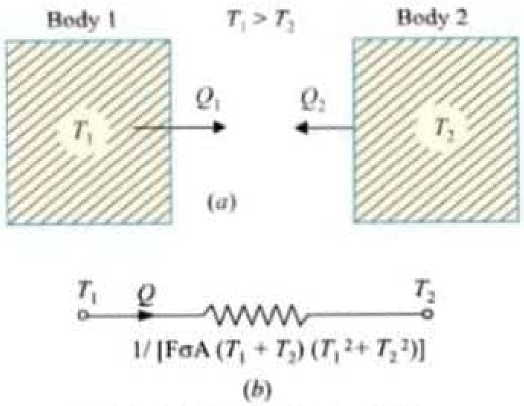


Fig. 1.10. Heat transfer by radiation.

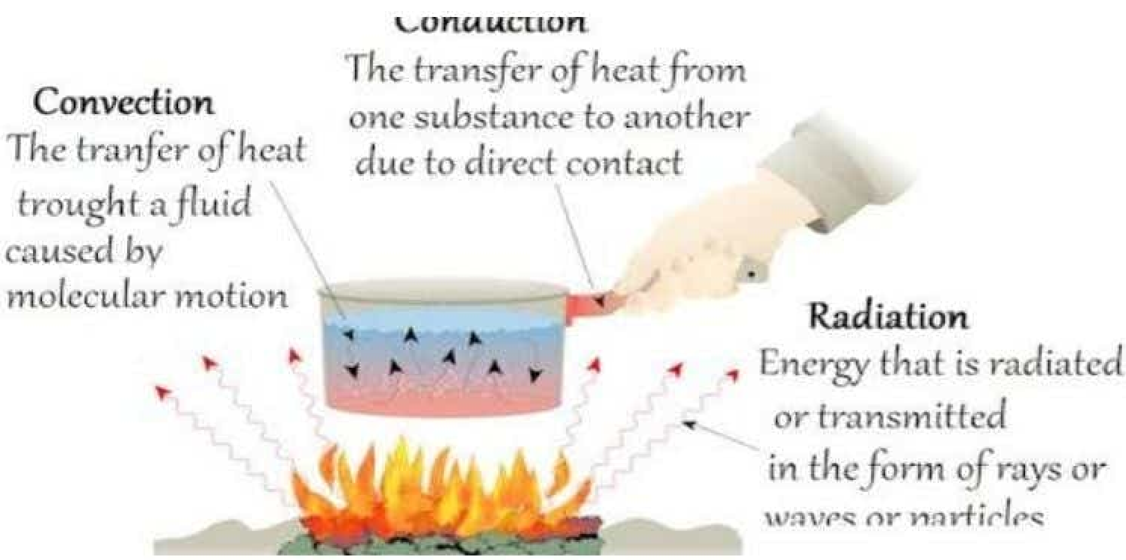
Examples where three modes of heat transfer(Conduction , Convection and Radiation)occur :

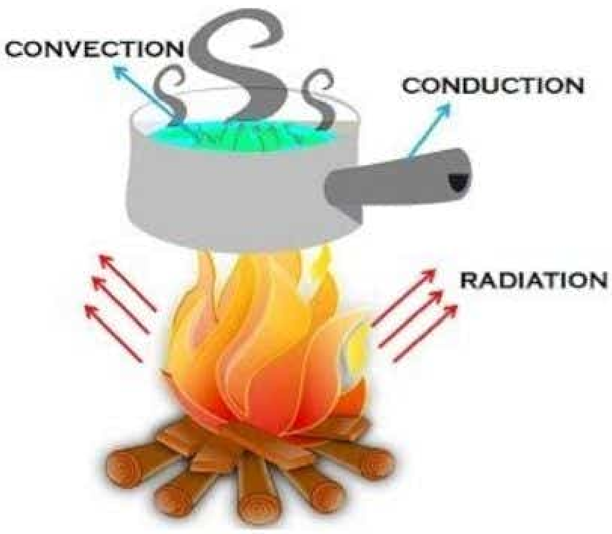
Example-1 :

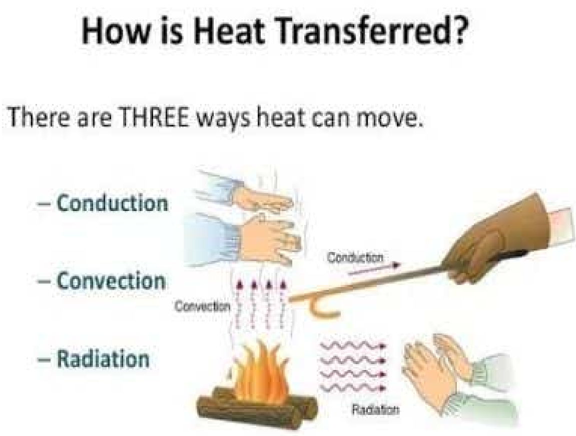
A good example would be heating a tin can of water using a Bunsen burner. Initially the flame produces radiation which heats the tin can. The tin can then transfers heat to the water through conduction. The hot water then rises to the top, in the convection process.

Example -2 :

The atmosphere would be another example. The atmosphere is heated by radiation from the Sun, the atmosphere exhibits convection as hot air near the equator rises producing winds, and finally there is conduction between air molecules, and small amounts of air-land conduction.







Heat Transfer within Boiler

- A steam boiler is designed to absorb the maximum amount of heat released from the process of combustion.
- Heat transfer within steam boiler is accomplished by three methods: **radiation, convection, and conduction**.
- The relative percentage of each heat transfer within steam boiler is dependent on the type of steam boiler, the designed transfer surface, and fuels.

CHAPTER - 2

Steady State Heat Conduction

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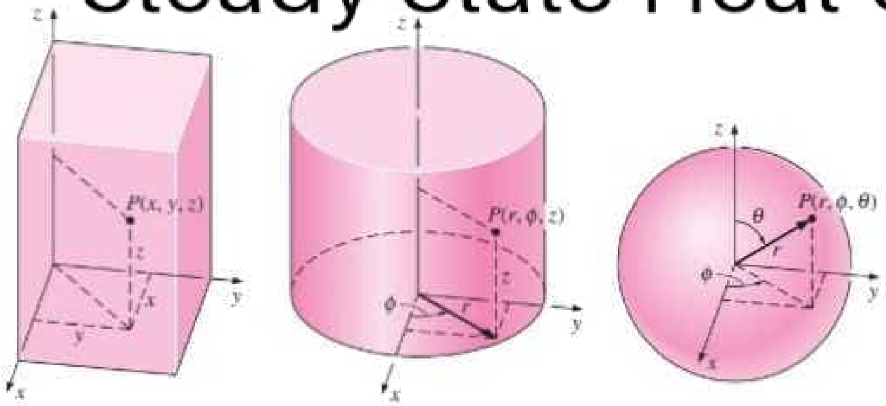
Chapter-2
Steady State Heat Conduction



(a) Rectangular coordinates (b) Cylindrical coordinates (c) Spherical coordinates

FIGURE 2–3
The various distances and angles involved when describing the location of a point in different coordinate systems.

notation $T(x)$, on the other hand, indicates that the temperature varies in the x-direction only and there is no variation with the other two space coordinates or time.

Steady versus Transient Heat Transfer

Heat transfer problems are often classified as being **steady** (also called *steady-state*) or **transient** (also called *unsteady*). The term *steady* implies *no change* with time at any point within the medium, while *transient* implies *variation with time* or *time dependence.* Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location, although both quantities may vary from one location to another (Fig. 2–4). For example, heat transfer through the walls of a house will be steady when the conditions inside the house and the outdoors remain constant for several hours. But even in this case, the temperatures on the inner and outer surfaces of the wall will be different unless the temperatures inside and outside the house are the same. The cooling of an apple in a refrigerator, on the other hand, is a transient heat transfer process since the temperature at any fixed point within the apple will change with time during cooling. During transient heat transfer, the temperature normally varies with time as well as position. In the special case of variation with time but not with position, the temperature of the medium changes *uniformly* with time. Such heat transfer systems are called **lumped systems.** A small metal object such as a thermocouple junction or a thin copper wire, for example, can be analyzed as a lumped system during a heating or cooling process.

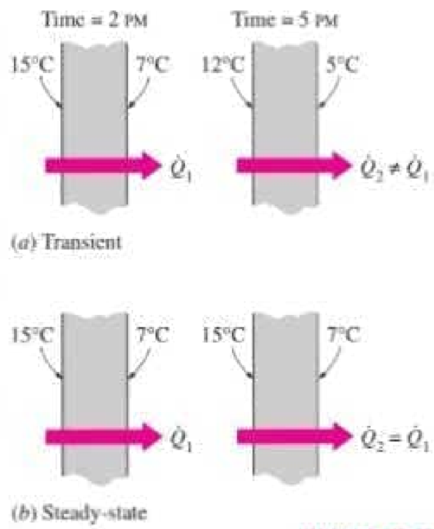


FIGURE 2–4
Steady and transient heat conduction in a plane wall.

STEADY STATE HEAT CONDUCTION

⇒ Steady state implies that the temperature(t) at each point of the system remains constant in the course of time(τ) and it is a function of only space coordinates (x, y, z):

$$t = f(x, y, z) \; ; \; \frac{dt}{d\tau} = 0$$

Steady state results in a constant rate of heat exchange (heat influx equals heat efflux) and there is no change in internal energy of the system during such a process.

⇒ Unsteady state results in heat transfer rate which changes with time. Under unsteady thermal conditions, temp. of the system changes continuously with time.
Further, a change in temperature indicates a change of internal energy of the system. Energy storage is thus a part and parcel of unsteady heat flow.

$$t = f(x, y, z, \tau) \; ; \; \frac{dt}{d\tau} \neq 0$$

Fourier's Law Of Heat Conduction :

$$\dot{Q} = -KA \cdot \frac{dt}{dx} = KA \frac{t_1 - t_2}{dx}$$

dx = thickness of material along the path of heat flow, m
A = area ⊥ar to the dirⁿ of heat flow, m^2.

Heat flux $= q = \frac{\dot{Q}}{A}$, W/m^2

Temperature gradient $= \frac{dt}{dx}$, °c/m ⇒ change in temp. per unit thickness

Assumptions In Fourier's Law of Conduction :

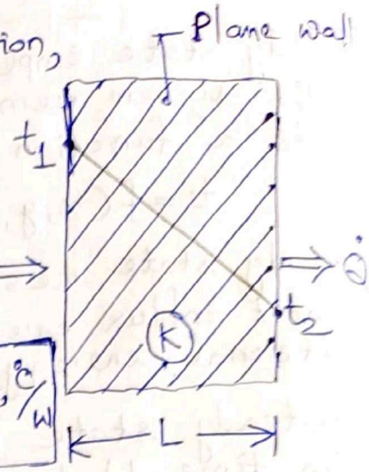
- Steady state heat conduction.
- One-directional heat flow
- Homogeneous and isotropic material
  ['K' has a constant value in all the directions]
- Bounding surfaces are Isothermal in character.
  [Constant and Uniform temp.s are maintained at the two faces]
- Constant temp. gradient and a linear temp. profile.
- No internal heat generation.
- Fourier law is valid for all matter regardless of its state Solid, Liquid or Gas.
- Fourier law is a vector expression indicating that heat flow rate is normal to an isotherm and is in the dirⁿ of decreasing temperature.
- Fourier law can not be derived from first principles; it is a generalization based on experimental evidence.

Thermal Resistance (R_{th}) :

By Fourier's law of heat conduction,

$$\dot{Q} = KA \cdot \frac{t_1 - t_2}{L}$$

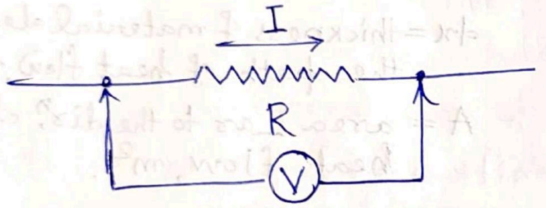
$$\Rightarrow \dot{Q} = \frac{t_1 - t_2}{\left(\frac{L}{KA}\right)} = \frac{t_1 - t_2}{R_{th}}$$

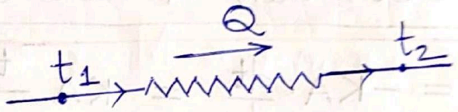


where, $R_{th} = \frac{L}{KA}$ = thermal resistance, °C/W

* The reciprocal of thermal resistance (R_{th}) is called thermal conductance and it represents the amount of heat conducted through a solid wall of area (A) and thickness (L) when a temperature difference of unit degree (1°) is maintained across the bounding surfaces.

$$\text{Thermal conductance} = \frac{1}{R_{th}} = \frac{KA}{L}, \; W/°C$$





$$R_{th} = \frac{dx}{KA} \text{ or } \frac{L}{KA}$$

Analogy betn Flow of Electricity (I) and Heat ($\dot{Q}$)

By Ohm's law of electricity,

$$\text{Current (I)} = \frac{\text{Potential difference (dV)}}{\text{Electrical resistance (R)}}$$

By Fourier's law of conduction,

$$\text{Rate of heat flow } (\dot{Q}) = \frac{\text{temperature potential (dt)}}{\text{thermal resistance } (R_{th})}$$

Hence,

→ I is analogous to $\dot{Q}$

→ dV is analogous to dt

→ R is analogous to R_{th}

* Unit conductance ~ heat transfer co-efficient (h)

* Capacitance ~ heat capacity (ρ.C)

Lorenz Number (L_0): The ratio of the thermal conductivity and electrical conductivity (σ) is same for all metals at the same temperature; and that the ratio is directly proportional to the absolute temperature of the metal (T)

Mathematically, $\frac{K}{\sigma} \propto T$

$\Rightarrow \frac{K}{\sigma} = L_0 T$

$\Rightarrow \boxed{\frac{K}{\sigma T} = L_0}$ where 'L_0' is a proportionality constant and is known as 'Lorenz number'.

Value of L_0: $L_0 = 2.45 \times 10^{-8} \frac{W\text{-}\Omega}{K^2}$

** This law holds good for a large number of metals betn. $-100°C$ to $100°C$. At low temperatures the ratio $\left(\frac{K}{\sigma}\right)$ decreases and the value tends to be zero at absolute zero.

** With decrease in temp, the K & σ of metals increases, but increase in 'σ' is higher and its value tends to infinity at abs. zero.

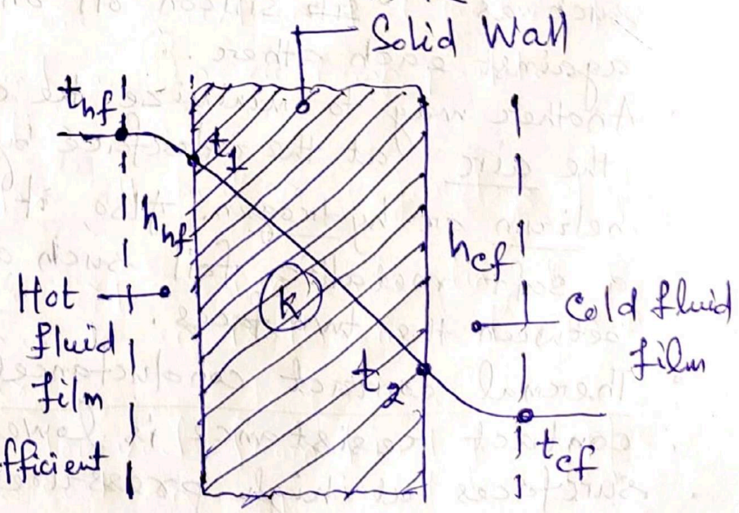
Overall heat transfer co-efficient (U)

→ It represents the intensity of heat transfer from one fluid to another through a wall separating them.

→ Numerically, it equals the quantity of heat passing through unit area of wall surface in unit time at a temp. diff. of unit degree.

→ Unit of 'U': $W/m^2\text{-}°C$ or $\frac{W}{m^2\text{-}K}$

$$Q = \frac{(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} A} + \frac{L}{KA} + \frac{1}{h_{cf} A}\right]}$$



$$\Rightarrow Q = UA(t_{hf} - t_{cf})$$

where $U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{cf}}}$

→ The overall heat transfer co-efficient depends upon the geometry of separating wall, its thermal properties, and the convective co-efficient at the two surfaces.

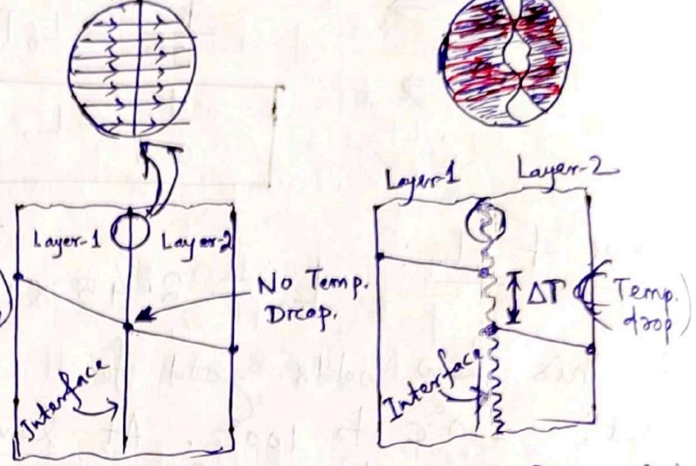
Thermal Contact Resistance

When two surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air in most cases. As a result, an interface contains numerous air gaps of varying sizes that acts as insulation because of the low 'K' value of air. Thus, an interface offers some resistance to heat transfer, and this resistance for a unit interface area is called the 'thermal contact resistance (R_c)'.

* R_c represents thermal contact resistance for a unit area.

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{(Q/A)} \left(m^2 \cdot K/W\right)$$

$$= \frac{\text{Temp. drop}}{\text{Heat flux}}$$



(a) Ideal (perfect) thermal contact

(b) Actual (Imperfect) thermal contact

- The value of thermal contact resistance (R_c) depends on the surface roughness, material properties, temperature and pressure at the interface and the type of fluid trapped at the interface.
- Thermal contact resistance decreases with decreasing surface roughness and increasing interface pressure.
- The thermal contact resistance can be minimized by applying a thermally conducting liquid called a thermal grease such as ~~sti~~ silicon oil on the surfaces before they are pressed against each other.
- Another way to minimize the contact resistance is, replacing the air at the interface by a better conducting gas, such as helium or hydrogen. Also, it can be decreased by inserting a soft metallic foil such as Tin, Silver, Copper, Nickel, Al between the two faces.
- Thermal contact conductance (h_c) is highest (and thus, the contact resistance is lowest) for soft metals with smooth surfaces at high pressures.

Problem-1 : The thermal conductance at the interface of two 1 cm thick aluminium plates is measured to be 11,000 $W/m^2 \cdot K$. Determine the thickness of the aluminium plate whose thermal resistance is equal to the thermal resistance of the interface between the plates. ($K_{AL} = 237 \, W/m\text{-}K$)

Solution : Given that

Thermal conductance at the interface, $h_c = 11000 \, W/m^2 \cdot K$

$\therefore$ Thermal contact resistance at the interface, $R_c = \frac{1}{h_c}$

$$\Rightarrow R_c = \frac{1}{11000} = 0.909 \times 10^{-4} \, m^2 \cdot K/W$$

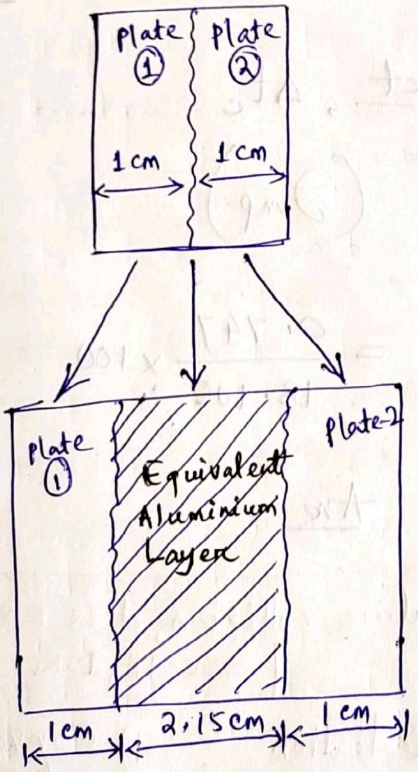
For a unit surface area ($A = 1 \, m^2$), the thermal resistance of a flat plate (Aluminium) is given by, $R = \frac{L}{K}$

Setting, $R = R_c$

$$\Rightarrow \frac{L}{K} = 0.909 \times 10^{-4}$$

$$\Rightarrow L = 237 \times 0.909 \times 10^{-4}$$

$$= 0.0215 \, m \text{ or } 2.15 \, cm \quad \underline{\text{Ans}}$$



NOTE :- The interface betn the two plates offers as much resistance to heat transfer as a 2.15 cm-thick aluminium plate.

But the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

Imp **Problem-2**: Two 3.0 cm diameter stainless steel bars (with $\frac{1}{h_c} = 5.28 \times 10^{-4} \frac{m^2 °C}{W}$), 10 cm long, have ground surfaces and are exposed to air with a surface roughness of about 1 μm. If the surfaces are pressed together with a pressure of 50 atm. and the two-bar combination is exposed to an overall temp. difference of 100°C, Calculate the axial heat flow and temperature drop across the contact surface. $K_{steel} = 16.3$ W/m°C.

Solution:- For each stainless steel bar,

$$R_{th} = \frac{L}{KA} = \frac{0.10}{16.3 \times \frac{\pi (0.03)^2}{4}} = 8.679 °C/W$$

$$\text{Contact resistance, } R_c = \frac{1}{h_c A} = 5.28 \times 10^{-4} \times \frac{1}{\frac{\pi (0.03)^2}{4}} = 0.747$$

$$\text{Total thermal resistance, } \Sigma R_{th} = 2R_{th} + R_c$$

$$= (2 \times 8.679) + 0.747$$

$$= 18.105 \ °C/W$$

$$\text{Overall heat flow, } Q = \frac{\Delta t}{\Sigma R_{th}} = \frac{100}{18.105} = 5.52 \text{ W}$$

Ans

Temperature drop across the contact, Δt_c

$$\frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t_c}{R_c} \quad \text{(Imp)}$$

$$\Rightarrow \Delta t_c = \frac{R_c}{\Sigma R_{th}} \times \Delta t = \frac{0.747}{18.105} \times 100$$

$$= 4.13 °C$$

Ans

Imp

Q. When heat is transferred by molecular collision, it is known as heat transfer by [SSC-JE, ISRO]

a) Conduction b) Convection c) Radiation d) Convection and Radiation

Ans: (a) In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.

Internal Heat Generation (q_g) :

A medium conducting heat energy may involve the conversion of mechanical, electrical, nuclear or chemical energy into heat energy. For example, when a resistance wire conducts electric current, it converts electrical energy into heat energy at a rate of I^2R, where I is the current and R is the electrical resistance of the wire.

Similarly, heat is generated in an exothermic chemical reaction in a medium. The reaction may also be an endothermic reaction. In such a case, the heat generation term will become a negative quantity.

Likewise, nuclear fission process in a nuclear reactor generates a large amount of heat in fuel elements.

Heat is also generated in a nuclear fusion reaction. For example, hydrogen atoms get fused into helium making the sun a large nuclear reactor.

One thing to note is that heat generation is a volumetric phenomenon. This means that heat generation occurs throughout the body of the medium. For this reason, the rate of heat generation in a medium is usually specified per unit volume, W/m^3.

The rate of heat generation may vary with respect to time as well as position within the medium. When variation with respect to position (x,y,z) is known, we can calculate total rate of heat generation in a medium of volume V by:

$$\dot{E}_{gen} = \int \dot{e}_{gen} dV$$

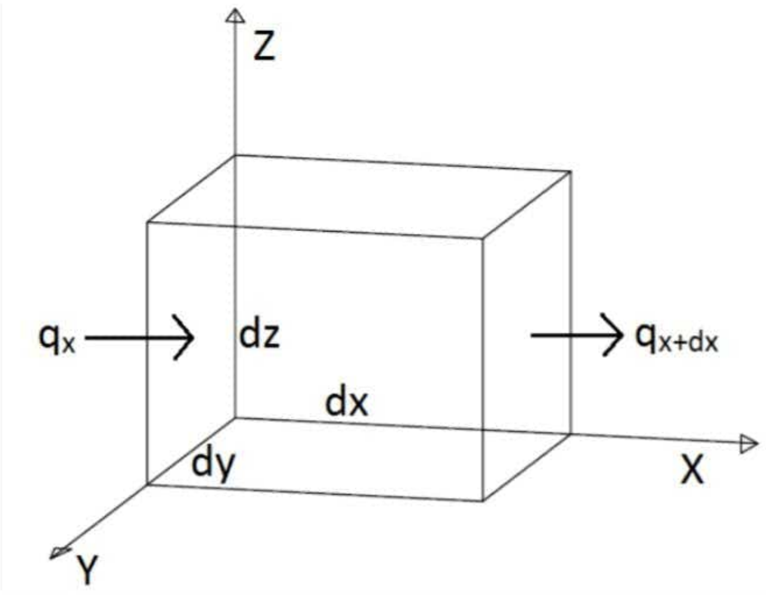
If the rate of heat generation is uniform throughout the medium then the above equation will become:

$$\dot{E}_{gen} = \dot{e}_{gen} * V$$

General heat conduction equation

What is the basic form of heat conduction equation?

The basic form of heat conduction equation is obtained by applying the first law of thermodynamics (principle of conservation of energy).



Consider a differential element in Cartesian coordinates. The energy balance for the differential element can be written as follows:

$$\begin{pmatrix} \text{Rate of} \\ \text{heat conduction} \\ \text{at } x, y, z \end{pmatrix} - \begin{pmatrix} \text{Rate of} \\ \text{heat conduction} \\ \text{at } x + dx, y + dy, z + dz \end{pmatrix} + \begin{pmatrix} \text{Rate of} \\ \text{heat generation} \\ \text{inside the element} \end{pmatrix} = \begin{pmatrix} \text{Rate of change of} \\ \text{energy content} \\ \text{of the element} \end{pmatrix}$$

The first term in the above equation represents the rate of heat energy coming into the element at x, y and z planes. The second term represents the rate of heat energy coming out of the element at x+dx, y+dy and z+dz planes.The third term represents the rate of heat generation inside the element.

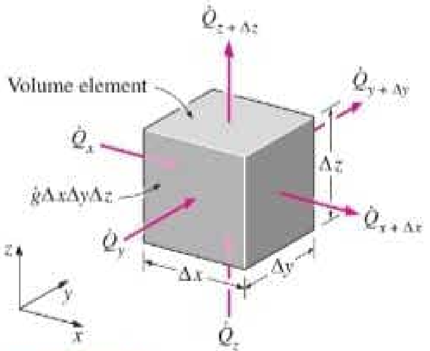


FIGURE 2–21
Three-dimensional heat conduction through a rectangular volume element.

Rectangular Coordinates

Consider a small rectangular element of length Δx, width Δy, and height Δz, as shown in Figure 2–21. Assume the density of the body is ρ and the specific heat is C. An *energy balance* on this element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ x, y, \text{and } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y, \text{and } z+\Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{pmatrix}$$

or

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \quad \textbf{(2-36)}$$

Noting that the volume of the element is $V_{\text{element}} = \Delta x\Delta y\Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C\Delta x\Delta y\Delta z(T_{t+\Delta t} - T_t)$$
$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}\Delta x\Delta y\Delta z$$

Substituting into Eq. 2–36, we get

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g}\Delta x\Delta y\Delta z = \rho C\Delta x\Delta y\Delta z\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x\Delta y\Delta z$ gives

$$-\frac{1}{\Delta y\Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x\Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x\Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C\frac{T_{t+\Delta t} - T_t}{\Delta t} \quad \textbf{(2-37)}$$

Noting that the heat transfer areas of the element for heat conduction in the x, y, and z directions are $A_x = \Delta y \Delta z$, $A_y = \Delta x \Delta z$, and $A_z = \Delta x \Delta y$, respectively, and taking the limit as Δx, Δy, Δz and $\Delta t \to 0$ yields

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \qquad \textbf{(2-38)}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0} \frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z}\frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z}\frac{\partial}{\partial x}\left(-k\Delta y \Delta z\frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

$$\lim_{\Delta y \to 0} \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z}\frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z}\frac{\partial}{\partial y}\left(-k\Delta x \Delta z\frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

$$\lim_{\Delta z \to 0} \frac{1}{\Delta x \Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{\Delta x \Delta y}\frac{\partial Q_z}{\partial z} = \frac{1}{\Delta x \Delta y}\frac{\partial}{\partial z}\left(-k\Delta x \Delta y\frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

Equation 2–38 is the general heat conduction equation in rectangular coordinates. In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \textbf{(2-39)}$$

where the property $\alpha = k/\rho C$ is again the *thermal diffusivity* of the material. Equation 2–39 is known as the **Fourier-Biot equation,** and it reduces to these forms under specified conditions:

(1) *Steady-state:* (called the **Poisson equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0 \qquad \textbf{(2-40)}$$

(2) *Transient, no heat generation:* (called the **diffusion equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \textbf{(2-41)}$$

(3) *Steady-state, no heat generation:* (called the **Laplace equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad \textbf{(2-42)}$$

Note that in the special case of one-dimensional heat transfer in the x-direction, the derivatives with respect to y and z drop out and the equations above reduce to the ones developed in the previous section for a plane wall (Fig. 2–22).

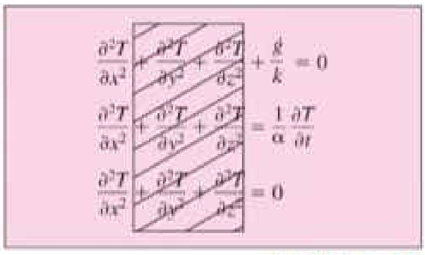


FIGURE 2–22
The three-dimensional heat conduction equations reduce to the one-dimensional ones when the temperature varies in one dimension only.

Cylindrical Coordinates

The general heat conduction equation in cylindrical coordinates can be obtained from an energy balance on a volume element in cylindrical coordinates, shown in Figure 2–23, by following the steps just outlined. It can also be obtained directly from Eq. 2–38 by coordinate transformation using the following relations between the coordinates of a point in rectangular and cylindrical coordinate systems:

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad \text{and} \qquad z = z$$

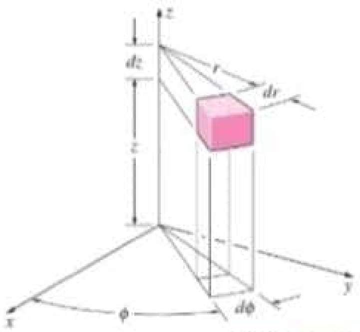


FIGURE 2–23
A differential volume element in cylindrical coordinates.

After lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \qquad \text{(2-43)}$$

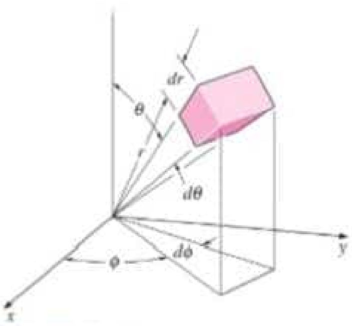


FIGURE 2–24
A differential volume element in spherical coordinates.

Spherical Coordinates

The general heat conduction equations in spherical coordinates can be obtained from an energy balance on a volume element in spherical coordinates, shown in Figure 2–24, by following the steps outlined above. It can also be obtained directly from Eq. 2–38 by coordinate transformation using the following relations between the coordinates of a point in rectangular and spherical coordinate systems:

$$x = r\cos\phi\sin\theta, \qquad y = r\sin\phi\sin\theta, \qquad \text{and} \qquad z = \cos\theta$$

Again after lengthy manipulations, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \qquad \text{(2-44)}$$

Special cases

(a) Steady state

Steady state refers to a stable condition that does not change over time. Time variation of temperature is zero. Hence,

$$\frac{\partial}{\partial x}(k\,\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\,\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\,\frac{\partial T}{\partial z}) + \dot{e}_{gen} = 0$$

(b) Uniform properties

If the material is homogeneous and isentropic, the thermal conductivity of the material would be constant.
{Comment: What do you mean by homogeneous and isentropic material? The term homogenous means, the values of physical properties of a material do not vary with position within the body of the material. E.g., The value of thermal conductivity at position (x1, y1, z1) will be same as that at some other position (x2, y2, z2). The term isentropic means, the value of physical properties at a point in different directions will be same. That is to say kx=ky=kz at a point. }

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{gen}}{k} = 0$$

Or,

$$\nabla^2 T = -\frac{\dot{e}_{gen}}{k}$$

The above equation is also known as **POISSON'S**

(c) No heat generation

When there is no heat generation inside the element, the differential heat conduction equation will become,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Or,

$$\nabla^2 T = 0$$

The above equation is also known as **LAPLACE** Equation.

(d) One dimensional form of equation

If heat conduction in any one direction is in dominance over heat conduction in other directions,

$$\frac{\partial^2 T}{\partial x^2} = 0$$

General Heat Conduction Equation

(A) Cartesian Co-ordinate System (x, y, z):

1. $$\frac{\partial}{\partial x}\left(K_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial t}{\partial z}\right) + q_g = \rho c \frac{\partial t}{\partial \tau}$$

The above equation represent the general heat conduction equation for three dimensional unsteady state heat flow, with internal heat generation through a non-homogeneous and anisotropic material.

2. $$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{K} = \frac{\rho c}{K}\frac{\partial t}{\partial \tau} \text{ or } \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$ ← Fourier-Biot Equation

Here, α = thermal diffusivity of material, m^2/s

$= \frac{K}{\rho c}$, ρc = heat capacity.

The above equation represents the general heat conduction equation for three dimensional unsteady state heat flow, with internal heat generation (q_g) through a homogeneous and isotropic material (K = constant)

$$\nabla^2 t + \frac{q_g}{K} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$ (Vector notation)

3. $$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$

or, $$\nabla^2 t = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$ (Fourier's Equation or, Diffusion Equation)

The above equation represents the general heat conduction equation for three dimensional unsteady state heat flow, without internal heat generation ($q_g = 0$) through a homogeneous, isotropic material.

4. $$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{K} = 0$$

or, $$\nabla^2 t + \frac{q_g}{K} = 0$$ (Poisson's Equation)

The above equation represents the general heat conduction equation for three dimensional, steady state heat flow, with internal heat generation (q_g) through a homogeneous, isotropic material.

5. $\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$

or, $\boxed{\nabla^2 t = 0}$ (Laplace Equation)

The above equation represents the general heat conduction equation for three dimensional, steady state without internal heat generation ($q_g = 0$) through a homogeneous and isotropic material.

Q. The Poisson's heat conduction equation represents

(a) 3D, Steady state heat flow with internal heat generation

(b) 3D, Steady state heat flow without internal heat generation

(c) 3D, unsteady state heat flow with internal heat generation

(d) 3D, unsteady state heat flow without internal heat generation

Ans: (a)

Q. In rectangular coordinates, the following equation

$$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{is called}$$

(a) Fourier equation (b) Laplace equation

(c) Poisson equation (d) None of the above

Ans: (a)

Q. In Cylindrical coordinates, the following relation

$$\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial t}{\partial r}\right) + \frac{1}{r^2} \cdot \frac{\partial^2 t}{\partial \theta^2} + \frac{\partial^2 t}{\partial z^2} = 0 \quad \text{is called}$$

(a) Laplace equation (b) Fourier equation

(c) Poisson equation (d) None of the above

Ans - (a)

* **Thermal Diffusivity (α):** The ratio of thermal conductivity (K) of a material to its heat capacity (ρC) is called as thermal diffusivity.

$$\boxed{\alpha = \frac{K}{\rho C}}$$

Unit: m^2/s, Dimension: $[M^0 L^2 T^{-1}]$

The larger the value of α the faster will the heat diffuse through the material and its temp. will change with time. It is a material property. Metals and gases have high value of α and their response to temp changes is quite rapid. The non-metallic solids and liquids respond slowly to temp changes because of their relatively

(B) Cylindrical Co-ordinates (r, ϕ, z)

1. $$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{K} = \frac{\rho C}{K}\cdot\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$

The above equation represents the general heat conduction equation in cylindrical co-ordinates. (for homogeneous & isotropic materials)

2. In case, there is no heat source $(q_g=0)$ present and the heat flow is Steady and One-dimensional, then the equation reduces to

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} = 0$$

(C) Spherical Coordinates (r, ϕ, θ)

1. $$\left[\frac{1}{r^2 \sin^2\theta}\cdot\frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\cdot\frac{\partial}{\partial \theta}\left(\sin\theta \cdot \frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial t}{\partial r}\right)\right] + \frac{q_g}{K} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau}$$

2. In case, there is no heat source $(q_g=0)$ present and the heat flow is Steady and One-dimensional, then the eqn. reduces to

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(r^2 \cdot \frac{dt}{dr}\right) = 0$$

** Combined One-Dimensional (1-D) Heat Conduction Eqn

$$\frac{1}{r^n}\cdot\frac{\partial}{\partial r}\left(r^n K \frac{\partial t}{\partial r}\right) + q_g = \rho C \frac{\partial t}{\partial \tau}$$

$n=0$: Plane wall
$n=1$: cylinder
$n=2$: Sphere

where, the variable 'r' is to replace by 'x' in case of a plane wall.

---

small value of thermal diffusivity (α).

* Both the specific heat (C_p) and the heat capacity (ρC_p) represent the heat storage capability of a material. But C_p expresses it per unit Mass whereas ρC_p expresses it per unit volume.

$$C_p = \frac{J}{kg\text{-}K}, \quad \rho . C_p = J/M^3\text{-}K$$

* Larger the value of 'α', faster is the propagation of heat into the medium.

K → represents how well a material conducts heat

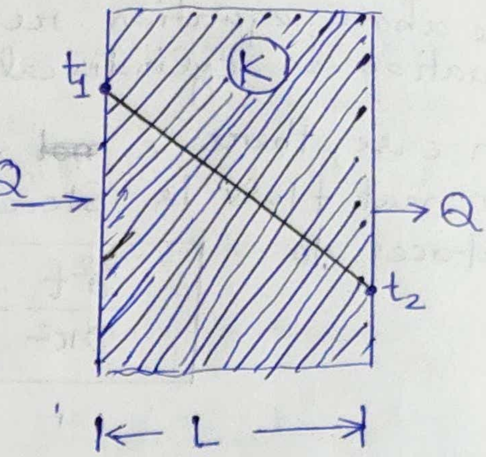
ρC_p → represents how much heat energy a material stores per unit volume

Heat Conduction Through a Plane Wall and Composite Wall

(A) Plane Wall (Uniform Thermal Conductivity and $q_g = 0$)

By Fourier's law of conduction

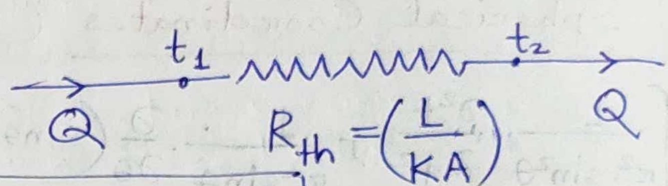
$$Q = KA\left(\frac{t_1 - t_2}{L}\right)$$



$$\Rightarrow Q = \frac{t_1 - t_2}{\left(\frac{L}{KA}\right)} = \frac{t_1 - t_2}{(R_{th})_{Cond.}}$$

Where, $(R_{th})_{Conduction}$ = Thermal conduction resistance

$$= \frac{L}{KA}$$



Temperature distribution

$$\boxed{t = \left(\frac{t_2 - t_1}{L}\right)x + t_1} \quad \text{(linear)}$$

i.e., temperature distribution across a plane wall is Linear and is independent of thermal conductivity (K).

* Plane Wall With Variable Thermal Conductivity (K)

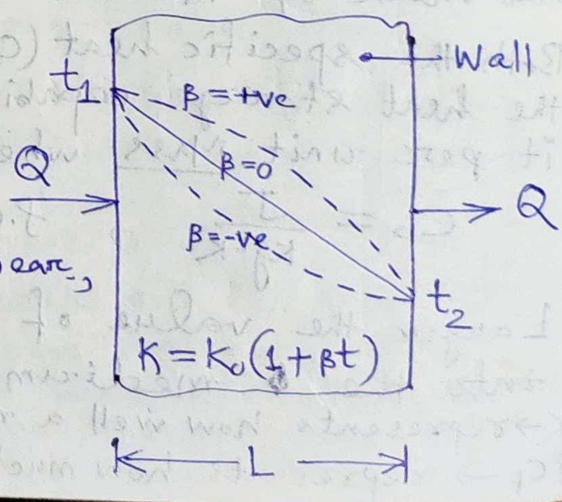
Suppose the variation of 'K' with temp is linear, then

$$K = K_o(1 + \beta t), \quad \beta = \text{temp. coefficient of thermal conductivity}$$

$$Q = K_m A\left(\frac{t_1 - t_2}{L}\right), \quad K_m = \text{Mean thermal conductivity of wall material}$$

$$= K_o(1 + \beta t_m), \quad t_m = \frac{t_1 + t_2}{2}$$

* Temperature distribution across a plane wall having thermal conductivity K (which varies with 't') is parabolic.



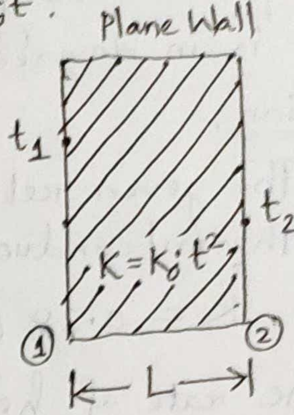
* If variation of K with temp. is not linear, then $K = K_o f(t)$

and $$K_M = \frac{1}{t_1 - t_2}\int_{t_2}^{t_1} K_o f(t)\, dt$$

Then the rate of heat flow, $\dot{Q} = K_m A\left(\frac{t_1 - t_2}{L}\right)$.

- β is POSITIVE for Non-metals and Insulation materials (Exception is Manganese bronze)
- β is NEGATIVE for Metallic conductors (Exceptions are Aluminium and certain non-ferrous alloys)

Problem-1: The surfaces of a plane wall of thickness L are maintained at temp. t_1 and t_2. The thermal conductivity of wall material varies according to the relation: $K = K_0 t^2$.



(i) Expression for heat conduction through wall

$$\dot{Q} = -KA\frac{dt}{dx}$$

$$= -K_0 t^2 . A\frac{dt}{dx}$$

$$\Rightarrow \quad \dot{Q}.dx = -K_0 t^2 . A\, dt$$

By integrating both sides, $\int_0^L \dot{Q}.dx = -K_0 A \int_{t_1}^{t_2} t^2 dt$

$$\Rightarrow \quad \dot{Q}.L = -\frac{K_0 A}{3}(t_2^3 - t_1^3)$$

$$\Rightarrow \quad \boxed{\dot{Q} = \frac{K_0 A}{3L}(t_1^3 - t_2^3)}$$ (Required expression)

(ii) Temperature (t_m) at which mean thermal conductivity be evaluated:

$$\dot{Q} = K_m A\left(\frac{t_1 - t_2}{L}\right)$$

$$\Rightarrow \quad \frac{K_0 A}{3L}(t_1^3 - t_2^3) = K_0 t_m^2 A\left(\frac{t_1 - t_2}{L}\right)$$

$$\Rightarrow \quad t_m^2 = \frac{1}{3}\left(\frac{t_1^3 - t_2^3}{t_1 - t_2}\right) = \frac{(t_1 - t_2)(t_1^2 + t_1 t_2 + t_2^2)}{3(t_1 - t_2)}$$

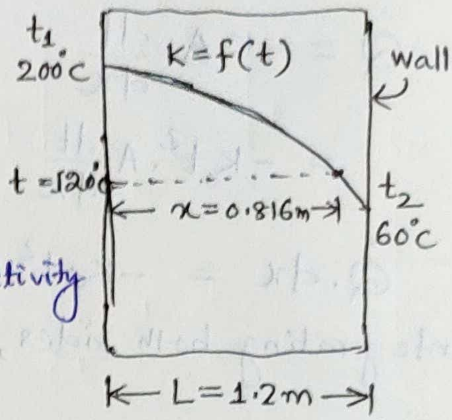
$$\Rightarrow \quad \boxed{t_m = \sqrt{\frac{t_1^2 + t_1 t_2 + t_2^2}{3}}}$$ (Required expression).

Problem-2: It is proposed to carry pressurized water through a pipe imbeded in a 1.2 m thick wall whose surfaces are held at constant temp. of 200°c and 60°c respectively. It is desired to locate the pipe in wall where the temp. is 120°c, find how far from the hot surface should the pipe be imbeded? The thermal conductivity of the wall material varies with the temp. according to the relation, $K = 0.28(1+0.036t)$, where 't' is in degree celsius and K is in $W/m\text{-}°C$.

Solution :-

The given relationship for thermal conductivity is

$$K = 0.28(1 + 0.036t)$$



The rate of heat transfer through a plane wall of variable thermal conductivity is given by

$$\dot{Q} = K_m A\left(\frac{t_1 - t_2}{L}\right)$$

$$= K_0(1+\beta t_M)\cdot\left(\frac{t_1-t_2}{L}\right)A$$

$$= K_0\left(1+\beta\frac{t_1+t_2}{2}\right)\left(\frac{t_1-t_2}{L}\right)A$$

At $L = 1.2m$, $t = 60°c$

At $L = x$, $t = 120°c$

$$\therefore \dot{Q} = K_0\left[1+\beta\left(\frac{200+120}{2}\right)\right]A\left(\frac{200-60}{1.2}\right) = K_0\left[1+\beta\left(\frac{120+60}{2}\right)\right]A\left(\frac{200-60}{x}\right)$$

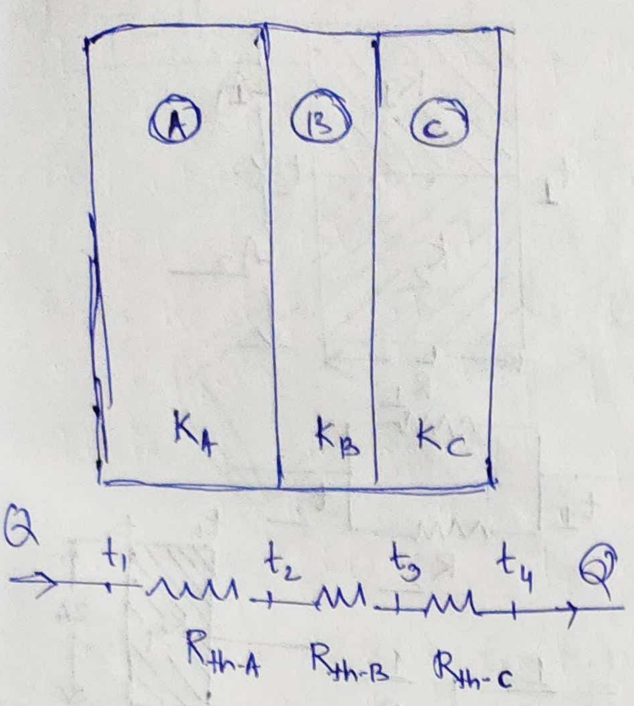
$$\Rightarrow 185.54 = \frac{151.42}{x}$$

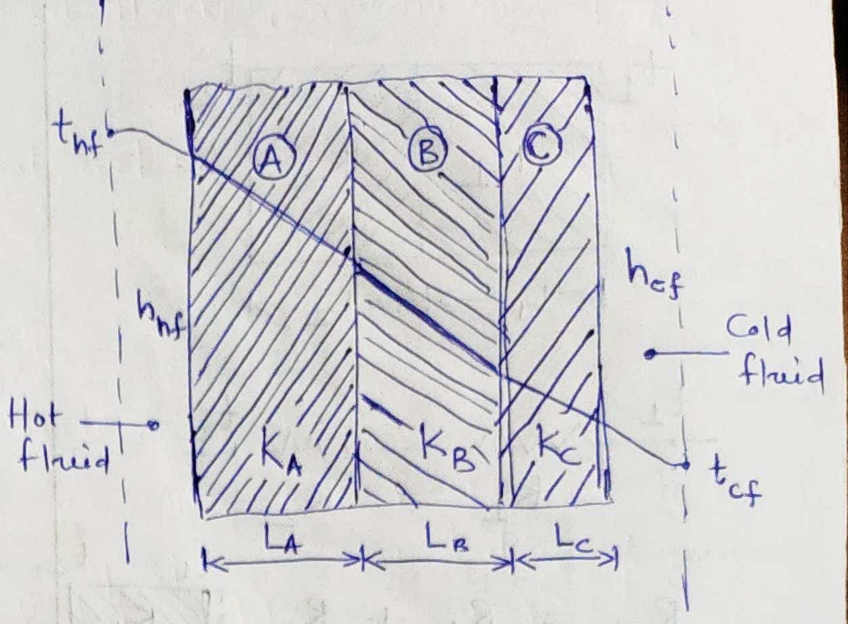
$$\Rightarrow \boxed{x = 0.816m}$$

Hence, the pipe should be embeded 0.816m from the hot wall surface.

Ans

(B) Heat Conduction Through a Composite Wall





$$U = \frac{1}{\left[\frac{1}{h_{hf} A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_{cf} A}\right]}$$

$$Q = U(t_1 - t_2)$$

$$Q = UA(t_1 - t_2)$$

$$= \frac{(t_1 - t_2)}{(R_{th-A} + R_{th-B} + R_{th-C})}$$

$$= \frac{t_1 - t_2}{\left[\frac{L_A}{K_A \cdot A} + \frac{L_B}{K_B \cdot A} + \frac{L_C}{K_C \cdot A}\right]}$$

$$= \frac{A(t_1 - t_2)}{\left[\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{K_C}{K_C}\right]}$$

Q. Statement-1: The larger the value of ~~these~~ the thermal diffusivity the faster is the propagation of heat into the medium.

Statement-2: A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

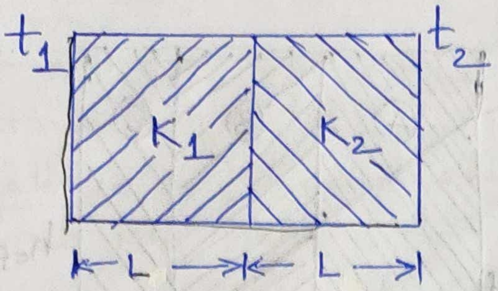
Choose the correct answer?

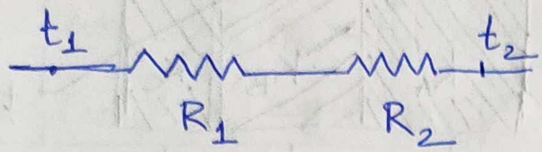
(a) Only statement-1 is true
(b) Both statement-1 & 2 are true
(c) Neither of statement-1 nor-2 is true
(d) Only statement-2 is true

[Ans: (b)]

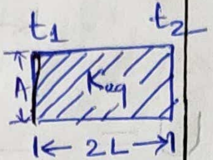
Imp

Calculation of Equivalent Thermal Conductivity (K_{eq})





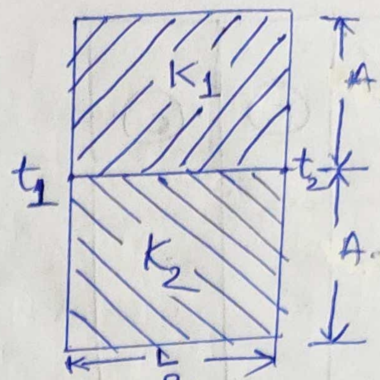
$$R_{eq} = R_1 + R_2$$

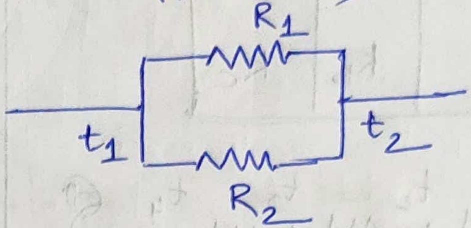


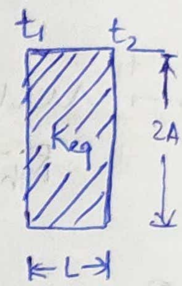
$$\Rightarrow \frac{2L}{K_{eq} A} = \frac{L}{K_1 A} + \frac{L}{K_2 A}$$

$$\Rightarrow \frac{2}{K_{eq}} = \frac{K_1 + K_2}{K_1 K_2}$$

$$\Rightarrow \boxed{K_{eq} = \frac{2 K_1 K_2}{K_1 + K_2}}$$







$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \frac{1}{\left(\frac{L}{K_{eq} \cdot 2A}\right)} = \frac{1}{\left(\frac{L}{K_1 A}\right)} + \frac{1}{\left(\frac{L}{K_2 A}\right)}$$

$$\Rightarrow 2 \cdot K_{eq} = K_1 + K_2$$

$$\Rightarrow \boxed{K_{eq} = \frac{K_1 + K_2}{2}}$$

Example 2.7. *A furnace wall consists of 200 mm layer of refractory bricks, 6 mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400 W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find :*

(i) To how many millimeters of insulation brick is the air layer equivalent?

(ii) What is the temperature of the outer surface of the steel plate?

(AMIE Winter, 1996)

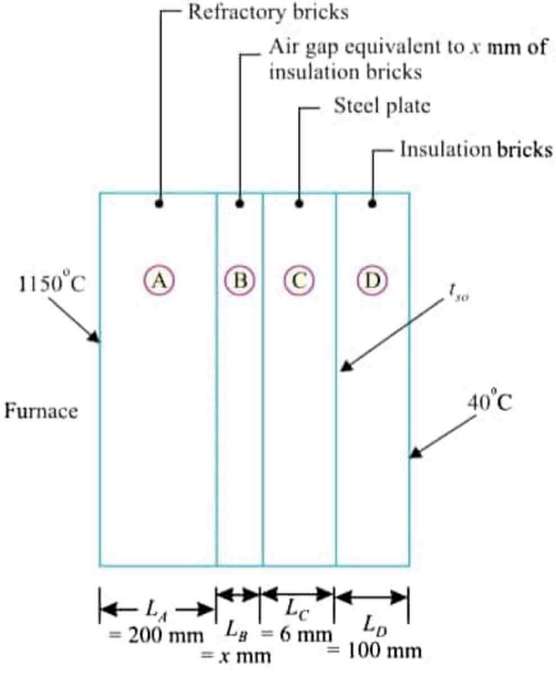


Fig. 2.15.

Solution. Refer Fig. 2.15.

Thickness of refractory bricks,

$L_A = 200 \text{ mm} = 0.2 \text{ m}$

Thickness of steel plate,

$L_C = 6 \text{ mm} = 0.006 \text{ m}$

Thickness of insulation bricks, $L_D = 100 \text{ mm} = 0.1 \text{ m}$

Difference of temperature beetween the innermost and outermost sides of the wall,

$$\Delta t = 1150 - 40 = 1110°C$$

Thermal conductivities :

$k_A = 1.52$ W/m°C; $k_B = k_D = 0.138$ W/m°C; $k_C = 45$ W/m°C

Heat loss from the wall, $q = 400$ W/m²

(*i*) **The value of x (= L_C) :**

We know,
$$Q = \frac{A \cdot \Delta t}{\Sigma \frac{L}{k}} \quad \text{or} \quad \frac{Q}{A} = q = \frac{\Delta t}{\Sigma \frac{L}{k}}$$

or,
$$400 = \frac{1110}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{L_D}{k_D}}$$

or,
$$400 = \frac{1110}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$

$$= \frac{1110}{0.1316 + 0.0072x + 0.00013 + 0.7246} = \frac{1110}{0.8563 + 0.0072x}$$

or,
$$0.8563 + 0.0072\,x = \frac{1110}{400} = 2.775$$

or,
$$x = \frac{2.775 - 0.8563}{0.0072} = \mathbf{266.5\ mm} \quad \textbf{(Ans.)}$$

(*ii*) **Temperature of the outer surface of the steel plate t_{so}:**

$$q = 400 = \frac{(t_{so} - 40)}{L_D / k_D}$$

or,
$$400 = \frac{(t_{so} - 40)}{(0.1/0.138)} = 1.38\,(t_{so} - 40)$$

or,
$$t_{so} = \frac{400}{1.38} + 40 = \mathbf{329.8°C} \quad \textbf{(Ans.)}$$

Example 2.8. *A furnace wall is composed of 220 mm of fire brick, 150 mm of common brick, 50 mm of 85% magnesia and 3 mm of steel plate on the outside. If the inside surface temperature is 1500°C and outside surface temperature is 90°C, estimate the temperatures between layers and calculate the heat loss in kJ/h- m². Assume, k (for fire brick) = 4kJ/m-h-°C, k(for common brick) = 2.8 kJ/m-h-°C, k (for 85% magnesia) = 0.24 kJ/m-h-°C, and k (steel) = 240 kJ/m-h-°C.*

(AMIE, Winter, 1997)

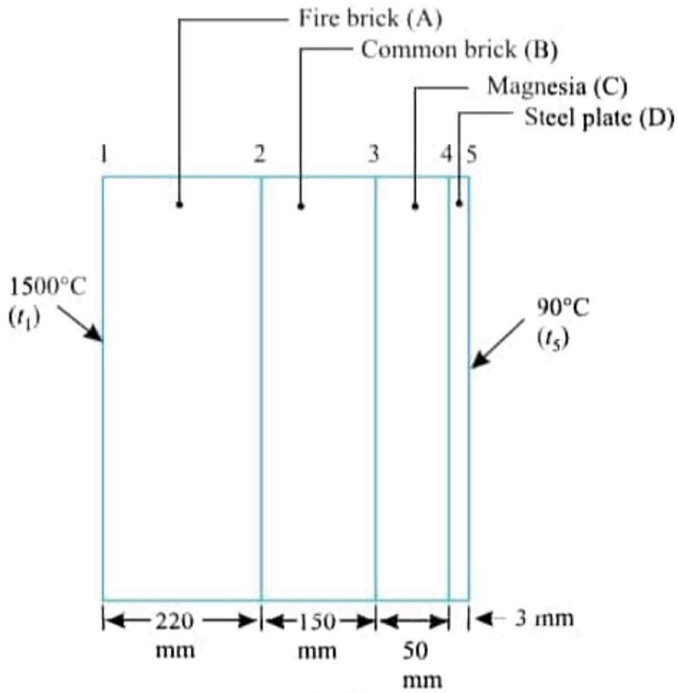


Fig. 2.16.

Solution. *Given* : L_A = 220 mm = 0.22 m; L_B = 150 mm = 0.15 m; L_C = 50 mm = 0.05m; L_D = 3 mm = 0.003 m

$$t_1 = 1500°\text{C}, t_5 = 90°\text{C};$$
$$k_A = 4 \text{ kJ/mh°C}; k_B = 2.8 \text{ kJ/mh°C}$$
$$k_C = 0.24 \text{ kJ/mh°C}; k_D = 240 \text{ kJ/mh°C}.$$

Heat loss in kJ/hm² :

The equivalent thermal resistances of various layers are :

$$R_{th-A} = \frac{L_A}{k_A} = \frac{0.22}{4} = 0.055 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-B} = \frac{L_B}{k_B} = \frac{0.15}{2.8} = 0.05357 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-C} = \frac{L_C}{k_C} = \frac{0.05}{0.24} = 0.2083 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-D} = \frac{L_D}{k_D} = \frac{0.003}{240} = 1.25 \times 10^{-5} \text{ m}^2\text{h°C/kJ}$$

Total thermal resistance,

$$(R_{th})_{total} = 0.055 + 0.05357 + 0.2083 + 1.25 \times 10^{-5} = 0.3169 \text{ m}^2\text{h°C/kJ}$$

Heat loss, $$q = \frac{(t_1 - t_5)}{(R_{th})_{total}} = \frac{(1500 - 90)}{0.3169} = \mathbf{4449.35 \text{ kJ/hm}^2} \quad \textbf{(Ans.)}$$

Temperatures between layers :

Also, $$q = \frac{t_4 - t_5}{R_{th-D}}$$

or $$t_4 = t_5 + q\,R_{th-D} = 90 + 4449.35 \times 1.25 \times 10^{-5} = \mathbf{90.056°C} \quad \textbf{(Ans.)}$$

Similarly, $$t_3 = t_4 + q\,R_{th-C} = 90.056 + 444.35 \times 0.2083 = \mathbf{1016.86°C}$$

and $$t_2 = t_3 + q\,R_{th-B} = 1016.86 + 4449.35 \times 0.05357 = \mathbf{1255.2°C}$$

[Check $$t_1 = t_2 + q\,R_{th-A} = 1255.2 + 4449.35 \times 0.55 \simeq 1500°\text{C}]$$

Exampel 2.29. *The furnace wall consists of 120 mm wide refractory brick and 120 mm wide insulating fire brick separated by an air gap. The outside wall is covered with a 12 mm thickness of plaster. The inner surface of the wall is at 1090°C and the room temperature is 20°C. The heat transfer coefficient from the outside wall surface to the air in the room is 18 W/m²°C, and the resistance to heat flow of the air gap is 0.16 K/W. If the thermal conductivities of the refractory brick, insulating fire brick, and plaster are 1.6, 0.3 and 0.14 W/mK, respectively calculate :*

(i) Rate at which heat is lost per m² of the wall surface;

(ii) Each interface temperature; and

(iii) Temperature of the outside surface of the wall.

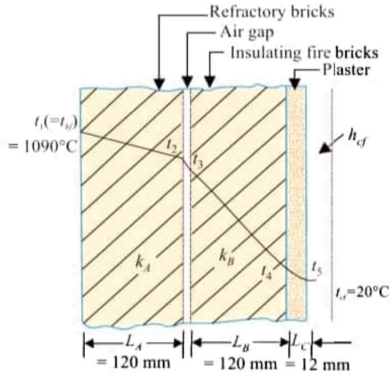


Fig. 2.39.

Solution. Refer Fig. 2.39.

Thickness of refractory brick, $L_A = 120 \text{ mm} = 0.12 \text{ m}$

Thickness of insulating fire brick, $L_B = 120 \text{ mm} = 0.12 \text{ m}$

Thickness of plaster, $L_C = 12 \text{ mm} = 0.012 \text{ m}$

Heat transfer coefficient from the outside wall surface to the air in the room,

$$h_{cf} = 18 \text{ W/m}^2\text{°C}$$

Resistance of air gap to heat flow = 0.16 K/W

Thermal conductivities :

Refractory brick, $k_A = 1.6$ W/m°C

Insulating fire brick, $k_B = 0.3$ W/m°C

Plaster, $k_C = 0.14$ W/m°C.

Temperatures : $t_{hf} = 1090$°C; $t_{cf} = 20$°C

Consider 1m² of surface area.

(i) Rate of heat loss per m² of surface area, q :

$$q = \frac{(t_{hf} - t_{cf})}{\frac{L_A}{k_A} + \text{air gap resistance} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

$$= \frac{(1090 - 20)}{\frac{0.12}{1.6} + 0.16 + \frac{0.12}{0.3} + \frac{0.012}{0.14} + \frac{1}{18}}$$

$$= \frac{1070}{0.075 + 0.16 + 0.4 + 0.0857 + 0.0555} = 1378.5 \text{ W or } 1.3785 \text{ kW}$$

i.e., Rate of heat loss per m² of surface area = **1.3785 kW** **(Ans.)**

(ii) Temperatures at interfaces, t_2, t_3, t_4:

$$Q = 1378.5 = \frac{1090 - t_2}{L_A / k_A} = \frac{1090 - t_2}{0.12/1.6} = \frac{1090 - t_2}{0.075}$$

∴ $t_2 = 1090 - 1378.5 \times 0.075 =$ **986.6°C** **(Ans.)**

Also, $Q = 1378.5 = \dfrac{t_2 - t_3}{\text{air gap resistance}} = \dfrac{986.6 - t_3}{0.16}$

∴ $t_3 = 986.6 - 1378.5 \times 0.16 =$ **766.04°C** **(Ans.)**

Again, $Q = 1378.5 = \dfrac{t_3 - t_4}{L_B / k_B} = \dfrac{766.04 - t_4}{0.12/0.3} = \dfrac{766.04 - t_4}{0.4}$

∴ $t_4 = 766.04 - 1378.5 \times 0.4 =$ **214.64°C** **(Ans.)**

(iii) Temperature of the outside surface of the wall, t_5 :

$$Q = 1378.5 = \frac{t_4 - t_5}{L_C / k_C} = \frac{214.64 - t_5}{0.012/0.14} = \frac{214.64 - t_5}{0.0857}$$

∴ $t_5 = 214.64 - 1378.5 \times 0.0857 =$ **96.5°C** **(Ans.)**

Example 2.30. *A furnace wall is made up of three layers of thicknesses 250 mm, 100 mm and 150 mm with thermal conductivities of 1.65, k and 9.2 W/m°C respectively. The inside is exposed to gases at 1250°C with a convection coefficient of 25 W/m²°C and the inside surface is at 1100°C, the outside surface is exposed to air at 25°C with convection coefficient of 12 W/m²°C. Determine :*

(i) The unknown thermal conductivity 'k';

(ii) The overall heat transfer coefficient;

(iii) All surface temperatures.

Solution. $L_A = 250 \text{ mm} = 0.25 \text{ m}$;

$L_B = 100 \text{ mm} = 0.1 \text{ m}$;

$L_C = 150 \text{ mm} = 0.15 \text{ m}$;

$k_A = 1.65$W/m°C;

$k_C = 9.2$ W/m°C;

$t_{hf} = 1250$°C, $t_1 = 1100$°C;

$h_{hf} = 25$ W/m²°C; $h_{cf} = 12$ W/m²°C.

(i) Thermal conductivity, k (= k_B) :

The rate of heat transfer per unit area of the furnace wall,

$$q = h_{hf}(t_{hf} - t_1)$$
$$= 25(1250 - 1100) = 3750 \text{ W/m}^2$$

Also,
$$q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}}$$

or
$$q = \frac{(t_{hf} - t_{cf})}{(R_{th})_{conv-hf} - R_{th-A} + R_{th-B} + R_{th-C} + (R_{th})_{conv-cf}}$$

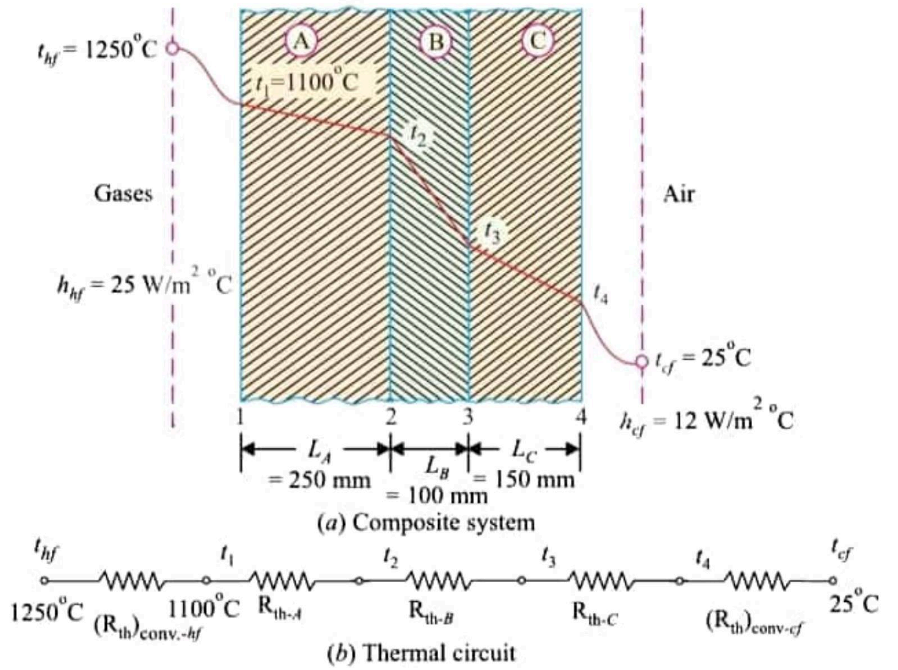


Fig. 2.40.

or,
$$3750 = \frac{(1250 - 25)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

or,
$$3750 = \frac{1225}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{k_B} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$= \frac{1225}{0.04 + 0.1515 + \frac{0.1}{k_B} + 0.0163 + 0.0833} = \frac{1225}{0.2911 + \frac{0.1}{k_B}}$$

or,
$$3750\left(0.289 + \frac{0.1}{k_B}\right) = 1225$$

or,
$$\frac{0.1}{k_B} = \frac{1225}{3750} - 0.2911 = 0.0355$$

∴
$$k_B = k = \frac{0.1}{0.0355}$$

= 2.817 W/m°C (Ans.)

(ii) The overall heat transfer coefficient, U :

The overall heat transfer coefficient, $U = \frac{1}{(R_{th})_{total}}$

***(ii)* The overall heat transfer coefficient, *U* :**

The overall heat transfer coefficient, $U = \frac{1}{(R_{th})_{total}}$

Now, $$(R_{th})_{total} = \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.817} + \frac{0.15}{9.2} + \frac{1}{12}$$

$$= 0.04 + 0.1515 + 0.0355 + 0.0163 + 0.0833 = 0.3266\ °\text{C m}^2/\text{W}$$

∴ $$U = \frac{1}{(R_{th})_{total}} = \frac{1}{0.3266} = \mathbf{3.06\ W/m^2\ °C\ (Ans.)}$$

***(iii)* All surface temperatures; t_1, t_2, t_3, t_4 :**

$$q = q_A = q_B = q_C$$

or, $$3750 = \frac{(t_1 - t_2)}{L_A / k_A} = \frac{(t_2 - t_3)}{L_B / k_B} = \frac{(t_3 - t_4)}{L_C / k_C}$$

or, $$3750 = \frac{(1100 - t_2)}{0.25/1.65}$$

or, $$t_2 = 1100 - 3750 \times \frac{0.25}{1.65} = \mathbf{531.8°\ C\ (Ans.)}$$

Similarly, $$3750 = \frac{(531.8 - t_3)}{0.1/2.817}$$

or, $$t_3 = 531.8 - 3750 \times \frac{0.1}{2.817} = \mathbf{398.6°\ C\ (Ans.)}$$

and $$3750 = \frac{(398.6 - t_4)}{(0.15/9.2)}$$

or, $$t_4 = 398.6 - 3750 \times \frac{0.5}{9.2} = \mathbf{337.5°\ C\ (Ans.)}$$

[Check using outside convection,

$$q = \frac{(337.5 - 25)}{1/h_{cf}} = \frac{(337.5 - 25)}{1/12} = 3750\ \text{W/m}^2\ ]$$

). HEAT CONDUCTION WITH INTERNAL HEAT GENERATION

Following are some of the cases where heat generation and heat conduction are encountered :

(*i*) Fuel rods – nuclear reactor;

(*ii*) Electrical conductors;

(*iii*) Chemical and combustion processes;

(*iv*) Drying and setting of concrete.

It is of paramount importance that the heat generation rate be controlled otherwise the equipment may fail (*e.g.*, some nuclear accidents, electrical fuses blowing out). Thus, in the design of the thermal systems temperature distribution within the medium and the rate of heat dissipation to the surroundings assumes ample importance / significance.

PLANE WALL WITH UNIFORM HEAT GENERATION

Refer to Fig. 2.91. Consider a plane wall of thickness L (small in comparison with other dimension) of uniform thermal conductivity k and in which heat sources are uniformly distributed in the whole volume. Let the wall surfaces are maintained at temperatures t_1 and t_2.

Let us assume that heat flow is one-dimensional, under steady state conditions, and there is a *uniform volumetric heat generation* within the wall.

Consider an element of thickness at a distance x from the left hand face of the wall.

Heat conducted in at distance x,

$$Q_x = -kA\frac{dt}{dx}$$

Heat generated in the element,

$$Q_g = A \,.\, dx \,.\, q_g$$

(where q_g = heat generated per unit volume per unit time in the element)

Heat conducted out at distance $(x + dx)$, $Q_{(x+dx)} = Q_x + \frac{d}{dx}(Q_x)\,dx$

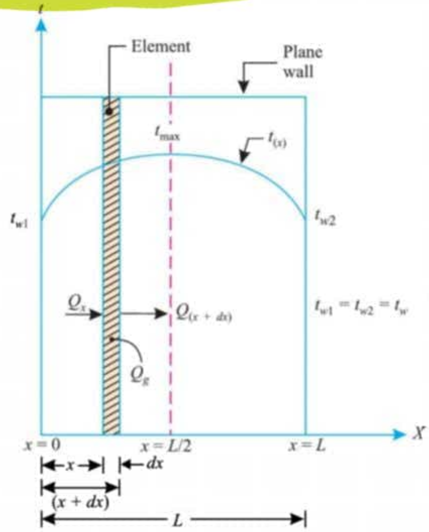


Fig. 2.91. Plane wall uniform heat generation. Both the surfaces maintained at a common temperature.

As Q_g represents an energy increase in the volume element, an energy balance on the element of thick dx is given by

$$Q_x + Q_g = Q_{(x+dx)}$$

$$= Q_x + \frac{d}{dx}(Q_x)\,dx$$

or, $$Q_g = \frac{d}{dx}(Q_x)\,dx$$

or, $$q_g \,.\, A \,.\, dx = \frac{d}{dx}\left[-k\,A\frac{dt}{dx}\right]dx$$

$$= -k\,A.\frac{d^2t}{dx^2}.dx$$

or, $$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \qquad \text{...(2.88)}$$

Eqn. (2.88) may also be obtained from eqn. (2.8) by assuming one-dimensional steady state conditions.

The first and second integration of Eqn. (2.88) gives respectively

$$\frac{dt}{dx} = -\frac{q_g}{k}x + C_1 \qquad \text{...(2.89)}$$

$$t = -\frac{q_g}{2k}\cdot x^2 + C_1\,x + C_2 \qquad \text{... (2.90)}$$

Case I. Both the surfaces have the same temperature :

Refer to Fig. 2.92.

At $x = 0$ $\quad t = t_1 = t_w$, and

At $x = L$ $\quad t = t_2 = t_w$

(where t_w = temperature of the wall surface).

Using these boundary conditions in eqn. (2.90), we get

$$C_2 = t_w \text{ and } C_1 = \frac{q_g}{2k}\cdot L$$

Substituting these values of C_1 and C_2 in eqn. (2.90), we have

$$t = -\frac{q_g}{2k}x^2 + \frac{q_g}{2k}\cdot L\cdot x + t_w$$

or, $$t = \frac{q_g}{2k}(L - x)\,x + t_w \qquad \text{...(2.91)}$$

In order to determine the location of the maximum temperature, differentiating the eqn. (2.91) *w.r.t x* and equating the derivative to *zero*, we have

$$\frac{dt}{dx} = \frac{q_g}{2k}(L - 2x) = 0$$

Since, $$\frac{q_g}{2k} \neq 0, \text{ therefore,}$$

$$L - 2x = 0 \qquad \text{or} \qquad x = \frac{L}{2}$$

$$L - 2x = 0 \qquad \text{or} \qquad x = \frac{L}{2}$$

Thus the *distribution of temperature* given by eqn. (2.91) is the *parabolic* and *symmetrical* about the midplane. The maximum temperature occurs at $x = \frac{L}{2}$ and its value equals

$$t_{max} = \left[\frac{q_g}{2k}(L - x)\,x\right]_{x = \frac{L}{2}} + t_w$$

or,
$$= \left[\frac{q_g}{2k}\left(L - \frac{L}{2}\right)\frac{L}{2}\right] + t_w$$

i.e.
$$t_{max} = \frac{q_g}{8k}\cdot L^2 + t_w \qquad ...(2.92)$$

Heat transfer then takes place towards both the surfaces, and for each surface it is given by

$$Q = -kA\left(\frac{dt}{dx}\right)_{x = 0 \text{ or } x = L}$$

$$= -kA\left[\frac{q_g}{2k}(L - 2x)\right]_{x = 0 \text{ or } x = L}$$

i.e.,
$$Q = \frac{AL}{2}\cdot q_g \qquad ...(2.93)$$

When both the surfaces are considered,

$$Q = 2 \times \frac{AL}{2} q_g = A.L.q_g \qquad ...[2.93\ (a)]$$

Also heat conducted to *each wall surface* is further dissipated to the surrounding atmosphere at temperature t_a,

Thus,
$$\frac{AL}{2}\cdot q_g = hA\,(t_w - t_a)$$

or,
$$t_w = t_a + \frac{q_g}{2h}\cdot L \qquad ...(2.94)$$

Substituting this value of t_w in eqn. (2.91), we obtain

$$t = t_a + \frac{q_g}{2h}\cdot L + \frac{q_g}{2k}(L - x)\,x \qquad ...(2.95)$$

At $x = L/2$ *i.e.*, at the midplane :

$$t = t_{max} = t_a + \frac{q_g}{2h}.L + \frac{q_g\,L^2}{8k}$$

or,
$$t_{max} = t_a + q_g\left[\frac{L}{2h} + \frac{L^2}{8k}\right] \qquad ...[2.95\ (a)]$$

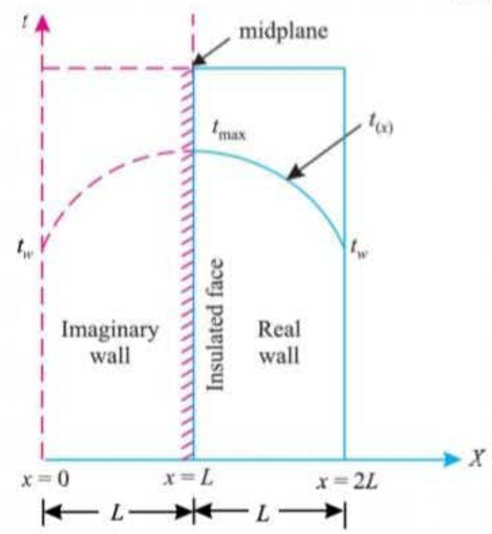


Fig. 2.92. Heat conduction in an insulated wall.

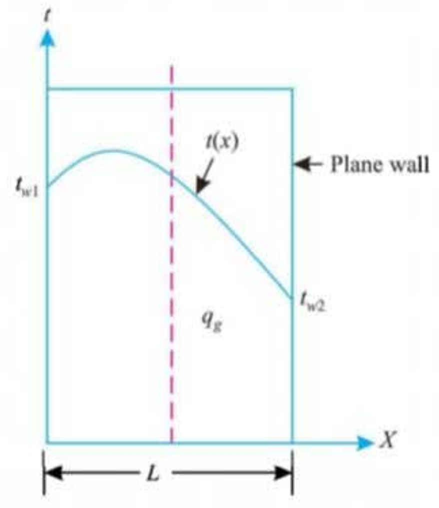


Fig. 2.93. Plane wall with uniform heat generation—Both the surfaces of the wall having different temperatures.

The eqn. (2.95) also works well in case of conduction in an *insulated wall* Fig. (2.92).

The following boundary conditions apply in the full *hypothetical wall* of thickness $2L$:

$$\text{At } x = L \qquad \frac{dt}{dx} = 0$$

$$\text{At } x = 2L \qquad t = t_w$$

The location $x = L$ refers to the mid-plane of the hypothetical wall (or insulated face of given wall).

Eqns. (2.91) and (2.92) for temperature distribution and maximum temperature at the mid- plane (insulated end of the given wall) respectively can be written as

$$t = \frac{q_g}{2k}(2L - x)\,x + t_w \qquad ...(2.96)$$

$$t_{max} = \frac{q_g}{2k}L^2 + t_w \qquad ...(2.97)$$

[Substituting L = 2L in eqn. (2.91) and (2.92)]

Case II. Both the surfaces of the wall have different temperatures :

Refer to Fig. 2.93

The boundary conditions are :

$$\text{At } x = 0 \qquad t = t_{w1}$$

$$\text{At } x = L \qquad t = t_{w2}$$

Substituting these values in eqn. (2.90), we obtain the values of constant C_1 and C_2 as :

$$C_2 = t_{w1}; \qquad C_1 = \frac{t_{w2} - t_{w1}}{L} + \frac{q_g}{2k}\cdot L$$

Inserting these values in eqn. (2.90), we get

$$t = -\frac{q_g}{2k}x^2 + \frac{t_{w2} - t_{w1}}{L}x + \frac{q_g}{2k}L.x + t_{w1}$$

$$= \frac{q_g}{2k}L\cdot x - \frac{q_g}{2k}x^2 + \frac{x}{L}(t_{w2} - t_{w1}) + t_1$$

or, $$t = \left[\frac{q_g}{2k}(L - x) + \frac{t_{w2} - t_{w1}}{L}\right]x + t_{w1} \qquad ...(2.98)$$

The temperature distribution, in dimensionless form can be obtained by making the following transformations :

$$t - t_{w2} = \frac{q_g}{2k}L^2\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] + \frac{x}{L}(t_{w2} - t_{w1}) + (t_{w1} - t_{w2})$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \frac{q_g}{2k}\cdot\frac{L^2}{(t_{w1} - t_{w2})}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{x}{L} + 1$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \frac{q_g}{2k}\cdot\frac{L^2}{(t_{w1} - t_{w2})}\cdot\frac{x}{L}\left[1 - \frac{x}{L}\right] + \left[1 - \frac{x}{L}\right]$$

Replacing the parameter $\frac{q_g}{2k}\cdot\frac{L^2}{(t_{w1} - t_{w2})}$ (a constant) by a factor Z, we have

$$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = Z.\frac{x}{L}\left[1 - \frac{x}{L}\right] + \left[1 - \frac{x}{L}\right]$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{x}{L}\right]\left[\frac{Zx}{L} + 1\right] \qquad ...(2.99)$$

In order to get maximum temperature and its location, differentiating Eqn. (2.99) w.r.t x and equating the derivative to zero, we have

$$\frac{dt}{d(x/L)} = \left(1 - \frac{x}{L}\right)Z + \left(\frac{Zx}{L} + 1\right)(-1) = 0$$

or, $$Z - \frac{Zx}{L} - \frac{Zx}{L} - 1 = 0$$

or, $$\frac{2Zx}{L} = Z - 1$$

or, $$\frac{x}{L} = \frac{Z - 1}{2Z} \qquad ...(2.100)$$

Thus the maximum value of temperature occurs at $\frac{x}{L} = \frac{Z - 1}{2Z}$ and its value is given by:

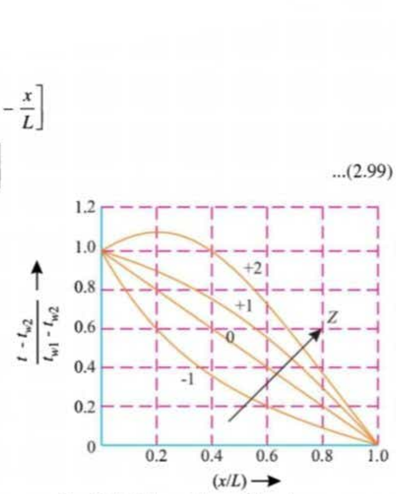


Fig. 2.94. Effect of factor Z on the temperature distribution in the plane wall.

$$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{Z - 1}{2Z}\right]\left[Z \times \left(\frac{Z - 1}{2Z}\right) + 1\right]$$

or, $$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left(\frac{Z + 1}{2Z}\right)\left(\frac{Z + 1}{2}\right)$$

$$= \frac{(Z + 1)^2}{4Z} \qquad ...(2.101)$$

Fig. 2.94 shows the effect of factor Z on the temperature distribution in the plane wall.The following points emerge :

- As the value of Z *increases* the slope of the curve changes; obviously the direction of heat flow can be reversed by an adequately large value of q_g.
- When Z = 0, the temperature distribution is *linear* (*i.e.*, no internal heat generation).
- When the value of Z is *negative*, q_g represents *absorption of heat* within the wall/body.

Example 2.81. *A plane wall 90 mm thick (k = 0.18 W/m°C) is insulated on one side while the other side is exposed to environment at 80°C. The rate of heat generation within the wall is 1.3 × 105 W/m³. If the convective heat transfer coefficient between the wall and the environment is 520 W/m²°C, determine the maximum temperature to which the wall will be subjected.*

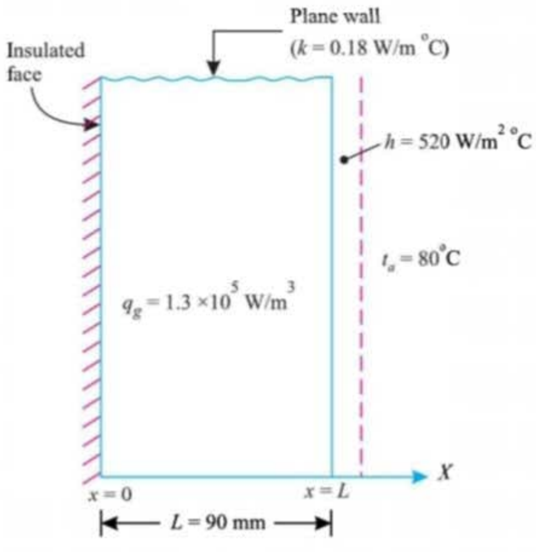


Fig. 2.98.

Solution. Refer Fig. 2.98

L = 90 mm = 0.09 m,

k = 0.18 W/m°C

h = 520 W/m²°C

Temperature of environment,

t_a = 80°C

The rate of heat generation,

$q_g = 1.3 \times 10^5$ W/m³

Maximum temperature, t_{max} :

One dimensional, steady state heat conduction equation is given by

$$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \quad \text{..(i)}$$

Integrating the above equation twice, we have

$$\frac{dt}{dx} = -\frac{q_g}{k}\cdot x + C_1 \quad \text{...(ii)}$$

$$t = -\frac{q_g}{k}\cdot\frac{x^2}{2} + C_1x + C_2 \quad \text{...(iii)}$$

(where C_1, C_2 = constants of integration)

In order to evaluate C_1 and C_2, using the following boundary conditions, we have

(*i*) At $x = 0, \frac{dt}{dx} = 0 \quad \therefore C_1 = 0$

(*ii*) At $x = L$, the heat conduction equals the convective heat flow to the environment.

i.e $$-kA\left|\frac{dt}{dx}\right|_{x=L} = hA[t_{(L)} - t_a]$$

$$-\left|\frac{dt}{dx}\right|_{x=L} = \frac{h}{k}[t_{(L)} - t_a] \quad \text{...(iv)}$$

Again from eqn (*ii*),

$$\left|\frac{dt}{dx}\right|_{x=L} = -\frac{q_g}{k}.L \quad \text{...(v)}$$

From eqns. (*iv*) and (*v*), we obtain

$$-\frac{h}{k}\left[t_{(L)} - t_a\right] = -\frac{q_g}{k}\cdot L$$

$$t_{(L)} = t_a + \frac{q_g}{h}.L$$

Substituting into eqn. (*iii*), we have

$$t_{(L)} = t_a + \frac{q_g}{h}\cdot L = -\frac{q_g}{2k}\cdot L^2 + C_2$$

$$\therefore \quad C_2 = t_a + \frac{q_g}{h}\cdot L + \frac{q_g}{2k}\cdot L^2 \quad \text{...(vi)}$$

Inserting the values of constants C_1 and C_2 in eqn. (*iii*), we get

$$t = -\frac{q_g}{2k}\cdot x^2 + t_a + \frac{q_g}{h}\cdot L + \frac{q_g}{2k}\cdot L^2$$

or, $$t = t_a + \frac{q_g}{h}\cdot L + \frac{q_g}{2k}(L^2 - x^2) \quad \text{...(vii)}$$

The maximum temperature occurs at the insulated wall boundary *i.e.*, $x = 0$

$$\therefore \quad t_{max} = t_a + \frac{q_g}{h}.L + \frac{q_g}{2k}\cdot L^2 \quad \text{...(viii)}$$

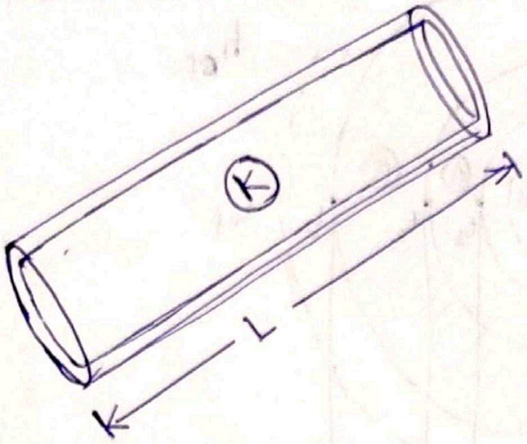
Substituting the proper values, we have

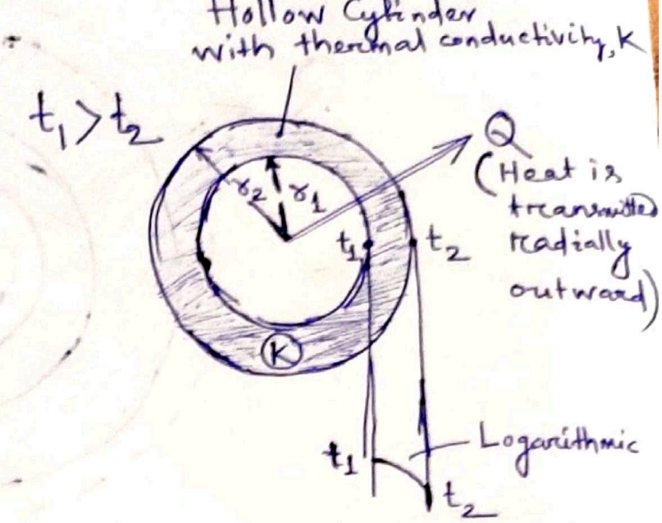
$$t_{max} = 80 + \frac{1.3\times10^5}{520}\times 0.09 + \frac{1.3\times10^5}{2\times0.18}\times(0.09)^2$$

= **3027.5°C (Ans.)**

Heat Conduction Through a Hollow and Composite Cylinder

(A) Hollow Cylinder :-





$$\dot{Q} = K(2\pi L)\frac{(t_1 - t_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$Q \xrightarrow{t_1} \text{WWW} \xrightarrow{t_2} Q$, $(R_{th})_{Cond.}$

$$\Rightarrow Q = \frac{t_1 - t_2}{\left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi KL}\right]} = \frac{t_1 - t_2}{(R_{th})_{Cond.}}$$

Temperature dist. across a hollow cylinder is Logarithmic.

* $$\frac{t - t_1}{t_2 - t_1} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}$$ (Dimensionless form)

Temperature at any point in the cylinder can be expressed as a function of radius only. Isotherms (or lines of T=c) are then concentric circles lying between inner and outer boundaries of the hollow cylinder.

→ If $\frac{r_2}{r_1} \to 1$, the temp. profile is nearly linear but non-linear for large value of $\left(\frac{r_2}{r_1}\right)$.

* Cylinder with variable thermal conductivity $[K = K_0(1+\beta t)]$, the temperature profile is also Logarithmic.

.2. HEAT CONDUCTION THROUGH A COMPOSITE CYLINDER

Consider flow of heat through a composite cylinder as shown in Fig. 2.47.

Let,
t_{hf} = The temperature of the hot fluid flowing inside the cylinder,
t_{cf} = The temperature of the cold fluid (atmospheric air),
k_A = Thermal conductivity of the inside layer A,
k_B = Thermal conductivity of the outside layer B,
t_1, t_2, t_3 = Temperatures at the points 1, 2, and 3 (see Fig. 2.47)
L = Length of the composite cylinder, and
h_{hf}, h_{cf} = Inside and outside heat transfer coefficients.

The rate of heat transfer is given by

$$Q = h_{hf}\,.\,2\pi r_1\,.\,L(t_{hf} - t_1) = \frac{k_A\,.\,2\pi L(t_1 - t_2)}{\ln(r_2/r_1)}$$

$$= \frac{k_B\,.\,2\pi L(t_2 - t_3)}{\ln(r_3/r_2)} = h_{cf}\,.\,2\pi r_3\,.\,L(t_3 - t_{cf})$$

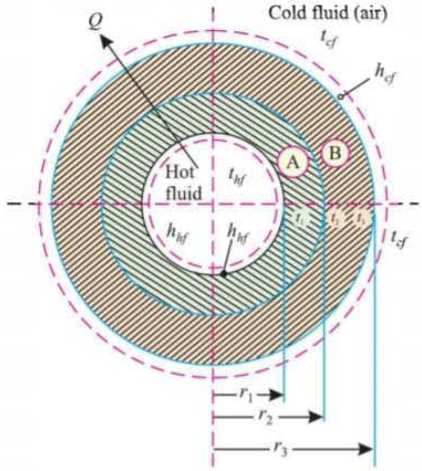


Fig. 2.47.

Rearranging the above expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf}\,.\,r_1\,.\,2\pi L} \quad ...(i)$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A\,.\,2\pi L}{\ln(r_2/r_1)}} \quad ...(ii)$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B\,.\,2\pi L}{\ln(r_3/r_2)}} \quad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf}\,.\,r_3\,.\,2\pi L} \quad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf}\,.\,r_1} + \frac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \frac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \frac{1}{h_{cf}\,.\,r_3}\right] = t_{hf} - t_{cf}$$

$\therefore$

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1} + \dfrac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \dfrac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \dfrac{1}{h_{cf}\,.\,r_3}\right]}$$

or,

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}\,.\,r_3}\right]} \quad ...(2.69)$$

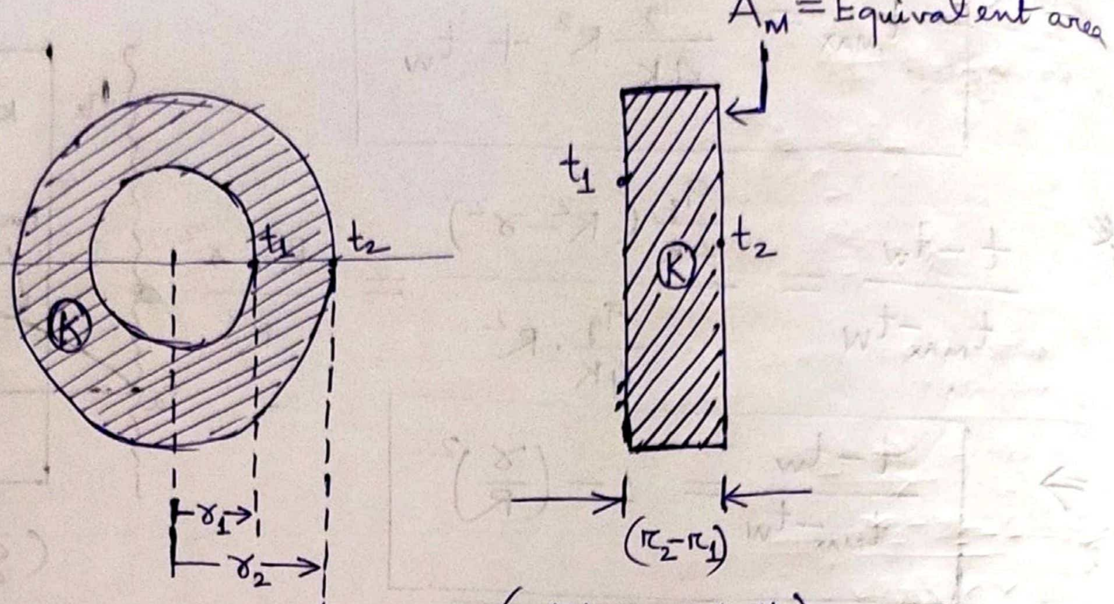
If there are '*n*' concentric cylinders, then

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}\,.\,r_1} + \displaystyle\sum_{n=1}^{n=n}\frac{1}{k_n}\ln\{r_{(n+1)}/r_n\} + \dfrac{1}{h_{cf}\,.\,r_{(n+1)}}\right]} \quad ...(2.70)$$

If inside and outside heat transfer coefficients are not considered then the above equation can be written as

$$Q = \frac{2\pi L\left[t_1 - t_{(n+1)}\right]}{\displaystyle\sum_{n=1}^{n=n}\frac{1}{k_n}\ln\left[r_{(n+1)}/r_n\right]} \quad ...(2.71)$$

Logarithmic Mean Area For the Hollow Cylinder



(Hollow Cylinder) (Plane Wall)

$$Q = \frac{t_1 - t_2}{\left[\frac{\ln(r_2/r_1)}{2\pi KL}\right]}$$ (Heat flow through cylinder)

$$Q = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{K \cdot A_M}\right)}$$ (Heat flow through plane wall)

A_m is so chosen that the heat flow through cylinder and plane wall will be equal for the same temp. difference $(t_1 - t_2)$.

$$\therefore \frac{t_1 - t_2}{\left[\frac{\ln(r_2/r_1)}{2\pi KL}\right]} = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{K A_m}\right)}$$

$$\Rightarrow \boxed{A_M = \frac{A_0 - A_i}{\ln(A_0/A_i)}}$$

where A_i and A_o are inside and outside surface areas of the cylinder.

The above expression is known as logarithmic mean area (A_m) of the plane wall and the hollow cylinder.

Example 2.47. *An insulated steam pipe having outside diameter of 30 mm is to be covered with two layers of insulation, each having thickness of 20 mm. The thermal conductivity of one material is 5 times that of the other.*

Assuming that the inner and outer surface temperatures of composite insulation are fixed, how much will heat transfer be increased when better insulation material is next to the pipe than it is outer layer? **(M.U.)**

Solution. Case I. When better insulation is inside :

Refer to Fig. 2.59.

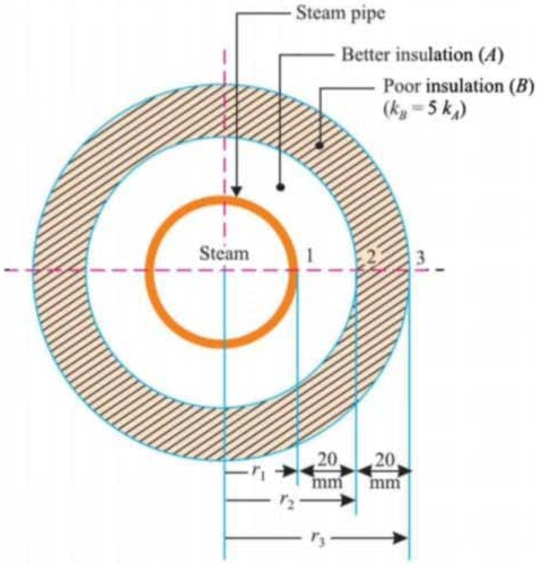


Fig. 2.59.

$$r_1 = \frac{30}{2} = 15\,\text{mm} = 0.015\,\text{m};$$

$$r_2 = 15 + 20 = 35\ \text{mm} = 0.035\ \text{m};$$

$$r_3 = 35 + 20 = 55\ \text{mm} = 0.055\text{m}$$

$$k_B = 5k_A$$

Heat lost through the pipe is given by

$$Q_1 = \frac{2\pi L(t_1 - t_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi L(t_1 - t_3)}{\dfrac{\ln(0.035/0.015)}{k_A} + \dfrac{\ln(0.055/0.035)}{5k_A}}$$

or,
$$Q_1 = \frac{k_A\,.\,2\pi L(t_1 - t_3)}{0.8473 + 0.0904} = \frac{k_A\,.2\pi L(t_1 - t_3)}{0.9377} = 1.066\,2\pi L k_A\,(t_1 - t_3) \qquad \text{...}(i)$$

Case II. When better insulation is outside : Refer to Fig. 260.

$$Q_2 = \frac{2\pi L(t_1 - t_3)}{\dfrac{\ln(r_2/r_1)}{k_B} + \dfrac{\ln(r_3/r_2)}{k_A}} = \frac{2\pi L(t_1 - t_3)}{\dfrac{\ln(0.035/0.015)}{5k_A} + \dfrac{\ln(0.055/0.035)}{k_A}}$$

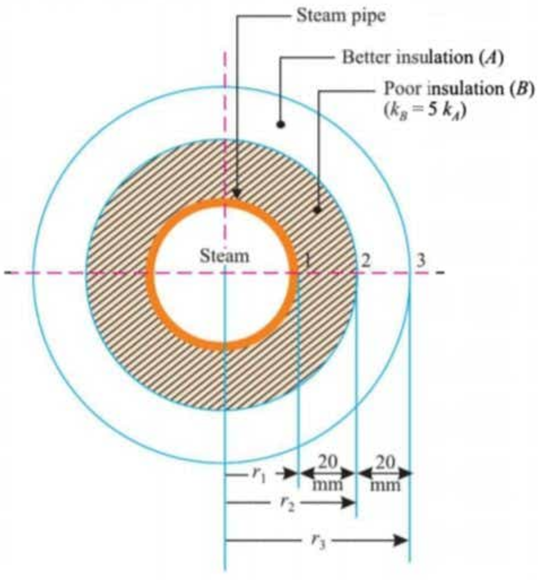


Fig. 2.60.

or,
$$Q_2 = \frac{k_A\,.2\pi L(t_1 - t_3)}{0.1694 + 0.452} = \frac{k_A\,.2\pi L(t_1 - t_3)}{0.6214} = 1.609\ 2\pi L.k_A\,(t_1 - t_3) \qquad \text{...}(ii)$$

From expression (*i*) and (*ii*), we have,

$$\frac{Q_2}{Q_1} = \frac{1.609 \times 2\pi L.k_A\,(t_1 - t_3)}{1.066 \times 2\pi L.k_A\,(t_1 - t_3)} = 1.509$$

As $Q_2 > Q_1$, therefore, *putting the better insulation next to the pipe decreases the heat flow.*

∴ Percentage decrease in heat transfer

$$= \frac{Q_2 - Q_1}{Q_1} = \frac{Q_2}{Q_1} - 1 = 1.509 - 1 = 0.509 \text{ or } \mathbf{50.9\%} \ \textbf{(Ans.)}$$

Example 2.50. *An aluminium pipe carries steam at 110°C. The pipe (k = 185 W/m°C) has an inner diameter of 100 mm and outer diameter of 120 mm. The pipe is located in a room where the ambient air temperature is 30°C and the convective heat transfer coefficient between the pipe and air is 15 W/m²°C. Determine the heat transfer rate per unit length of pipe.*

To reduce the heat loss from the pipe, it is covered with a 50 mm thick layer of insulation (k = 0.20 W/m°C). Determine the heat transfer rate per unit length from the insulated pipe. Assume that the convective resistance of the steam is negligible. **(AMIE Summer, 1999)**

Solution. Case I. Refer to Fig. 2.63.

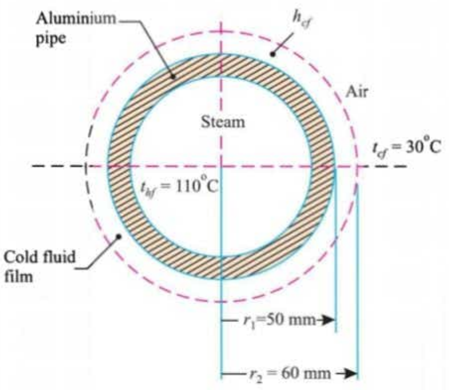


Fig. 2.63.

Given : $r_1 = \dfrac{100}{2} = 50\text{ mm} = 0.05\text{ m}$

$r_2 = \dfrac{120}{2} = 60\text{ mm} = 0.06\text{ m}$

Temperature of steam (hot fluid), $t_{hf} = 110°\text{C}$

Temperature of ambient air (cold fluid), $t_{cf} = 30°\text{C}$

Thermal conductivity of pipe material, $k = 185\text{ W/m°C}$

Heat transfer coefficient between the pipe and air, $h_{cf} = 15\text{ W/m}^2\text{°C}$

Heat transfer rate per unit length of pipe, *Q/L* :

Heat transfer rate is given by,

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{1}{h_{cf}.r_2}\right]} \qquad \text{[Eqn. (2.69)]}$$

or,
$$\frac{Q}{L} = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{1}{h_{cf}.r_2}\right]} = \frac{2\pi(110 - 30)}{\left[\dfrac{\ln(0.06/0.05)}{185} + \dfrac{1}{15 \times 0.06}\right]} = 451.99\text{ W/m}$$

i.e., Heat transfer rate per unit length of pipe = **451.99 W/m** **(Ans.)**

Case II : Refer to Fig. 2.64.

$r_1 = 50\text{ mm} = 0.05\text{ m};$ $\quad r_2 = 60\text{ mm} = 0.06\text{ m}$

$r_3 = 60 + 50 = 110\text{ mm} = 0.11\text{ m};$ $\quad k_A = 185\text{ W/m°C}$

$k_B = 0.20\text{ W/m°C};$ $\quad h_{cf} = 15\text{ W/m}^2\text{°C}$

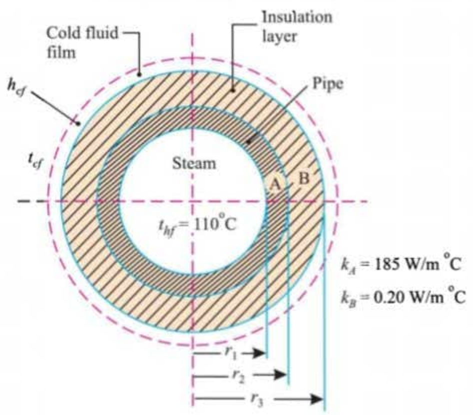


Fig. 2.64.

Heat transfer rate per unit length from the insulated pipe, *Q/L* :

Heat transfer rate in this case will be given by

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}.r_3}\right]}$$

$$\frac{Q}{L} = \frac{2\pi(t_{hf} - t_{cf})}{\left[\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}.r_3}\right]}$$

Substituting the given data is the above equation, we have,

$$\frac{Q}{L} = \frac{2\pi(110 - 30)}{\left[\dfrac{\ln(0.06/0.05)}{185} + \dfrac{\ln(0.11/0.06)}{0.20} + \dfrac{1}{15 \times 0.11}\right]}$$

$$= \frac{502.65}{0.000985 + 3.030679 + 0.606060} = 138.18\text{ W/m}$$

i.e., Heat transfer rate per unit length of insulated pipe = **138.18 W/m** **(Ans.)**

Example 2.53. *A steam pipe (k = 45 W/m°C) having 70 mm inside diameter and 85 mm outside diameter is lagged with two insulation layers; the layer in contact with the pipe is 35 mm asbestos (k = 0.15 W/m°C) and it is covered with 25 mm thick magnesia insulation (k = 0.075 W/m°C). The heat transfer coefficients for the inside and outside surfaces are 220 W/m²°C and 6.5 W/m²°C respectively. If the temperature of steam is 350°C and the ambient temperature is 30°C, calculate :*

(i) The steady loss of heat for 50 m length of the pipe;

(ii) The overall heat transfer coefficients based on inside and outside surfaces of the lagged steam main.

Solution. Refer to Fig. 2.67.

$$r_1 = \frac{70}{2} = 35\,\text{mm or } 0.035\,\text{m}$$

$$r_2 = \frac{85}{2} = 42.5\,\text{mm or } 0.0425\,\text{m}$$

$$r_3 = 42.5 + 35 = 77.5\,\text{mm or } 0.0775\,\text{m}$$

$$r_4 = 77.5 + 25 = 102.5\,\text{mm or } 0.1025\,\text{m}$$

$L = 50$ m

$k_A = 45$ W/m°C

$k_B = 0.15$ W/m°C

$k_C = 0.075$ W/m°C

Temperature of steam,

$t_{hf} = 350$°C

Ambient temperature,

$t_{cf} = 30$°C

$h_{hf} = 220$ W/m²°C,

$h_{cf} = 6.5$ W/m²°C.

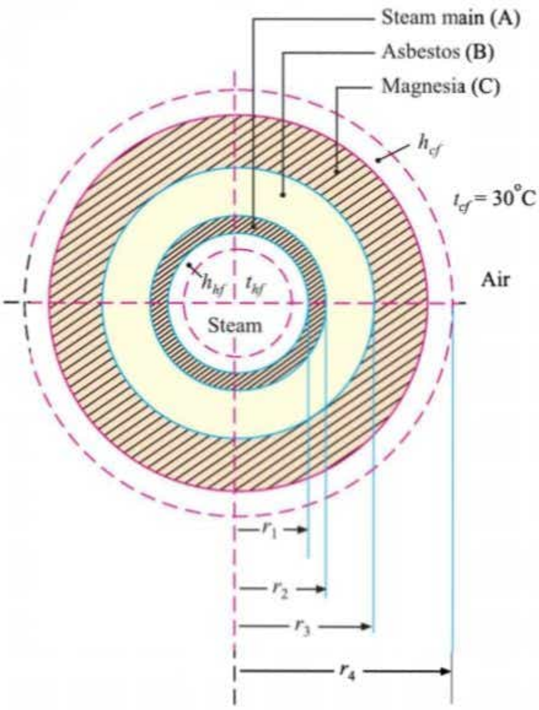


Fig. 2.67.

(*i*) Loss of heat, *Q* :

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\dfrac{1}{h_{hf} \cdot r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{\ln(r_4/r_3)}{k_C} + \dfrac{1}{h_{cf} \cdot r_4}}$$

$$= \frac{2\pi \times 50(350 - 30)}{\dfrac{1}{220 \times 0.035} + \dfrac{\ln(0.0425/0.035)}{45} + \dfrac{\ln(0.0775/0.0425)}{0.15} + \dfrac{\ln(0.1025/0.0775)}{0.075} + \dfrac{1}{6.5 \times 0.1025}}$$

$$= \frac{100530.96}{0.129870 + 0.00431 + 4.00516 + 3.72779 + 1.50094} = 10731.23\,\text{W}$$

i.e., Loss of heat for 50 m of length = **10731.23 W (Ans.)**

(*ii*) The overall heat transfer coefficients, U_o, U_i :

The loss of heat can also be expressed as follows :

$$Q = U_o A_o \Delta t = U_i A_i \Delta t$$

Where U_o and U_i are the overall heat transfer co-efficients based on the outside area A_o and inside area A_i respectively.

$$\therefore \qquad U_0 = \frac{Q}{A_o \cdot \Delta t} = \frac{10731.23}{2\pi r_4 L \times \Delta t}$$

$$= \frac{10731.23}{2\pi \times 0.1025 \times 50(350 - 30)} = \mathbf{1.0414\ W/m^2 °C} \quad \textbf{(Ans.)}$$

Similarly,

$$U_i = \frac{Q}{A_i \cdot \Delta t} = \frac{10731.23}{2\pi r_1 L \times \Delta t} = \frac{10731.23}{2\pi \times 0.035 \times 50(350 - 30)}$$

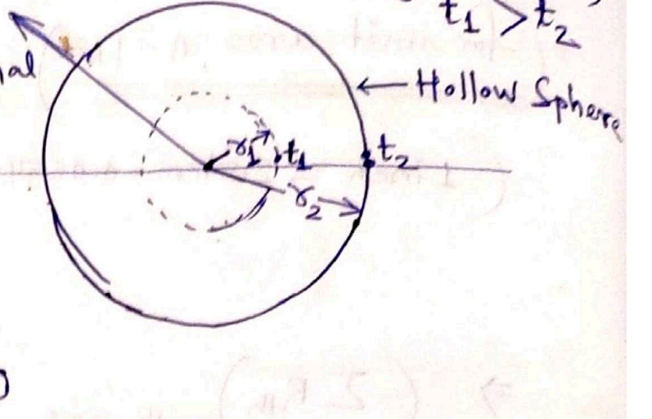
$$= \mathbf{3.05\ W/m^2 °C} \quad \textbf{(Ans.)}$$

Heat Conduction Through a Hollow and Composite Sphere

(A) Hollow Sphere:

Q (Heat flows radially outwards)

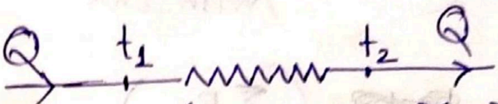
$t_1 > t_2$



For steady state, unidirectional heat flow in the radial direction and with no heat generation, the equation is given by,

$$\frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \frac{dt}{dr}\right) = 0$$

By integration & simplification,



$$(R_{th})_{cond.} = \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

$$t = t_1 + \frac{t_1 - t_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}\left[\frac{1}{r_1} - \frac{1}{r}\right]$$

or

$$\frac{t - t_1}{t_2 - t_1} = \frac{r_2}{r}\left[\frac{r - r_1}{r_2 - r_1}\right]$$ (Dimensionless form)

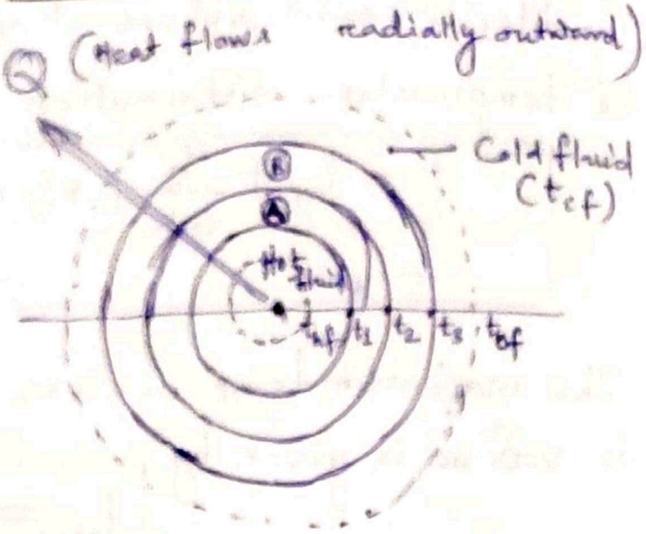
The temperature distribution associated with radial conduction through a sphere is represented by a <u>hyperbola</u>.

By Fourier's law of conduction,

$$Q = \frac{4\pi K r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{t_1 - t_2}{\left[\frac{r_2 - r_1}{4\pi K r_1 r_2}\right]}$$

$$(R_{th})_{conduction} = \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

(B) Heat Conduction through a Composite Sphere

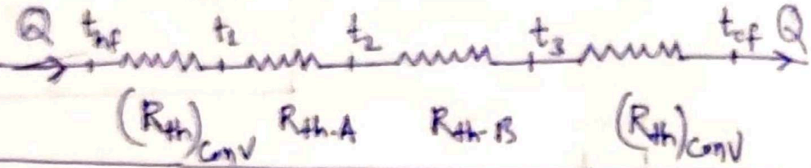


$$(R_{th})_{conv} = \frac{1}{h_{hf}\, 4\pi r_1^2}$$

$$(R_{th\text{-}A})_{cond.} = \frac{r_2 - r_1}{4\pi K_A r_1 r_2}$$

$$(R_{th\text{-}B})_{cond.} = \frac{r_3 - r_2}{4\pi K_B\, r_3 \cdot r_2}$$

$$(R_{th})_{conv} = \frac{1}{h_{cf} \cdot 4\pi r_3^2}$$



$$Q = \frac{(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} 4\pi r_1^2} + \frac{r_2 - r_1}{4\pi K_A r_1 \cdot r_2} + \frac{r_3 - r_2}{4\pi K_B r_2 \cdot r_3} + \frac{1}{h_{cf} 4\pi r_3^2}\right]}$$

* Logarithmic Mean Area for the Hollow Sphere

$$Q_{sphere} = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{4\pi K r_1 r_2}\right)}, \quad Q_{plane\ wall} = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{K A_M}\right)}$$

'A_m' is so chosen that the heat flow through a hollow cylinder and plane wall will be equal for the same temp. difference.

i.e., $Q_{sphere} = Q_{plane\ wall}$

$$\Rightarrow \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{4\pi K r_1 r_2}\right)} = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{K A_M}\right)}$$

$$\Rightarrow \boxed{A_M = 4\pi r_1 r_2 = 4\pi r_M^2} \quad (\because r_M^2 = r_1 \cdot r_2)$$

$$\Rightarrow A_m^2 = (4\pi r_1 r_2)^2 = 4\pi r_1^2 \times 4\pi r_2^2$$

$$\Rightarrow \boxed{A_m = \sqrt{A_i \cdot A_o}}$$

A_i = inner surface area of sphere
A_o = outer surface area of sphere
r_M = logarithmi mean radius of hollow sphere

Example 2.62. *A spherical thin walled metallic container is used to store liquid N_2 at – 196° C. The container has a diameter of 0.5 m and is covered with an evacuated reflective insulation composed of silica powder. The insulation is 25 mm thick and its outer layer is exposed to air at 27°C. The convective heat transfer coefficient on outer surface = 20 W/m²° C. Latent heat of evaporation of $N_2 = 2 \times 10^5$ J/kg. Density of N_2 = 804 kg/m³.*

k (silica powder) = 0.0017 W/m°C.

Find out the rate of heat transfer and rate of N_2 boil-off. **(N.U., 1998)**

Solution. *Given :* $t_1 = -196°C; t_2 = 27°C; r_1 = \frac{0.5}{2} = 0.25 \text{ m};$

$$r_2 = r_1 + 0.025 = 0.25 + 0.025 = 0.275 \text{ m};$$

$h_0 = 20 \text{ W/m}^{2}°\text{C}; h_{fg\,N_2} = 2 \times 10^5 \text{ J/kg}; \rho_{N_2} = 804 \text{ kg/m}^3;$

$k = 0.0017$ W/m°C.

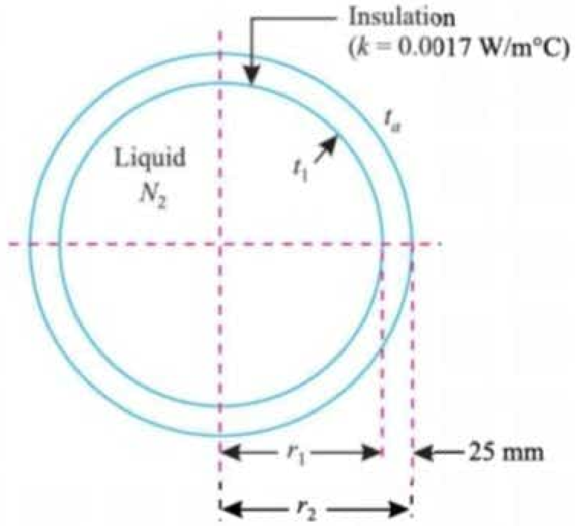


Fig. 2.79.

Rate of heat transfer, *Q* :

The heat flow is given by

$$Q = \frac{(t_1 - t_a)}{\frac{(r_2 - r_1)}{4\pi k\, r_1 r_2} + \frac{1}{h_0 \times 4\pi r_2^2}}$$

$$= \frac{(-196 - 27)}{\frac{(0.275 - 0.25)}{4\pi \times 0.0017 \times 0.25 \times 0.275} + \frac{1}{20 \times 4\pi \times 0.275^2}}$$

$$= \frac{-223}{17.022 + 0.0526} = -13.1 \text{ W}$$

The – ve sign indicates that the heat *flows in*.

$$\therefore \quad m_{N_2} \times h_{fg} = 13.1$$

or $$m_{N_2} = \frac{13.1}{2 \times 10^5} \times 3600 \text{ kg/h} = \mathbf{0.2358 \text{ kg/h}} \text{ (Ans.)}$$

Example 2.63. *Determine the rate of heat flow through a spherical boiler wall which is 2 m in diameter and 2 cm thick steel (k = 58 W/m K). The outside surface of boiler wall is covered with asbestos (k = 0.116 W/m K) 5mm thick. The temperature of outer surface and that of fluid inside are 50°C and 300°C respectively. Take inner film resistance as 0.0023 K/W.* **(N.U. Summer, 2000)**

Solution. *Given :* $r_1 = \frac{2}{2} = 1\,\text{m};\ r_2 = 1 + \frac{2}{100} = 1.02\,\text{m};\ k_A = 58$ W/m K;

$$k_B = 0.116 \text{ W/m k};\ r_3 = r_2 + \frac{5}{100} = 1.02 + 0.005 = 0.025\,\text{m}$$

$Q = h_i A_i (t_i - t_1)$ as heat flows from fluid to inner surface by convection only.

or,
$$Q = \frac{t_i - t_1}{\frac{1}{h_i A_i}}$$

where, $\frac{1}{h_i A_i}$ is inner film resistance.

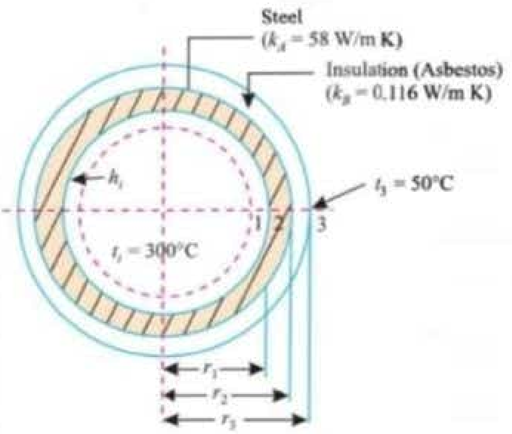


Fig. 2.80.

∴
$$Q = \frac{(t_i - t_3)}{\frac{1}{h_i A_i} + \frac{(r_2 - r_1)}{4\pi k_A r_1 r_2} + \frac{(r_3 - r_2)}{4\pi k_B r_2 r_3}}$$

$$= \frac{(300 - 50)}{0.0023 + \frac{(1.02 - 1.0)}{4\pi \times 58 \times 1.0 \times 1.02} + \frac{(1.025 - 1.02)}{4\pi \times 0.116 \times 1.02 \times 1.025}}$$

$$= \frac{250}{0.0023 + 2.6902 \times 10^{-5} + 0.0032808} = 44581\,\text{W} = \mathbf{4.581\,kW} \quad \textbf{(Ans.)}$$

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CYLINDER WITH UNIFORM HEAT GENERATION

Refer to Fig. 2.105. Consider a cylindrical rod in which one-dimensional radial conduction is taking place under steady state conditions.

Let, R = Radius of the rod,

L = Length of the rod,

k = Thermal conductivity (uniform),

q_g = Uniform volumetric heat generation per unit volume per unit time,

h = Heat transfer coefficient, and

t_a = Ambient temperature.

In order to obtain temperature distribution, consider an *element* of radius r and thickness dr as shown in Fig. 2.105.

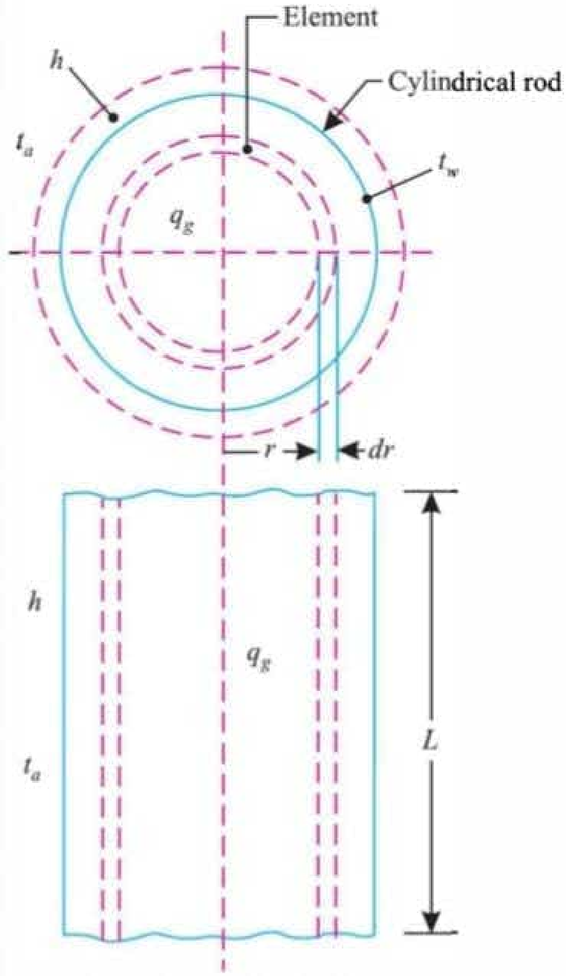


Fig. 2.105. Heat conduction in a solid cylinder with heat generation.

Heat conducted in at radius r,

$$Q_r = -k \,.\, 2\pi r L . \frac{dt}{dr}$$

Heat generated in the element,

$$Q_g = q_g . 2\pi r . dr . L$$

Heat conducted out at radius, $r + dr$,

$$Q_{(r+dr)} = Q_r + \frac{d}{dr}(Q_r)\,dr$$

Under steady state conditions,

$$Q_r + Q_g = Q_{(r+dr)}$$

$$= Q_r + \frac{d}{dr}(Q_r)\,dr$$

$$\therefore \quad Q_g = \frac{d}{dr}(Q_r)\,dr$$

$$q_g . 2\pi r . dr . L = \frac{d}{dr}\left[-k . 2\pi r L . \frac{dt}{dr}\right] dr$$

or $$\frac{d}{dr}\left[r . \frac{dt}{dr}\right] = -\frac{q_g}{k} \cdot r \qquad ...(2.105)$$

[Eqn. (2.105) may also be obtained from eqn. 2.22 assuming steady state uni-directional heat conduction in radial direction].

Intergrating the above equation twice, we obtain

$$r . \frac{dt}{dr} = -\frac{q_g}{k} \cdot \frac{r^2}{2} + C_1$$

or, $$\frac{dt}{dr} = -\frac{q_g}{k} \cdot \frac{r}{2} + \frac{C_1}{r} \qquad ...(2.106)$$

$$t = -\frac{q_g}{k} \cdot \frac{r^2}{4} + C_1 \log_e{}^r + C_2 \qquad ...(2.107)$$

(where C_1 and C_2 = constants of integration).

The constants C_1 and C_2 are evaluated from the boundary conditions, as follows :

(*i*) At $r = R$, $t = t_w$

(*ii*) Heat generated = Heat lost by conduction at the rod surface i.e. $$q_g \times (\pi R^2 \times L) = -k \times 2\pi RL \times \left[\frac{dt}{dr}\right]_{r=R}$$

Also, at $$r = 0,\ \frac{dt}{dr} = 0$$

Since in case of a cylinder, centre line is line of symmetry for temperature distribution and as such $\frac{dt}{dr}$ (temperature gradient) must be zero.

The temperature gradient $\left(\frac{dt}{dr}\right)$ at the surface (*i.e.* at $r = R$) is given by

$$\left[\frac{dt}{dr}\right]_{r=R} = -\frac{q_g}{k}\cdot\frac{R}{2} + \frac{C_1}{R}$$

Also from boundary condition (*ii*), we have

$$\left[\frac{dt}{dr}\right]_{r=R} = -\frac{q_g}{k}\cdot\frac{R}{2}$$

$$\therefore \quad -\frac{q_g}{k}\cdot\frac{R}{2} + \frac{C_1}{R} = -\frac{q_g}{k}\cdot\frac{R}{2} \quad \text{or} \quad C_1 = 0$$

Applying the boundary condition (*i*) [*i.e.* at $r = R$, $t = t_w$] to eqn. (2.107), we obtain,

$$t_w = -\frac{q_g}{k}\cdot\frac{R^2}{4} + C_2$$

or, $$C_2 = t_w + \frac{q_g}{k}\cdot\frac{R^2}{4}$$

Substituting the values of C_1 and C_2 in eqn. (2.107), we have the general solution for temperature distribution as

$$t = -\frac{q_g}{k}\cdot\frac{r^2}{4} + t_w + \frac{q_g}{k}\cdot\frac{R^2}{4}$$

or, $$t = t_w + \frac{q_g}{4k}[R^2 - r^2] \qquad \text{...(2.108)}$$

It is evident from eqn. (2.108) that temperature distribution is *parabolic* and the maximum temperature occurs at the centre of the rod ($r = 0$) and its value is given by

$$t_{max} = t_w + \frac{q_g}{4k}\cdot R^2 \qquad \text{...(2.109)}$$

By combining eqns. (2.108) and (2.109), we arrive at the following dimensionless form of temperature distribution :

$$\frac{t - t_w}{t_{max} - t_w} = \frac{\frac{q_g}{4k}(R^2 - r^2)}{\frac{q_g}{4k}\cdot R^2} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

i.e. $$\frac{t - t_w}{t_{max} - t_w} = 1 - \left(\frac{r}{R}\right)^2 \qquad \text{...(2.110)}$$

Also, energy generated within the rod (per unit time)

= Energy dissipated (per unit time) by convection at the rod boundary

i.e. $$q_g \times (\pi R^2 \times L) = h \times 2\pi RL\,(t_w - t_a)$$

or, $$t_w = t_a + \frac{q_g}{2h}.R \qquad \text{...(2.111)}$$

Inserting the value of t_w in eqn. (2.108), we obtain the temperature distribution (in terms of t_a) as

$$t = t_a + \frac{q_g}{2h}\cdot R + \frac{q_g}{4k}[R^2 - r^2] \qquad \text{...(2.112)}$$

The value of t_{max}, at $r = 0$, is given by

$$t_{max} = t_a + \frac{q_g}{2h}\cdot R + \frac{q_g}{4k}\cdot R^2 \qquad \text{...(2.113)}$$

Example 2.88. *A current of 300 amperes passes through a stainless steel wire of 2.5 mm diameter and k = 20 W/m°C. The resistivity of the wire is 70 × 10⁻⁸ Ωm and the length of the wire is 2m. If the wire is submerged in a fluid maintained at 50°C and convective heat transfer coefficient at the wire surface is 4000 W/m²°C, calculate the steady state temperature at the centre and at the surface of wire.* **(M.U)**

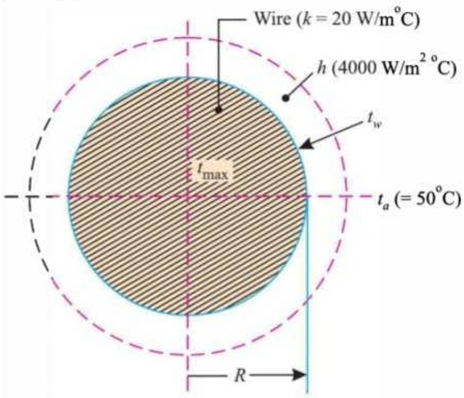


Fig. 2.106.

Solution. Refer to Fig. 2.106.

$$R = \frac{2.5}{2} = 1.25\,\text{mm} = 0.00125\,\text{m};$$

$k = 20$ W/m°C,

resistivity, $\rho = 70 \times 10^{-8}$ Ωm

$L = 2$m, $t_a = 50$°C, Current, $I = 300$ amp.

Temperature at the surface of wire (t_w) and at the centre of wire (t_{max}) :

Rate of heat generation,

$$Q_g = I^2 R_e = I^2 \times \frac{\rho L}{A}$$

(where R_e = electrical resistance)

Rate of heat generation per unit volume,

$$q_g = \frac{Q_g}{AL} = I^2 \times \frac{\rho L}{A} \times \frac{1}{AL} = \rho\left(\frac{I}{A}\right)^2$$

$$= 70 \times 10^{-8}\left[\frac{300}{\pi \times 0.00125^2}\right]^2$$

$$= 26.14 \times 10^8 \text{ W/m}^3$$

Temperature at the surface of wire is given by

$$t_w = t_a + \frac{q_g}{2h} \cdot R \quad \text{...[Eqn. (2.111)]}$$

or, $$t_w = 50 + \frac{26.14 \times 10^8}{2 \times 4000} \times 0.00125 = \mathbf{458.44°\,C} \quad \textbf{(Ans.)}$$

Temperature at the centre of wire is given by

$$t_{max} = t_w + \frac{q_g}{4k} \cdot R^2 \quad \text{[Eqn. (12.109)]}$$

or, $$t_{max} = 458.44 + \frac{26.14 \times 10^8}{4 \times 20} \times (0.00125)^2 = \mathbf{509.5°\,C} \quad \textbf{(Ans.)}$$

Example 2.89. *A 3 mm diameter stainless steel wire (k = 20 W/m°C, resistivity, ρ = 10 × 10⁻⁸ Ωm) 100 metres long has a voltage of 100 V impressed on it. The outer surface of the wire is maintained at 100°C. Calculate the centre temperature of the wire. If the heated wire is submerged in a fluid maintained at 50°C, find the heat transfer coefficient on the surface of the wire.* **(M.U.)**

Solution. Radius of stainless steel wire, $R = \frac{3}{2} = 1.5 \text{ mm} = 0.0015 \text{ m}$

Length of the wire, $L = 100$ m

Voltage impressed $= 100$ V

Thermal conductivity, $k = 20$ W/m°C

Resistivity, $\rho = 10 \times 10^{-8}$ Ωm

The temperature of the outer surface of the wire,

$t_w = 100$°C

Fluid temperature, $t_a = 50$°C.

Centre temperature of the wire, t_{max} :

Electrical resistance of the wire, $R_e = \frac{\rho L}{A} = \frac{10 \times 10^{-8} \times 100}{\pi \times 0.0015^2} = 1.415\,\Omega$

Rate of heat generation, $Q_g = VI = \frac{V^2}{R_e} = \frac{100^2}{1.415} = 7067$ W

∴ Rate of heat generation per unit volume

$$q_g = \frac{Q_g}{AL} = \frac{7067}{\pi \times 0.0015^2 \times 100} = 9.998 \times 10^6 \text{ W/m}^3$$

The centre temperature is given by

$$t_{max} = t_w + \frac{q_g}{4k} \cdot R^2 \quad \text{...[Fig. (2.109)]}$$

$$= 100 + \frac{9.998 \times 10^6}{4 \times 20} \times 0.0015^2 = 100.28°\text{C}$$

Heat transfer coefficient, h :

$$t_w = t_a + \frac{q_g}{2h} \cdot R \quad \text{...[Eqn. (2.111)]}$$

$$100 = 50 + \frac{9.998 \times 10^6}{2h} \times 0.0015$$

or, $$(100 - 50) = \frac{7498.5}{h}$$

∴ $$h = \frac{7498.5}{50} = \mathbf{149.97\ W/m^2\,°C} \quad \textbf{(Ans.)}$$

Problem-1 :- The ~~metal~~ meat rolls of 25 mm diameter having $K = 1\ W/m^\circ C$ are heated up with the help of microwave heating for roasting. The centre temp. of the rolls is maintained at 100°c when the surrounding temp is 30°c. The heat transfer co-efficient on the surface of the meat roll is $20\ W/m^2\ ^\circ C$. Find, the microwave heating capacity required in W/m^3.

Solution:- $K = 1\ W/m^\circ C$, $h = 20\ W/m^2\ ^\circ C$, $t_{max} = 100^\circ C$

$R = \frac{25}{2} = 12.5\ mm = 0.0125\ m$, $t_a = 30^\circ C$

The maximum temperature occurs at the centre and is given by

$$t_{max} = t_a + \frac{q_g}{2h} R + \frac{q_g}{4K} R^2$$

$$\Rightarrow \quad 100 = 30 + q_g \left(\frac{0.0125}{2 \times 20} + \frac{0.0125^2}{4 \times 1} \right)$$

$$\Rightarrow \quad q_g = 1.991 \times 10^5\ W/m^3$$

Ans

Problem-1: The resistance wire of a 1200 W hair dryer is 80 cm long and has a diameter of D = 0.3 cm. Determine the rate of heat generation in the wire per unit volume, in W/cm^3, and the heat flux on the outer surface of the wire as a result of this heat generation.

Solution:- $\dot{Q} = 1200\,W$, $D = 0.3\,cm$, $L = 80\,cm$

$\dot{q}_{gen} = ?$

$q = \frac{Q}{A} = ?$

Rate of heat generation in the wire per unit volume, $\dot{q}_{gen} = \frac{\text{Power consumption of resistance wire}}{\text{Volume of the wire}}$

$$\Rightarrow \dot{q}_{gen} = \frac{\dot{Q}}{\frac{\pi D^2}{4} \times L} = \frac{1200}{\frac{\pi (0.3)^2}{4} \times 80} = 212\ W/cm^3$$

Ans

Heat flux on the outer surface of wire, $q = \frac{\dot{Q}}{\text{Outer surface area}}$

$$\Rightarrow q = \frac{1200}{\pi D L} = \frac{1200}{\pi \times 0.3 \times 80} = 15.9\ W/cm^2$$

Ans

Imp

Problem-2: (Composite Wall) : An exterior wall of a house may be approximated by a 4-in layer of common brick ($K = 0.7\ W/m\cdot°c$) followed by a 1.5-in layer of gypsum plaster ($K = 0.48\ W/m\cdot°c$). What thickness of loosely packed rock-wool insulation ($K = 0.065\ W/m\cdot°c$) should be added to reduce the heat loss (or gain) through the wall by 80 percent?

Sol?: Overall heat loss from the wall, $\dot{Q}_{Without\ ins.} = \frac{\Delta t}{\Sigma R_{th}}$

It is given that,

$$\dot{Q}_{With\ ins} = 0.20 \times \dot{Q}_{Without\ ins} \qquad \left(\because \frac{Q_{without} - Q_{with}}{Q_{without}} = 80\% = 0.80\right)$$

(i.e., the heat loss with the rock-wool insulation will be only 20%)

$$\Rightarrow \frac{\dot{Q}_{With\ ins}}{\dot{Q}_{Without\ ins}} = 0.20$$

$$\Rightarrow \frac{(\Sigma R_{th})_{Without\ ins}}{(\Sigma R_{th})_{with\ ins}} = 0.20$$

$$\Rightarrow (\Sigma R_{th})_{with\ ins} = \frac{(\Sigma R_{th})_{without\ ins}}{0.20}$$

$$\left(\text{for unit area, } A = 1\,m^2\right) = \frac{\left(\frac{L}{KA}\right)_{brick} + \left(\frac{L}{KA}\right)_{plaster}}{0.20}$$

$$(1\text{ inch} = 2.54\text{cm} = 0.0254\text{m}) = \frac{\left(\frac{4\times 0.0254}{0.7}\right) + \left(\frac{1.5\times 0.0254}{0.48}\right)}{0.20}$$

$$= 0.224/0.20$$

$$\Rightarrow (\Sigma R_{th})_{with\ ins} = 1.122\ m^2\text{-}°C/W$$

$$\Rightarrow (\Sigma R_{th})_{without\ ins} + (R_{th})_{rw} = 1.122$$

$$\Rightarrow 0.224 + (R_{th})_{rw} = 1.122$$

$$\Rightarrow (R_{th})_{rw} = 0.898$$

$$\Rightarrow \frac{L}{K} = 0.898$$

$$\Rightarrow \frac{L}{0.065} = 0.898$$

$$\Rightarrow L = 0.0584\,m \text{ or } 2.30 \text{ in}$$ Ans

SPHERE WITH UNIFORM HEAT GENERATION

Consider one dimensional radial conduction of heat, under steady state conditions, through a sphere having uniform heat generation.

Let, R = Outside radius of sphere,

k = Thermal conductivity (uniform),

q_g = Uniform heat generation per unit volume, per unit time within the solid,

t_w = Temperature of the outside surface (wall) of the sphere, and

t_a = Ambient temperature.

Consider an element at radius r and thickness, dr as shown in Fig. 2.119.

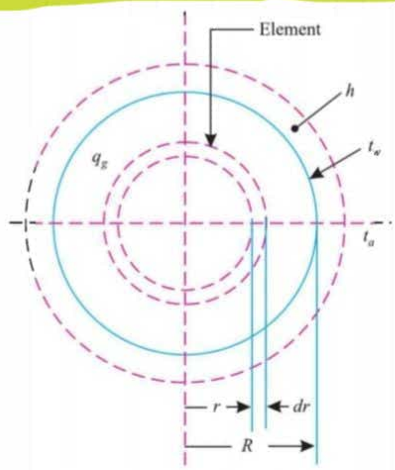


Fig. 2.119. Sphere with uniform heat generation.

Heat conducted in at radius r,

$$Q_r = -kA\frac{dt}{dr} = -k \times 4\pi r^2 \cdot \frac{dt}{dr}$$

Heat generated in the element,

$$Q_g = q_g \times A \times dr = q_g \times 4\pi r^2 \times dr$$

Heat conducted out at radius $(r + dr)$

$$Q_{(r+dr)} = Q_r + \frac{d}{dr}(Q_r)\, dr$$

Under steady state conditions, we have

$$Q_r + Q_g = Q_{(r+dr)}$$

$$= Q_r + \frac{d}{dr}(Q_r).dr$$

or, $$Q_g = \frac{d}{dr}(Q_r)\, dr$$

or, $$q_g \times 4\pi r^2 \times dr = \frac{d}{dr}\left[-4\pi k\, r^2 \cdot \frac{dt}{dr}\right] dr$$

or, $$q_g \times 4\pi r^2 \times dr = -4\pi k \frac{d}{dr}\left[r^2 \cdot \frac{dt}{dr}\right] dr$$

or, $$\frac{1}{r^2}\frac{d}{dr}\left[r^2 \cdot \frac{dt}{dr}\right] + \frac{q_g}{k} = 0 \qquad ...(i)$$

(Heat flow equation)

or, $$\frac{1}{r^2}\left[r^2 \cdot \frac{d^2t}{dr^2} + 2r \cdot \frac{dt}{dr}\right] + \frac{q_g}{k} = 0$$

or, $$\frac{d^2t}{dr^2} + \frac{2}{r} \times \frac{dt}{dr} + \frac{q_g}{k} = 0$$

or, $$r\frac{d^2t}{dt^2} + 2\frac{dt}{dr} + \frac{q_g r}{k} = 0 \qquad \text{(multiplying both sides by } r\text{)}$$

or, $$r\frac{d^2t}{dr^2} + \frac{dt}{dr} + \frac{dt}{dr} + \frac{q_g r}{k} = 0$$

or, $$\frac{dt}{dr}\left(r\frac{dt}{dr}\right) + \frac{dt}{dr} + \frac{q_g r}{k} = 0$$

Integrating both sides, we have

$$r\frac{dt}{dr} + t + \frac{q_g}{k} \cdot \frac{r^2}{2} = C_1 \qquad ...(ii)$$

or, $$\frac{d}{dr}(rt) + \frac{q_g}{k} \cdot \frac{r^2}{2} = C_1$$

Integrating again, we have

$$rt + \frac{q_g}{k} \cdot \frac{r^3}{6} = C_1 r + C_2 \qquad ...(iii)$$

(where C_1, C_2 = constants of integration).

At the centre of sphere, $r = 0$ $\quad \therefore \quad C_2 = 0$ [From eqn. (*iii*)]

Applying boundary condition, at $r = R$, $t = t_w$ to eqn. (*iii*), we have

$$Rt_w + \frac{q_g}{k} \cdot \frac{R^3}{6} = C_1 R \qquad (\because C_2 = 0)$$

or, $$C_1 = t_w + \frac{q_g}{6k} R^2$$

By substituting the values of C_1 and C_2 in eqn. (*iii*), we have the temperature distribution as

$$rt + \frac{q_g}{k} \cdot \frac{r^3}{6} = \left[t_w + \frac{q_g}{6k} R^2\right] r$$

or, $$t + \frac{q_g}{6k} \cdot r^2 = t_w + \frac{q_g}{6k} R^2$$

or, $$t = t_w + \frac{q_g}{6k}(R^2 - r^2) \qquad ...(2.122)$$

or, $$t = t_w + \frac{q_g}{6k}(R^2 - r^2) \quad ...(2.122)$$

From eqn. (2.122) it is evident that the temperature distribution is *parabolic*; the maximum temperature occurs at the centre ($r = 0$) and its value is given by

$$t_{max} = t_w + \frac{q_g}{6k}R^2 \quad ...(2.123)$$

From eqns. (2.122) and (2.123), we have

$$\frac{t - t_w}{t_{max} - t_w} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

i.e., $$\frac{t - t_w}{t_{max} - t_w} = 1 - \left(\frac{r}{R}\right)^2 \quad ...(2.124)$$

(temperature distribution in *dimensionless form*)

Invoking Fourier's equation (to evaluate heat flow), we have

$$Q = -kA\left(\frac{dt}{dr}\right)_{r=R}$$

$$= -k \times 4\pi R^2 \times \frac{d}{dr}\left[t_w + \frac{q_g}{6k}(R^2 - r^2)\right]_{r=R}$$

[substituting the value of t from eqn. (2.122)]

$$= -k \times 4\pi R^2 \left[\frac{q_g}{6k}(-2r)\right]_{r=R} = k \times 4\pi R^2 \times \frac{q_g}{3k} \cdot R$$

or, $$Q = \frac{4}{3}\pi R^3 \times q_g$$

(= volume of sphere × heat generation capacity) ...(*iv*)

Thus heat conducted is equal to heat generated. Under steady state conditions the heat conducted (or generated) should be equal to the heat convected from the outer surface of the sphere.

i.e., $$q_g \times \frac{4}{3}\pi R^3 = h \times 4\pi R^2 (t_w - t_a)$$

or, $$t_w = t_a + \frac{q_g R}{3h} \quad ...(2.125)$$

Inserting this value of t_w in eqn. 2.122, we have

$$t = t_a + \frac{q_g R}{3h} + \frac{q_g}{6k}(R^2 - r^2) \quad ...(2.126)$$

The maximum temperature, $t_{max} = t_a + \frac{q_g}{3k} \cdot R + \frac{q_g}{6k} \cdot R^2$ (at $r = 0$) ...(2.127)

Example 2.104. *An approximately spherical shaped orange (k = 0.23 W/m°C), 90 mm in diameter, undergoes riping process and generates 5100 W/m³ of energy. If external surface of the orange is at 8°C, determine :*

(i) Temperature at the centre of the orange, and

(ii) Heat flow from the outer surface of the orange.

Solution. Outside radius of the orange, $R = \frac{90}{2} = 45$ mm = 0.045 m

Rate of heat generation, $q_g = 5100$ W/m³

The temperature at the outer surface of the orange, $t_w = 8$°C

(*i*) Temperature at the centre of the orange, t_{max} :

$$t_{max} = t_w + \frac{q_g}{6k}R^2 \quad ...[\text{Eqn. } (2.123)]$$

or, $$t_{max} = 8 + \frac{5100}{(6 \times 0.23)} \times (0.045)^2 = \mathbf{15.48°C} \quad \textbf{(Ans.)}$$

(*ii*) Heat flow from the outer surface of the orange, Q :

Heat conducted = Heat generated

∴ $$Q = q_g \times \frac{4}{3}\pi R^3$$

or, $$Q = 5100 \times \frac{4}{3}\pi \times (0.045)^3 = \mathbf{1.946\ W} \quad \textbf{(Ans.)}$$

Problem-1: A solid sphere of radius 0.5m has an internal heat generation rate of 2×10^6 W/m³. If the thermal conductivity of the material is 40 W/m-K and the convective heat transfer co-efficient at the surface of the sphere is 10 W/m²K, calculate the temperatures at the outer surface and at the centre. Take ambient temp. as 30°C.

Solution: Rate of heat generation, $Q = q_g \times \frac{4}{3}\pi R^3$

$$= 2 \times 10^6 \times \frac{4}{3}\pi \times (0.5)^3$$

$$= 1.047 \times 10^6 \text{ W}$$

Also, $Q = h \cdot A (t_w - t_a)$

$$\Rightarrow 1.047 \times 10^6 = 10 \times 4\pi (0.5)^2 \times (t_w - 30)$$

$$\Rightarrow t_w = 63.32^\circ C \quad \text{Ans}$$

Temperature at the centre (i.e., $r = 0$)

$$t_{max} = t_w + \frac{q_g}{6K} R^2 = 63.32 + \frac{2 \times 10^6}{6 \times 40} (0.5)^2$$

$$= 2146.65^\circ C \quad \text{Ans}$$

Problem-2: A 3 mm diameter stainless steel wire ($K = 20$ W/m°C, resistivity, $\rho = 10 \times 10^{-8}\ \Omega m$), 100 metres long has a voltage of 100V impressed on it. The outer surface of the wire is maintained at 100°C. Calculate the centre temp. of the wire. If the heated wire is submerged in a fluid maintained at 50°C, find the heat transfer co-efficient on the surface of the wire.

Solution: Centre temp. of the wire, t_{max}:

Electrical resistance of the wire, $R_e = \rho \frac{L}{A} = 10 \times 10^{-8} \times \frac{100}{\pi (0.0015)^2}$

$$= 1.415\ \Omega$$

Rate of heat generation, $Q = VI = \frac{V^2}{R_e} = \frac{100^2}{1.415} = 7067 \text{ W}$

$$\therefore q_g = \frac{Q}{AL} = \frac{7067}{\pi \times 0.0015^2 \times 100} = 9.998 \times 10^6 \text{ W/m}^3$$

The centre temp. is given by, $t_{max} = \frac{q_g}{4K} R^2 + t_w$

$$= \frac{9.998 \times 10^6}{4 \times 20} \times (0.0015)^2 + 100$$

$$= 100.28^\circ C \quad \text{Ans}$$

Heat transfer co-efficient, h

Wall temp., $t_w = t_a + \frac{q_g}{2h} \cdot R$

$\Rightarrow 100 = 50 + \frac{9.998 \times 10^6}{2 \times h} \times 0.0015$

$\Rightarrow \boxed{h = 149.97 \, W/m^2 \, ^\circ C}$ Ans

or $Q = hA(t_w - t_a)$

$\Rightarrow 7067 = h \times \pi (0.0015)^2 \times (100-50)$

$\Rightarrow h = 149.97 \, W/m^2 \, ^\circ C$

Problem-3: An electric current of 34,000A flows along a flat steel plate 12.5 mm thick and 100 mm wide. Calculate the internal heat generation in the plate, if the resistivity of steel is $12 \times 10^{-6} \, \Omega$-cm and $K = 52.4 \, W/m\text{-}K$.

Solution:- Current density, $J = \frac{I}{A} = \frac{34000}{(100 \times 12.5) \times 10^{-6}}$

$= 2.72 \times 10^7 \, A/m^2$

Internal heat generation, $q_g = J^2 \rho = (2.72 \times 10^7)^2 \times 12 \times 10^{-8}$

$= 88.78 \times 10^6 \, W/m^3$ Ans

Imp

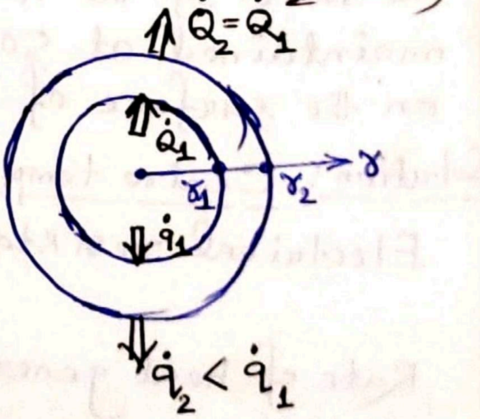
Question: The steady state temperature profile in hollow cylinders when heat flow radially with isothermal surface and thickness of cylinders is almost negligible

(a) Linear (b) Logarithmic

(c) Hyperbolic (d) Parabolic

Ans: (a)

Imp During steady, 1-D heat conduction in a spherical or cylindrical container, the total rate of heat transfer remains constant (i.e., $\dot{Q}_2 = \dot{Q}_1$) but the heat flux decreases ($\dot{q}_2 < \dot{q}_1$) with increasing radius ($r_2 > r_1$).



$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.1 \, kW}{4\pi (0.08 \, m)^2} = 337 \, kW/m^2$

$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.1 \, kW}{4\pi (0.10 \, m)^2} = 216 \, kW/m^2$

$\dot{q}_1 > \dot{q}_2$

Important Note :

* **Internal Heat Generation (q_g)**

Wall or Surface Temp.

$$(t_w)_{\text{Plane wall (L)}} = t_a + \frac{q_g}{2h} L$$

$$(t_w)_{\text{cylinder}} = t_a + \frac{q_g}{2h} R$$

$$(t_w)_{\text{sphere}} = t_a + \frac{q_g}{3h} R$$

Max. temp rise betⁿ. the surface and a mid-section of a medium

$$(t_{Max})_{\text{Plane wall (L)}} = \frac{q_g}{8K} L^2 + t_w$$

$$(t_{Max})_{\text{cylinder}} = \frac{q_g}{4K} \cdot R^2 + t_w$$

$$(t_{Max})_{\text{sphere}} = \frac{q_g}{6K} R^2 + t_w$$

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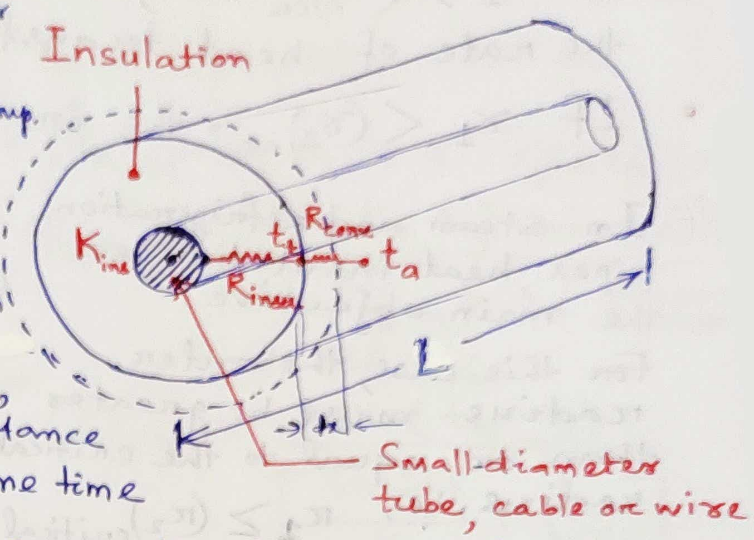
Critical Radius of Insulation

The thickness of insulation (or, radius) upto which heat flow increases and after which heat flow decreases termed as Critical thickness of insulation (or, some times, critical radius of insulation).

Let us consider a small-diameter tube, cable or wire, the outside surface of which has a constant temp. and dissipates heat by convection into the surrounding air. Let the surface be covered with a layer of insulation. As the insulation is added to the tube, the outer exposed surface temp. will decrease, because of higher conduction resistance ($R_{th} \propto dx$; $\because R_{th} = \frac{dx}{KA}$), but at the same time the surface area available for convective heat transfer/dissipation will increase causing more heat loss. These two opposing effects lead to an optimum insulation thickness.



As 'r_2' increases, $(r_2 - r_1)$ will increase because of which

$$\dot{Q}_{conduction} = \frac{(t_t - t_a)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L}\right]}$$ decreases

and simultaneously, $\dot{Q}_{conv.} = (2\pi r_2 L) \cdot h_a (t_t - t_a)$ increases

Thus, we have to choose an optimum value of 'r_2' upto which $\dot{Q}$ increases and after which $\dot{Q}$ decreases.

$$\dot{Q} = \frac{(t_t - t_a)}{\left[\frac{\ln(r_2/r_1)}{2\pi K L} + \frac{1}{h_a(2\pi r_2 L)}\right]} = \frac{2\pi L (t_t - t_a)}{\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_a r_2}\right]}$$

Thus, $\dot{Q}$ becomes maximum, when the denominator becomes minimum. The required condition is

$$\frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_a r_2}\right] = 0$$

Cond^n of Minimum resistance and consequently Max^m heat flow rate.

or $\boxed{r_2 = \frac{K}{h_a}}$ or $\boxed{r_c = \frac{K}{h_a}}$

The critical radius (r_2 or r_c) is dependent on the thermal quantities K & h and is independent of tube/wire radius (i.e, r_1).

- Therefore at $r_2 = r_c$, the heat loss will be maximum.

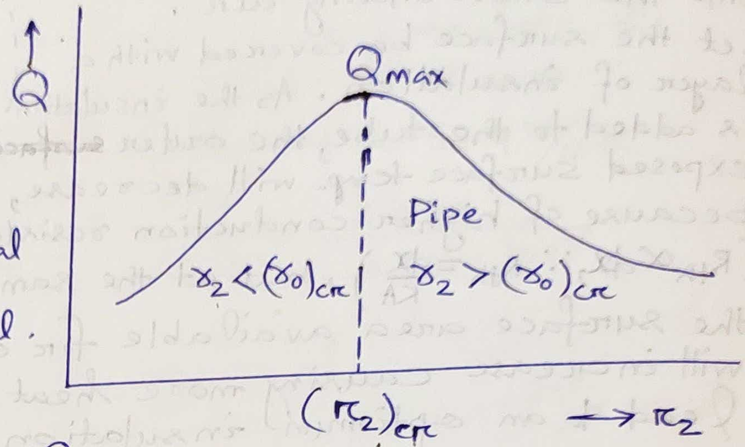
If $r_1 < (r_2)_{cr}$, as r_2 increases, Q increases till $r_2 = (r_2)_{cr}$

If $r_1 > (r_2)_{cr}$, as r_2 increases, Q decreases

- If $r_1 > (r_2)_{cr}$, any increase of insulation will decrease the rate of heat transfer.
- If $r_1 < (r_2)_{cr}$, the increase of insulation will increase Q till $Q = Q_{max}$.

In steam and refrigeration pipes, heat insulation is the main objective.

For this case, the outer radius must be greater than or equal to the critical radius, i.e., $r_1 \geq (r_2)_{critical}$.



Variation of insulation radius influencing heat loss to the outside

For wires and cables, r_1 is kept lower than $(r_2)_{cr}$ so that insulation added increases the heat loss from the wire or cable.

Therefore, an insulated small diameter wire has a higher current carrying capacity than an uninsulated one.

If the current flowing through an uninsulated wire increases I^2R increases and if heat dissipation from the wire is not equal to I^2R, the temp of wire goes on increasing till it exceeds the melting point and the wire snaps. If the wire is insulated, it can dissipate more heat [till $r_1 = (r_2)_{cr}$] and the wire temp. remains below the melting point.

For a sphere, critical radius of insulation $(r_2)_{cr} = \frac{2k}{h_a}$

* If the effect of thermal radiation taken into account

$$(r_2)_{cr} = \frac{k}{h_a + h_r} \quad \text{for a cylinder}$$

$$(r_2)_{cr} = \frac{2k}{h_a + h_r} \quad \text{for a sphere}$$

Problem-1: A wire of 6.5 mm diameter at 60°C is covered with insulation of $K = 0.174 \text{ W/m·K}$. $h_{air} = 8.722 \text{ W/m}^2\text{K}$, $t_{air} = 20°C$
Find: (i) critical radius of insulation, (ii) critical thickness of insulation and (iii) percentage increase in heat loss due to insulation.

Solution:– (i) For maximum heat loss, the critical radius of insulation,

$$(r_2)_{cr} = \frac{K}{h_a} = \frac{0.174}{8.722} = 0.1995 \text{ m} = 19.95 \text{ mm}$$

(ii) Critical thickness of insulation $= (r_2)_{cr} - r_1 = 19.95 - \frac{6.5}{2} = 16.70 \text{ mm}$

(iii) Heat loss without insulation, $Q_{without} = \dfrac{2\pi L(t_w - t_a)}{\left(\frac{1}{h_a r_1}\right)} = \dfrac{2\pi \times 1\,(60-20)}{\left(\frac{1}{8.722 \times 0.00325}\right)}$

$= 7.124 \text{ W/m}$

Heat loss (maximum) with insulation, $Q_{with} = \dfrac{2\pi L(t_w - t_a)}{\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_a r_1}\right]}$

$= 15.537$

∴ %ge increase in heat loss

$$= \frac{Q_{with} - Q_{without}}{Q_{without}} \times 100$$

$$= \frac{15.537 - 7.124}{7.124} \times 100$$

$= 118.1 \%$ **Ans**

Q.2 An insulated steam pipe is to be covered with two layers of insulation, each having same thickness; one material is having thermal conductivity 5 times that of the other material.
In order to decrease the heat loss from the steam pipe

Better insulation is to be put next to the pipe (✓)

or

Material with ~~higher~~ 'K' is to be put next to the pipe (✗)

Q.3 Critical radius of insulation implies that

(a) the conduction (insulation) thermal resistance decreases and simultaneously convective thermal resistance increases with increasing insulation radius 'r_2'.

(b) Only heat conduction increases with increasing r_2

(c) only heat loss by convection increases with increases with increasing r_2

(d) The rate of heat conduction through the insulation is same that of convection from the surface of the pipe.

(e) at $r_2 = r_{cr}$, the rate of increase of conductive resistance of insulation is equal to the rate of decrease of convective resistance, thus giving a minimum value for the sum of thermal resistances.

Ans: (e)

Problem-4: A 10 mm cable is to be laid in atmosphere of 20°C with outside heat transfer co-efficient 8.5 W/m²°C. The surface temperature of cable is likely to be 65°C due to heat generation within. Will the rubber insulation with K = 0.155 W/m°C, be effective? If yes, how much?

Solution:- For a cable (cylinder), $r_c = \frac{K}{h} = \frac{0.155}{8.5} = 0.018235\,m$

$= 18.235\ mm$

Here $r_2 = r_c$ is greater than the radius of cable.
Hence, rubber insulation upto a thickness of 13.235 mm (18.235 − 5) will be effective in heat dissipation. Ans

∴ Max^m heat transfer, $Q_{max} = \dfrac{2\pi L(t_c - t_{air})}{\left[\dfrac{\ln(r_2/r_1)}{K} + \dfrac{1}{h_a \times r_2}\right]}$

$= \dfrac{2\pi \times 1 \times (65-20)}{\left[\dfrac{\ln\left(\frac{18.235}{5}\right)}{0.155} + \dfrac{1}{8.5 \times 0.018235}\right]}$

$= 19.1\ W/m$

Q.5: If the radius of a current carrying wire is less than the critical radius, then the addition of electrical insulation will enable the wire to carry a higher current because,

(a) the thermal resistance of the insulation is reduced
(b) the thermal resistance of the insulation is increased
(c) the heat loss from the wire would decrease
(d) the heat loss from the wire would increase. Ans: (d)

imp
If we increase thickness of insulation inside in a given pipe or sphere, then

→ Both Conduction resistance and Convection resistance increase

CHAPTER - 3

Extended Surfaces : Fins

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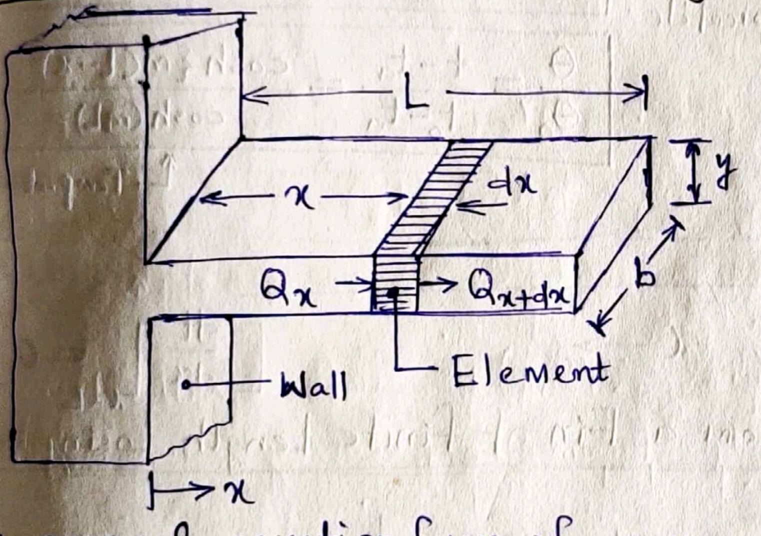
Extended Surfaces : Fins

When the available surface is inadequate to transfer required quantity of heat with the ~~avaia~~ available temp. drop and convective heat transfer co-efficient, extended surfaces or fins are used.

Applications of Fins

(i) Engines of automobiles
(ii) Radiator of automobiles
(iii) Electric Motor bodies
(iv) Economisers and Steam power plants
(v) Small capacity compressors
(vi) Transformers and other electric equipments

Heat flows Through Rectangular Fins



$A_{fin} = by + 2bL \approx 2bL$

$A_{cs} = b \times y$

$P = 2(b+y)$

$A_{fin\,(Lateral)} = PL$

$m = \sqrt{\frac{hP}{A_{cs}K}}$

The general ~~equation~~ form of energy equation for one-dimensional heat dissipation from an extended surface (fin), is given by

$$\frac{d^2t}{dx^2} - \frac{Ph}{A_{cs}K}(t - t_a) = 0 \quad \text{or} \quad \frac{d^2t}{dx^2} - m^2\theta = 0$$

The general solution of this homogeneous second order differential equ? is of the form :

$$\theta = (t - t_a) = C_1 \cdot e^{mx} + C_2 e^{-mx}$$

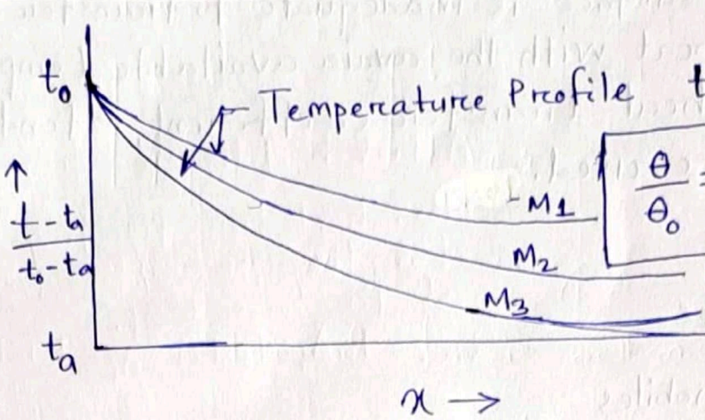
The following three cases of fins are to be considered here.

CASE-I : The fin is infinitely long and the temp. at the end of fin is essentially that of the ambient/surrounding fluid.

CASE-II : The end of the fin is insulated (Adiabatic fin)

CASE-III : The fin of finite length and loses heat by Convection.

① Heat Dissipation from an Infinitely Long fins ($L \to \infty$)



$$t = t_a + (t_0 - t_a)e^{-Mx}$$

$$\frac{\theta}{\theta_0} = \frac{t-t_a}{t_0-t_a} = e^{-Mx}$$

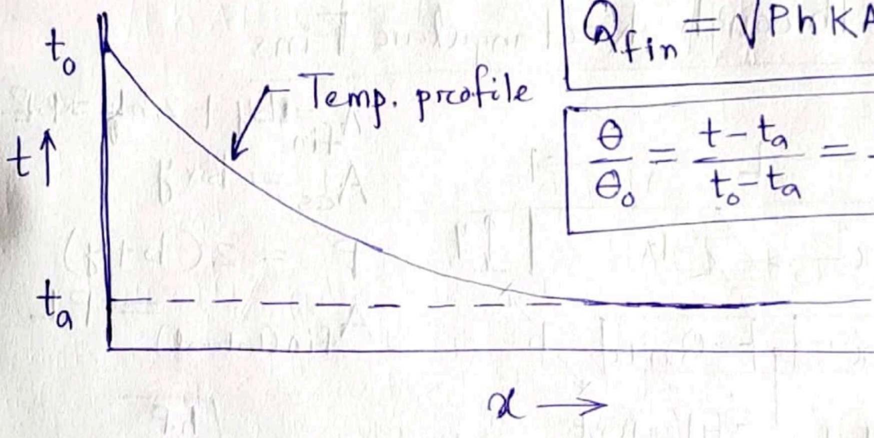
* As the value of 'M' increases, the dimensionless temp. falls.

$t = t_a$ (at $x = \infty$)

$$Q_{Fin} = \sqrt{PhKA_{cs}}\,(t_0 - t_a)$$

* As the length of fin increases to infinity(∞), all the curves approaches $\frac{t-t_a}{t_0-t_a} = 0$ asymptotic

② Heat dissipation from a fin insulated at the tip



$$Q_{fin} = \sqrt{PhKA_{cs}}\,(t_0 - t_a)\cdot\tanh(mL)$$

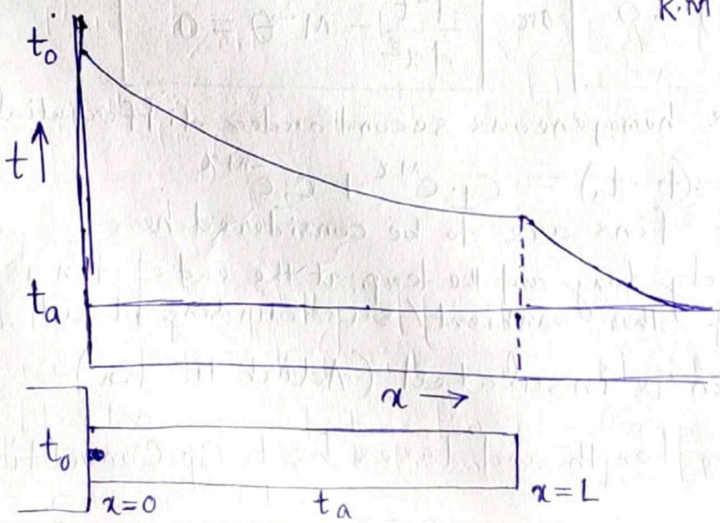
$$\frac{\theta}{\theta_0} = \frac{t-t_a}{t_0-t_a} = \frac{\cosh\{m(L-x)\}}{\cosh(mL)}$$

↳ Temp. distribut

$$\left[\frac{dt}{dx}\right]_{x=L} = 0$$

③ Heat dissipation from a Fin of finite Length Losing heat by Convection

$$Q_{fin} = \sqrt{PhKA_{cs}}\,(t_0 - t_a)\,\frac{\tanh(mL) + \frac{h}{K\cdot M}}{1 + \frac{h}{K\cdot M}\cdot\tanh(mL)}$$



Efficiency of Fin (η_{Fin}): The efficiency of a fin is defined as the ratio of actual heat transferred by the fin to the maximum heat transferred by the fin if the entire fin area were at base temp.

i.e. $$\boxed{\eta_{Fin} = \frac{Q_{actual}}{Q_{Max^m}}} = \frac{\text{Actual heat transfer from fin}}{\text{Max}^m \text{ heat transfer from fin if entire fin surface were at fin base temp.}}$$

$Q_{max} = h.A_f(t_o - t_a)$, A_f = total surface area of the fin

* **Infinitely long fin:**

$$\eta_{Fin} = \frac{\sqrt{PhKA_{cs}}\,(t_o - t_a)}{hPL\,(t_o - t_a)} = \frac{1}{mL}$$

* **Fin With Insulated tip at the end:**

For very thin rectangular fin,
$\therefore P = 2b + 2y \approx 2b$
$A_{cs} = b \cdot y$
$\therefore m = \left(\frac{h \times 2b}{Kby}\right)^{1/2} = \left(\frac{2h}{Ky}\right)^{1/2}$

$$\eta_{Fin} = \frac{\sqrt{PhKA_{cs}}\,(t_o - t_a).\tanh(mL)}{hPL\,(t_o - t_a)} = \frac{\tanh(mL)}{mL} = \left(\frac{Ky}{2h}\right)^{1/2}\frac{\tanh(mL)}{L}$$

Effectiveness of Fin (ε_{Fin}): Effectiveness of a fin is the ratio of the fin heat transfer rate to heat transfer rate that would exist without a fin.

$$\therefore \varepsilon_{fin} = \frac{Q_{With\ fins}}{Q_{Without\ fins}}$$

For a insulated tip fin,
$$\varepsilon_{fin} = \sqrt{\frac{PK}{hA_{cs}}}\tanh(mL)$$

For an infinitely long fin,

$$\varepsilon_{fin} = \frac{\sqrt{PhA_{cs}K}\,(t_o - t_a)}{hA_{cs}\,(t_o - t_a)} = \sqrt{\frac{PK}{hA_{cs}}}$$

$\varepsilon_{fin} \propto \sqrt{K}$

$\varepsilon_{fin} \propto \frac{1}{\sqrt{h}}$

$\varepsilon_{fin} \propto \sqrt{\frac{P}{A_{cs}}}$

* For a straight rectangular fin of thickness 'y' and width 'b'

$$\frac{P}{A_{cs}} = \frac{2(b+y)}{by} \approx \frac{2}{y}$$

$$\therefore \varepsilon_{fin} = \sqrt{\frac{2K}{hy}}$$

In terms of Biot number (Bi)

$$\varepsilon = \sqrt{\frac{PK}{hA_{cs}}} \qquad \left(\frac{P}{A_{cs}} = \frac{PL}{A_{cs}L} = \frac{A_s}{V} = \frac{1}{L_c}\right)$$

$$= \sqrt{\frac{K}{hL_c}}$$

$$\Rightarrow \boxed{\varepsilon = \frac{1}{\sqrt{Bi}}}$$

When $Bi > 1$, $\varepsilon < 1$
and $Bi < 1$, $\varepsilon > 1$

Relation between η_{Fin} & ε_{Fin} for Insulated tip fin

$$\eta_{Fin} = \frac{\sqrt{PhKA_{cs}}\,(t_0 - t_a).\tanh(mL)}{hPL\,(t_0 - t_a)}$$

$$\varepsilon_{Fin} = \frac{\sqrt{PhKA_{cs}}\,(t_0 - t_a).\tanh(mL)}{hA_{cs}\,(t_0 - t_a)}$$

$$\frac{\varepsilon_{fin}}{\eta_{fin}} = \frac{\text{Surface area of the fin}}{\text{Cross-sectional area of the fin}} = \frac{PL}{A_{cs}}$$

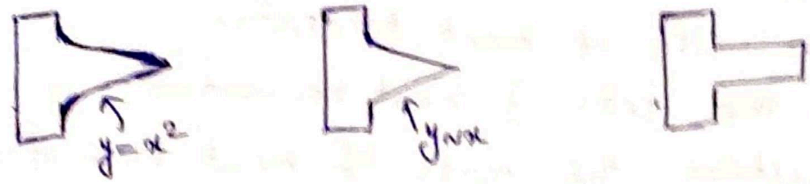
N.B. From an expression, an increase in fin effectiveness can be obtained by increasing the length of the fin but it decreases the efficiency of the fin on the other hand.

Thus, the conditions for fins to be effective are,

1. Thermal conductivity (K) of fin material should be Large.
2. Heat transfer coefficient (h) should be small.
3. Thickness of the fin(y) should be small.
4. Increasing the ratio of the perimeter to the cross-sectional area of the pin (i.e. $\frac{P}{A_{cs}}$). Thus, the use of thin (slender pin fin), but closely spaced fins is preferred to that of thick fins.
5. The ratio $\left(\frac{h}{K}\right)$ is to be kept low for a given geometry, so that the heat transfer would be more effective.
6. The fins are required when the fluid is gas rather than a liquid, particularly when the heat transfer from the surface is by natural convection. If fins are to be used on a surface separating a gas and a liquid, they are generally placed on the gas side, which is the side of lower heat transfer co-efficient ($h_{liquid} >> h_{gas}$, $h_{natural\ convection} << h_{forced\ convection}$)

* Fins are equally effective irrespective of whether they are on the hot side or cold side of the fluid. As the effectiveness of the fin is always highest when it is provided at the lower heat transfer co-efficient(h) side whether it is hot or cold.

* $\eta_{Parabolic} > \eta_{Triangular} > \eta_{Rectangular}$



* According to weight saving (Material saving)

Triangular fin is better than parabolic and rectangular fin.

* **Limitation of an Extended Surface :-**

The installation or use of fins on a heat transferring surface increases the heat transfer area but it is not necessary that the rate of heat transfer would increase.

For a long fin, the rate of heat loss from the fin is given by $\sqrt{PhAK}\,(t - t_a) = KA_{cs}\sqrt{\frac{hP}{KA_{cs}}}(t-t_a) = KA_{cs}\,M\,(t - t_a)$

When $\frac{h}{mK} = 1$, or $h = mK$ | For this condition $\varepsilon = 1$

$\therefore \dot{Q} = hA\,(t - t_a)$

which is equal to the heat loss from the primary surface with no extended surface.

Thus, when $h = MK$, an extended surface will not increase the heat transfer rate from the primary surface, whatever be the length of the extended surfaces.

→ For $\left(\frac{h}{mK}\right) > 1$, $\dot{Q} < hA\,(t_0 - t_a)$ and hence, addition a secondary surface reduces the heat transfer and the added surface will act as an insulation.

→ For $\left(\frac{h}{m \cdot K}\right) < 1$, $\dot{Q} > hA\,(t_0 - t_a)$, and the extended surface will increase the heat transfer. The heat transfer would be more effective when $\left(\frac{h}{K}\right)$ is low for a given geometry.

Q. For $\frac{h}{MK} > 1$, i.e. $h > MK$, $M = \sqrt{\frac{Ph}{A_{cs}K}}$, adding an extended surface

(a) reduces the rate of heat transfer

(b) increases the rate of heat transfer

(c) does not alter the rate of heat transfer

(d) obeys none of the above

Ans: (a)

Q. The temperature distribution along a fin with insulated tip is equal to

(a) e^{-Mx}

(b) $\frac{e^{Mx} + e^{-Mx}}{2}$

(c) $\frac{\cosh M(l-x)}{\cosh(ml)}$

(d) $\cosh M(l-x) + \cosh(ml)$

Ans: (c)

Q. For a rectangular fin of thickness 'b', then the fin efficiency is given by

(a) $\left(\frac{Kh}{2b}\right)^{1/2} \cdot \tanh(ML)$

(b) $\left(\frac{2K}{hb}\right)^{1/2} \cdot \tanh(mL)$

(c) $\left(\frac{2h}{Kb}\right)^{1/2} \tanh(mL)$

(d) $\left(\frac{Kb}{2h}\right)^{1/2} \cdot \frac{\tanh(mL)}{L}$

Ans: (d)

Q. When the convective heat transfer coefficient, $h = m \cdot K$ where $M = \sqrt{\frac{Ph}{KA_{cs}}}$, the incorporation of an extended surface will

(a) Increase the rate of heat flow

(b) decrease the rate of heat flow

(c) not alter the rate of heat transfer

(d) only increase the rate of heat transfer when the length of the fin is very large.

Ans: (c)

Q. Fins are provided on heat transfer surface

(a) to enhance heat transfer by increasing the turbulence in flow.

(b) to increase surface area in promoting the rate of heat transfer

(c) to increase the temp gradient in augmenting heat transfer

(d) to decrease the pressure drop of fluid.

Ans: (b)

Q. Two finned surfaces with long fins are identical, except that the convection heat transfer co-efficient for the first finned surface is twice that of the second one. What statement below is accurate for the efficiency and effectiveness of the first finned surface relative to the second one?

(a) Higher efficiency and higher effectiveness
(b) Higher efficiency and Lower effectiveness
(c) Lower efficiency and higher effectiveness
(d) Lower efficiency and Lower effectiveness
(e) Equal efficiency and equal effectiveness. Ans: (d)

** $\eta_{fin} = \frac{1}{ML}$ (for long fin) $= \frac{1}{\sqrt{\frac{Ph}{A_{cs}K}} \times L} = \sqrt{\frac{A_{cs}K}{Ph}} \cdot \frac{1}{L}$

i.e., $\boxed{\eta_{fin} \propto \frac{1}{\sqrt{h}}}$ Also $\boxed{\varepsilon_{fin} \propto \frac{1}{\sqrt{h}}}$

Q. (a) The longer the fin, the larger the heat transfer area and thus, the higher the rate of heat transfer from the fin and also the larger the fluid friction.
(b) The fin efficiency decreases with increasing fin length because of the decrease in fin temp.
(c) The efficiency of most fins used in practice is above 90%.

Which of the above statement(s) are correct?

(i) only a & b
(II) only b & c
(III) only a & c
(IV) a, b & c. Ans: (IV)

Q. Consider the following statements pertaining to heat transfer through fins:

1. Fins are equally effective irrespective of whether they are on the hot side or cold side of the fluid.
2. The temp. along the fin is variable and hence the rate of heat transfer varies along the fin.
3. The fins may be made of materials that have a higher thermal conductivity than the material of the wall.
4. Fins must be arranged at right angles to the direction of fluid flow

Of these statements,
(a) 1 & 2 are correct
(b) 2 & 4 are correct
(c) 1 & 3 are correct
(d) 2 & 3 are correct Ans- (b)

For the proper design of fins, the knowledge of temperature distribution along the fin is necessary. In this article the mathematical analysis for finding out the temperature distribution and heat flow from different types of fins is dealt with.

The following **assumptions** are made for the analysis of heat flow through the fin :

1. Steady state heat conduction.
2. No heat generation within the fin.
3. Uniform heat transfer coefficient (h) over the entire surface of the fin.
4. Homogeneous and isotropic fin material (*i.e.* thermal conductivity of material constant).
5. Negligible contact thermal resistance.
6. Heat conduction one-dimensional.
7. Negligible radiation.

2.10.2. HEAT FLOW THROUGH "RECTANGULAR FIN"

Consider a rectangular fin protruding from a wall surface as shown in Fig. 2.121.

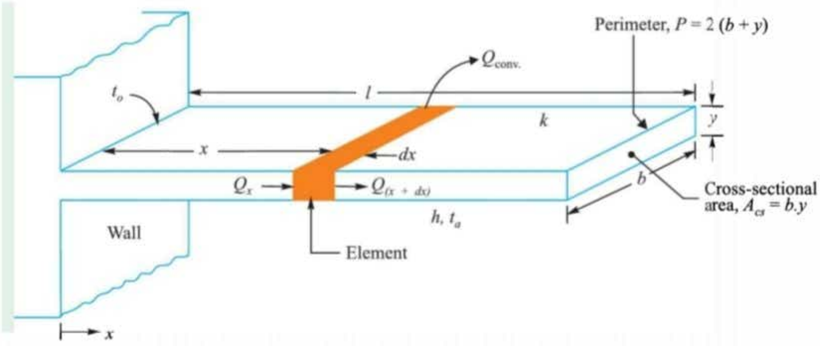


Fig. 2.121. Rectangular fin of uniform cross-section.

Let, l = Length of the fin (perpendicular to surface from which heat is to be removed),

b = Width of the fin (parallel to the surface from which heat is to be removed),

y = Thickness of the fin,

P = Perimeter of the fin [$=2(b + y)$],

A_{cs} = Area of cross-section ($=by$),

t_o = Temperature at the base of the fin, and

t_a = Temperature of the ambient/surrounding fluid,

k = Thermal conductivity (constant), and

h = Heat transfer coefficient (convective).

In order to determine the governing differential equation for the fins, shown in Fig. 2.121, consider the heat flow to and from an element dx thick at a distance x from the base.

Heat conducted into the element at plane x,

$$Q_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_x \quad ...(i)$$

Heat conducted out of the element at plane $(x + dx)$

$$Q_{(x+dx)} = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx} \quad ...(ii)$$

Heat convected out of the element between the planes x and $(x + dx)$,

$$Q_{conv} = h\,(P\,.\,dx)\,(t - t_a)$$

Applying an energy balance on the element, we can write

$$Q_x = Q_{(x+dx)} + Q_{conv.}$$

$$-k\,A_{cs}\left[\frac{dt}{dx}\right]_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx} + h(P.dx)(t - t_a) \quad ...(2.128)$$

Making a Taylor's expansion of the temperature gradient at $(x + dx)$ in terms of that at x, we get

$$\left(\frac{dt}{dx}\right)_{x+dx} = \left(\frac{dt}{dx}\right)_x + \frac{d}{dx}\left(\frac{dt}{dx}\right)_x dx + \frac{d^2}{dx^2}\left(\frac{dt}{dx}\right)\frac{(dx)^2}{2!} + ...$$

Substituting this in eqn. (2.128), we have

$$-k\,A_{cs}\left[\frac{dt}{dx}\right]_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_x - kA_{CS}\left[\frac{d^2t}{dx^2}\right]_x dx - k\,A_{cs}\left[\frac{d^3t}{dx^3}\right]\frac{(dx)^2}{2!} + .. + h(P.dx)(t - t_a)$$

Neglecting higher terms as $dx \to 0$, we have

$$-k\,A_{cs}\left[\frac{dt}{dx}\right] = -k\,A_{cs}\left[\frac{dt}{dx}\right] - k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx + h(P.dx)(t - t_a)$$

$$k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx - h(P.dx)(t - t_a) = 0$$

Dividing both sides by $A_{cs}\,dx$, we get,

$$k\frac{d^2t}{dx^2} - \frac{hP}{A_{cs}}(t - t_a) = 0$$

or,

$$\frac{d^2t}{dx^2} - \frac{h\,P}{k\,A_{cs}}(t - t_a) = 0 \quad ...(2.129)$$

or, $$\frac{d^2t}{dx^2} - \frac{hP}{kA_{cs}}(t - t_a) = 0 \qquad ...(2.129)$$

Eqn. (2.129) is further simplified by transforming the dependent variable by defining the *temperature excess* θ as,

$$\theta_{(x)} = t_{(x)} - t_{(a)}$$

As the ambient temperature t_a is constant, we get by differentiation

$$\frac{d\theta}{dx} = \frac{dt}{dx}; \quad \frac{d^2\theta}{dx^2} = \frac{d^2t}{dx^2}$$

Thus, $$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad ...(2.130)$$

where $$m = \sqrt{\frac{hP}{kA_{cs}}}$$

Eqns. (2.129) and (2.130) represent a general form of the energy equation for one-dimensional heat dissipation from an extended surface (fin). The parameter *m*, for a given fin, is constant provided the convective film coefficient *h* is constant over the whole surface and the thermal conductivity *k* is constant within the temperature range considered. Then the general solution of this linear and homogeneous second order differential equation is of the form :

$$\theta = C_1e^{mx} + C_2e^{-mx} \qquad ...(2.131)$$

or, $$[t - t_a = C_1e^{mx} + C_2e^{-mx}]$$

where C_1 and C_2 are the constants, these are to be determined by using proper boundary conditions.

One boundary condition is :

$$\theta = \theta_o = t_o - t_a \qquad \text{at } x = 0$$

The other boundary condtition depends on the *physical situation*. The following cases may be considered :

Case I. The *fin is infinitely long* and the temperature at the end of the fin is esentially that of the ambient/surrounding fluid.

Case II. The end of the *fin is insulated.*

Case III. The *fin is of finite length* and loses heat by convection.

.2.1. Heat dissipation from an infinitely long fin ($l \to \infty$) :

Refer to Fig. 2.122. In this case the boundary conditions are :

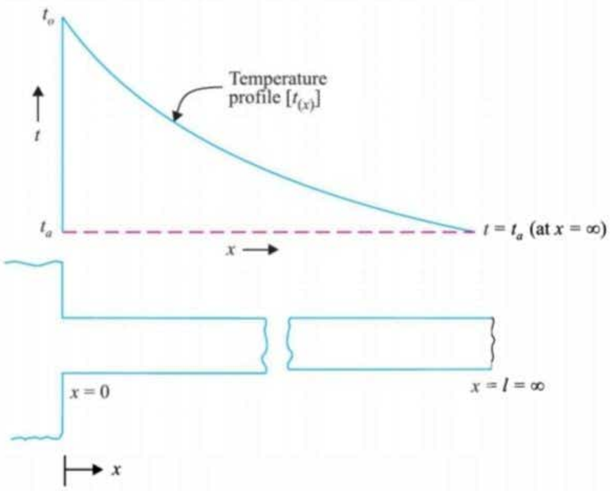


Fig. 2.122. Infinitely long fin (Case I).

(*i*) At $x = 0$, $t = t_o$ (Temperature at the base of fin equals the temperature of the surface to which fin is attached.)

$t - t_a = t_o - t_a$ (in terms of excess temperature)

or, At $x = 0$, $\theta = \theta_o$

(*ii*) At $x = \infty$, $t = t_a$

(Temperature at the end of an infinitely long fin equals that of the surroundings)

At $x = \infty$, $\theta = 0$ (in terms of *excess temperature*)

Substituting these boundary conditions in eqn. (2.131), we get,

$$C_1 + C_2 = \theta_0 \qquad ...(i)$$

$$C_1e^{m(\infty)} + C_2e^{-m(\infty)} = 0 \qquad ...(ii)$$

or, $$C_1e^{m(\infty)} + 0 = 0 \qquad \therefore C_1 = 0$$

and, $$C_2 = \theta_0 \qquad \text{[From eqn. } (i)]$$

Inserting these values of C_1 and C_2 in eqn. (2.131), we get the temperature distribution along the length of the fin,

$$\theta = \theta_0 \, e^{-mx}; \; (t - t_a) = (t_o - t_a) e^{-mx} \left[\text{or } \frac{t - t_a}{t_o - t_a} = e^{-mx} \right] \quad ...(2.132)$$

The dependence of dimensionless temperature $\left[\frac{t - t_a}{t_o - t_a}\right]$ along the fin length for different values of parameter m ($m_1 < m_2 < m_3$) is shown in Fig. 2.123; the plot indicates :

(*i*) As the value of m increases, the dimensionless temperature falls;

(*ii*) As the length of fin increases to infinity all the curves approach $\frac{t - t_a}{t_o - t_a} = 0$ *asymptotically.*

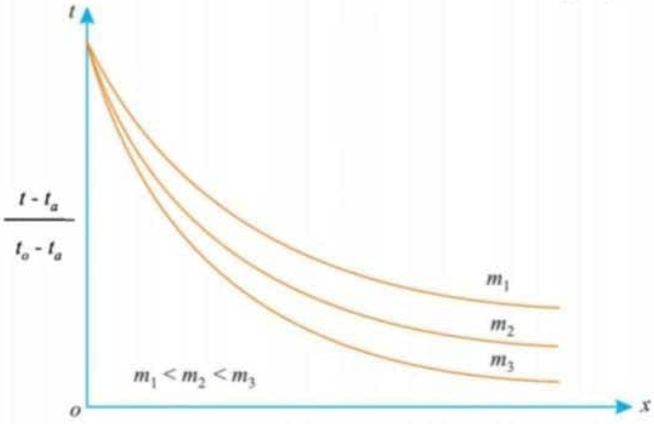


Fig. 2.123. Temperature distribution in a fin.

The heat flow rate can be determined in *either* of the two ways :

(*a*) By considering the heat flow across the root (or base) by conduction;

(*b*) By considering the heat which is transmitted by convection from the surface of the fin to the surrounding fluid.

(*a*) The rate of heat flow across the base of the fin is given by (Fourier's equation)

$$Q_{fin} = -k \, A_{cs} \left[\frac{dt}{dx}\right]_{x=0}$$

$$\left[\frac{dt}{dx}\right]_{x=0} = \left[-m(t_0 - t_a) e^{-mx}\right]_{x=0} = -m(t_0 - t_a) \quad \text{[From eqn. (2.132)]}$$

$\therefore$ $$Q_{fin} = -k \, A_{cs} \times [-m \, (t_o - t_a)] = k \, A_{cs} \, m \, (t_o - t_a)$$

i.e., $$Q_{fin} = k \, A_{cs} \, m \, (t_o - t_a) \quad ...(2.133)$$

or, $$Q_{fin} = k \, A_{cs} \sqrt{\frac{Ph}{kA_{cs}}} \, (t_0 - t_a) \quad \text{(Substituting for } m\text{)}$$

or, $$Q_{fin} = \sqrt{Ph \, k \, A_{cs}} \, (t_0 - t_a) \quad ...[2.133\,(a)]$$

(*b*) *Alternatively* :

$$Q_{fin} = \int_0^\infty h \, (P.dx) \, (t - t_a) \quad \text{...convective rate of heat flow}$$

$$= \int_0^\infty h \, P \, (t_o - t_a) e^{-mx} \, dx \quad \text{[From Eqn. (2.132)]}$$

$$= h \, P \, (t_o - t_a) \int_0^\infty e^{-mx} \, dx$$

$$= h \, P \, (t_o - t_a) \frac{1}{m} = h \, P \, (t_o - t_a) \sqrt{\frac{k \, A_{cs}}{Ph}} \quad \text{(Substituting for } m\text{)}$$

or, $$Q_{fin} = \sqrt{Ph \, kA_{cs}} \, (t_o - t_a) \quad \text{...[Same as Eqn. 2.133 (}a\text{)]}$$

[An infinitely long fin is one for which $m_l \to \infty$, and this condition may be approached when $ml > 5$]

From the Eqn. (2.132) it is evident that the temperature falls towards the tip of the fin, thus the area near the fin tip is *not utilised to the extent as the lateral area near the base.* Hence beyond a certain point the increase in the length of the fin *does not contribute much in respect of increase in the dissipation of heat.* Consequently a *tapered fin* is considered to be a *better design* since its lateral area is more near the base/root where temperature difference is high.

2.2. Heat dissipation from a fin insulated at the tip :

Fig. 2.126 illustrates a fin of finite length with insulated end (*i.e.* no heat loss from the end of the fin).

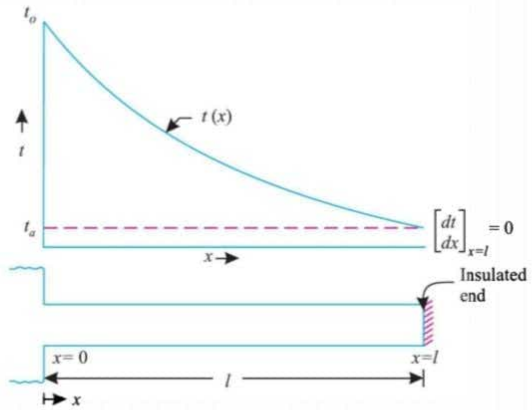


Fig. 2.126. The fin with insulated end (Case II).

The boundary conditions are :

(*i*) At $x = 0$, $\theta = \theta_o$

(*ii*) At $x = l$, $\frac{dt}{dx} = 0$

Applying these boundary conditions to eqn. (2.131), we have

$$C_1 + C_2 = \theta_o \quad ...(i)$$

Further

$$t - t_a = C_1 e^{mx} + C_2 e^{-mx} \quad \text{[Eqn. 2.131]}$$

$$\frac{dt}{dx} = m C_1 e^{mx} - m C_2 e^{-mx}$$

$$\left[\frac{dt}{dx}\right]_{x=l} = m C_1 e^{ml} - m C_2 e^{-ml} = 0$$

∴
$$C_1 e^{ml} - C_2 e^{-ml} = 0 \quad \text{[As per boundary condition (ii)]}$$

Solving eqns. (*i*) and (*ii*), we have

$$C_2 = \theta_o - C_1 \quad \text{...[From eqn. (i)]}$$

$$C_1 e^{ml} - (\theta_o - C_1) e^{-ml} = 0$$

or,
$$C_1 e^{ml} - \theta_o e^{-ml} + C_1 e^{-ml} = 0$$

or,
$$C_1 (e^{ml} + e^{-ml}) = \theta_o e^{-ml}$$

or,
$$C_1 = \theta_o \left[\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right]$$

∴
$$C_2 = \theta_o - \left[\theta_o \left\{\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right\}\right] \quad \text{...[From eqn. (i)]}$$

or,
$$C_2 = \theta_o \left[1 - \frac{e^{-ml}}{e^{ml} + e^{-ml}}\right] = \theta_o \left[\frac{e^{ml}}{e^{ml} + e^{-ml}}\right]$$

Inserting the values of C_1 and C_2 in eqn. (2.131), we have

$$\theta = \theta_o \left[\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right] e^{mx} + \theta_o \left[\frac{e^{ml}}{e^{ml} + e^{-ml}}\right] e^{-mx}$$

or,
$$\frac{\theta}{\theta_o} = \left[\frac{e^{m(x-l)} + e^{m(l-x)}}{e^{ml} + e^{-ml}}\right] = \left[\frac{e^{m(l-x)} + e^{[-m(l-x)]}}{e^{ml} + e^{-ml}}\right]$$

The above expression, in terms of hyperbolic functions, can be expensed as

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh\{m(l-x)\}}{\cosh(ml)} \quad ..(2.134)$$

...Expression for temperature distribution

$$\left[\because \cosh\{m(l-x)\} = \frac{e^{m(l-x)} + e^{[-m(l-x)]}}{2}, \text{ and } \cosh(ml) = \frac{e^{ml} + e^{-ml}}{2}\right]$$

The rate of heat flow from the fin is given by

$$Q_{fin} = -k A_{cs} \left[\frac{dt}{dx}\right]_{x=0}$$

Now,
$$t - t_a = (t_o - t_a)\left[\frac{\cosh\{m(l-x)\}}{\cosh(ml)}\right] \quad \text{...[From eqn. (2.134)]}$$

$$\frac{dt}{dx} = (t_o - t_a)\left[\frac{\sinh\{m(l-x)\}}{\cosh(ml)}\right](-m)$$

$$\left[\because \frac{d}{dx}[\cosh(mx)] = m \sinh(mx)\right]$$

$$\left[\frac{dt}{dx}\right]_{x=0} = -m(t_o - t_a)\tanh(ml)$$

∴
$$Q_{fin} = kA_{cs}\, m\, (t_o - t_a) \tanh(ml) \quad ...(2.135)$$

(Substituting for m)

or,
$$Q_{fin} = \sqrt{PhkA_{cs}}\,(t_o - t_a) \tanh(ml) \quad ...[2.135\,(a)]$$

..2.3. Heat dissipation from a fin losing heat at the tip

Fig. 2.132 illustrates a fin of finite length losing heat at the tip.

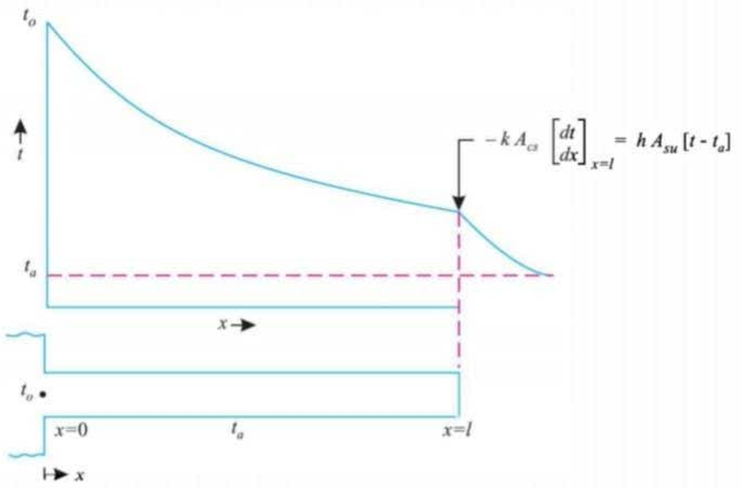


Fig. 2.132. A fin of finite length losing heat at tip (case III).

The boundary conditions are :

(*i*) At $x = 0, \quad \theta = \theta_0$

(*ii*) Heat conducted to the fin at $x = l$

= Heat convected from the end to the surroundings.

i.e.,
$$-k A_{cs} \left[\frac{dt}{dx}\right]_{x=l} = h A_{su} (t - t_a)$$

where A_{cs} (cross-sectional area for heat conduction) equals A_{su} (surface area from which the convective heat transport takes place), at the tip of the fin; *i.e.* $A_{cs} = A_{su}$.

Thus
$$\frac{dt}{dx} = -\frac{h\theta}{k} \quad \text{at } x = l$$

Applying these boundary conditions to eqn. (2.131), we get

$$C_1 + C_2 = \theta_0 \qquad ...(i)$$

Further
$$t - t_a = C_1 e^{mx} + C_2 e^{-mx} \qquad \text{[Eqn. 2.131]}$$

Differentiating this expression w.r.t. x, we have

$$\frac{dt}{dx} = m C_1 e^{mx} - m C_2 e^{-mx}$$

$$\left[\frac{dt}{dx}\right]_{x=l} = m C_1 e^{ml} - m C_2 e^{-ml} = -\frac{h\theta}{k}$$

or,
$$C_1 e^{ml} - C_2 e^{-ml} = -\frac{h\theta}{km}$$

or,
$$C_1 e^{ml} - C_2 e^{-ml} = -\frac{h}{km}[C_1 e^{ml} + C_2 e^{-ml}] \qquad ...(ii)$$

$$[\because \theta_{(x=1)} = C_1 e^{ml} + C_2 e^{-ml}]$$

Solving eqns. (*i*) and (*ii*), we have

$$C_2 = \theta_o - C_1 \qquad \text{...[From eqn. (}i\text{)]}$$

$$C_1 e^{ml} - (\theta_o - C_1) e^{-ml} = -\frac{h}{km}[C_1 e^{ml} + \theta_o - C_1) e^{-ml}] \qquad \text{...[From eqn. (}ii\text{)]}$$

$$C_1 e^{ml} - \theta_o e^{-ml} + C_1 e^{ml} = -\frac{h}{km} \cdot C_1 e^{ml} - \frac{h}{km} \cdot \theta_o \cdot e^{-ml} + \frac{h}{km} \cdot C_1 \cdot e^{-ml}$$

$$C_1 \left[(e^{ml} + e^{-ml}) + \frac{h}{km} e^{ml} - \frac{h}{km} e^{-ml}\right] = \theta_o e^{-ml} - \frac{h}{km} \theta_0 \cdot e^{-ml}$$

$$C_1 \left[(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})\right] = \theta_o e^{-ml} \left[1 - \frac{h}{km}\right]$$

$\therefore$
$$C_1 = \frac{\theta_0 \left[1 - \frac{h}{km}\right] e^{-ml}}{\left[(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})\right]}$$

and,
$$C_2 = \theta_o - \left[\frac{\theta_0 \left(1 - \frac{h}{km}\right) e^{-ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})}\right]$$

$$= \theta_o \left[1 - \frac{\left(1 - \frac{h}{km}\right) e^{-ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})} \right]$$

$$= \theta_o \left[\frac{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml}) - e^{-ml} + \frac{h}{km} e^{-ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})} \right]$$

$$= \theta_o \left[\frac{e^{ml} + e^{-ml} + \frac{h}{km} e^{ml} - \frac{h}{km} e^{-ml} - e^{-ml} + \frac{h}{km} \cdot e^{-ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})} \right]$$

or, $$C_2 = \frac{\theta_o \left[1 + \frac{h}{km}\right] e^{ml}}{\left[(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})\right]}$$

Substituting these values of constants C_1 and C_2 in eqn. (2.131), we get

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad \text{...[Eqn (2.129)]}$$

$$\theta = \left[\frac{\theta_o \left(1 - \frac{h}{km}\right) e^{-ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})} \right] e^{mx} + \left[\frac{\theta_o \left(1 + \frac{h}{km}\right) e^{ml}}{(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})} \right] e^{-mx}$$

or, $$\frac{\theta}{\theta_o} = \frac{[e^{m(l-x)} + e^{-m(l-x)}] + \frac{h}{km}[e^{m(l-x)} - e^{-m(l-x)}]}{[(e^{ml} + e^{-ml}) + \frac{h}{km}(e^{ml} - e^{-ml})]}$$

or, $$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh[m(l-x)] + \frac{h}{km}[\sinh\{m(l-x)\}]}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]} \quad \text{...(2.136)}$$

The rate of heat flow from the fin is given by

$$Q_{fin} = -k A_{cs} \left[\frac{dt}{dx}\right]_{x=0}$$

Now, $$t - t_a = (t_o - t_a) \left[\frac{\cosh\{m(l-x)\} + \frac{h}{km}[\sinh m(l-x)]}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}} \right]$$

Differentiating the above expression w.r.t. x, we get

$$\frac{dt}{dx} = (t_o - t_a) \left[\frac{-m \sinh\{m(l-x)\} - m\left[\frac{h}{km}\{\cosh[m(l-x)]\}\right]}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}} \right]$$

$$\left[\frac{dt}{dx}\right]_{x=0} = -(t_o - t_a) m \left[\frac{\sinh(ml) + \frac{h}{km}\{\cosh(ml)\}}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}} \right]$$

$$\therefore \quad Q_{fin} = k A_{cs} m (t_o - t_a) \left[\frac{\sinh(ml) + \frac{h}{km}\{\cosh(ml)\}}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}} \right]$$

$$= \sqrt{P h k A_{cs}} (t_o - t_a) \left[\frac{\sinh(ml) + \frac{h}{km}\{\cosh(ml)\}}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}} \right] \text{(Substituting for } m\text{)}$$

or, $$Q_{fin} = \sqrt{P h k A_{cs}} (t_o - t_a) \left[\frac{\tanh(ml) + \frac{h}{km}}{1 + \frac{h}{km} \cdot \tanh(ml)} \right] \quad \text{...(2.135)}$$

Example 2.126. *A longitudinal copper fin (k = 380 W/m°C) 600 mm long and 5 mm diameter is exposed to air stream at 20°C. The convective heat transfer coefficient is 20W/m²°C. If the fin base temperature is 150°C, determine :*

(i) *The heat transferred, and*

(ii) *The efficiency of the fin.* **[P.U., 1997]**

Solution. Length of the fin, l = 600 mm = 0.6 m

Diameter of the fin, d = 5 mm = 0.005 m

The fin base temperature, t_0 = 150°C

Air stream temperature, t_a = 20°C

Thermal conductivity of fin material, k = 380 W/m°C

Convective heat transfer coefficient h = 20 W/m²°C

(*i*) **The heat transferred, Q :**

Neglecting the heat loss from the end surface, t heat transfer from the fin is given by

$$Q = kA_{cs}\, m\, (t_o - t_a) \tanh (ml) \quad \text{...[Eqn. (2.135)]}$$

where

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h}{2} \times \frac{\pi d}{\frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 20}{380 \times 0.005}} = 6.49$$

$$\therefore \quad Q = 380 \times \left(\frac{\pi}{4} \times 0.005^2\right) \times 6.49 \times (150 - 20) \tanh (6.49 \times 0.6) = 6.29 \text{ W}$$

$$= 6.29 \times 3600 = 22644 \text{ J/h or } \mathbf{22.644\ kJ/h} \quad \textbf{(Ans.)}$$

(*ii*) **The efficiency of the fin, η_{fin} :**

For a fin which is insulated at the tip is given by

$$\eta_{fin} = \frac{\tanh (ml)}{ml} \quad \text{...[Eqn. (2.139)]}$$

$$= \frac{\tanh (6.49 \times 0.6)}{(6.49 \times 0.6)} = 0.2566 \text{ or } \mathbf{25.66\%} \quad \textbf{(Ans.)}$$

Example 2.127. *A steel rod (k = 32 W/m°C), 12 mm in diameter and 60 mm long, with an insulated end, is to be used as a spine. It is exposed to surroundings with a temperature of 60°C and a heat transfer coefficient of 55 W/m²°C. The temperature at the base of fin is 95°C. Determine :*

(i) The fin efficiency;

(ii) The temperature at the edge of the spine;

(iii) The heat dissipation.

Solution. *Given* : d = 12 mm = 0.012 m; l = 60 mm = 0.06 m; t_a = 60°C; (k = 32 W/m°C); h = 55 W/m²°C; t_o = 95°C.

(*i*) The fin efficiency, η_{fin} :

$$\eta_{fin} = \frac{\tanh (ml)}{(ml)} \quad \text{...[Eqn. (2.139)]}$$

where,

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h \times \pi d}{k \times \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 55}{32 \times 0.012}} = 23.93$$

$$\therefore \quad \eta_{fin} = \frac{\tanh (23.93 \times 0.06)}{(23.93 \times 0.06)} = 0.6218 \text{ or } \mathbf{62.18\%} \quad \textbf{(Ans.)}$$

(*ii*) **The temperature at the edge of the spine, t_1 :**

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh m(l - x)}{\cosh ml} \quad \text{[Eqn. (2.134)]}$$

At $x = l$, the above equation reduces to

$$\frac{\theta_l}{\theta_o} = \frac{t_l - t_a}{t_o - t_a} = \frac{1}{\cosh ml}$$

or,

$$\frac{t_l - 60}{95 - 60} = \frac{1}{\cosh (23.93 \times 0.06)} = 0.45$$

$$\therefore \quad t_1 = 60 + (95 - 60) \times 0.45 = \mathbf{75.75°C} \quad \textbf{(Ans.)}$$

(*iii*) The heat dissipation, Q_{fin} :

$$Q_{fin} = k_{Acs}\, m\, (t_o - t_a) \tanh (ml) \quad \text{...[Eqn. (2.135)]}$$

$$= 32 \times \frac{\pi}{4} \times (0.012)^2 \times 23.93(95 - 60) \tanh (23.93 \times 0.06) = \mathbf{2.7\ W} \quad \textbf{(Ans.)}$$

2.10.5. HEAT TRANSFER FROM A BAR CONNECTED TO THE TWO HEAT SOURCES AT DIFFERENT TEMPERATURES

Consider the system shown in Fig. 2.143.

Let, l = Length of the bar connecting two heat sources,

A_{cs} = Cross-sectional area of the bar (constant),

P = Perimeter of the bar,

t_1 = Temperature of heat source-1

t_2 = Temperature of heat source-2

t_a = Temperature of air surrounding the bar,

h = Heat transfer coefficient on the surface of the bar,

k = Thermal conductivity of the bar,

$\theta_1 = t_1 - t_a$, and

$\theta_2 = t_2 - t_a$.

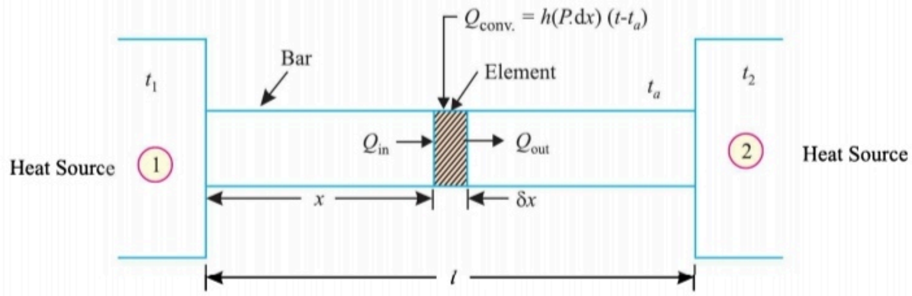


Fig. 2.143. Heeat transfer from a bar connected to heat sources at differernt temperatures.

Consider an infinitesimal element of the bar of thickness dx located at distance x from heat source–1 as shown in Fig. 2.143.

$$Q_{in} = -k\, A_{cs} \frac{dt}{dx};$$

$$Q_{out} = -k\, A_{cs} \left[t + \frac{dt}{dx}\delta x\right]$$

$$Q_{conv.} = hP.\delta x\,(t - t_a)$$

Applying energy balance on the element, we have

$$Q_{in} = Q_{out} + Q_{conv}.$$

or,
$$-kA_{cs} \cdot \frac{dt}{dx} = -kA_{cs} \frac{d}{dx}\left[t + \frac{dt}{dx}\delta x\right] + hP.\,\delta x\,(t - t_a)$$

Upon simplification and rearrangement, we obtain

$$\frac{d^2 t}{dx^2} - \frac{hP}{kA_{cs}}(t - t_a) = 0 \qquad ...(2.165)$$

Replacing the temperature excess $(t - t_a)$ by θ (since the ambient temperature is assumed constant), eqn. 2.165 gets transferred to

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA_{cs}}\theta = 0 \qquad ...(2.166)$$

The solution to the above differential equation is

$$\theta = C_1 e^{mx} + C_2\, e^{-mx} \qquad ...(2.167)$$

where,
$$m = \sqrt{\frac{hP}{kA_{cs}}}$$

The constant C_1 and C_2 are evaluated from the following boundary conditions :

(*i*) At $x = 0$, $\theta = \theta_1$

(*ii*) At $x = l$, $\theta = \theta_2$

$\therefore$ $\theta_1 = C_1 + C_2$ ...[From eqn. (2.167) using boundary condition (*i*) ...(*a*)

$\theta_2 = C_1 e^{ml} + C_2 e^{-ml}$...[From eqn. (2.167) using boundary condition (*ii*) ...(*b*)

Solving eqns. (*a*) and (*b*), we have

$$C_1 = \frac{\theta_2 - \theta_1 e^{-ml}}{e^{-ml} - e^{-ml}}, \qquad C_2 = \frac{\theta_1 e^{ml} - \theta_2}{e^{ml} - e^{-ml}}$$

The rate of heat loss is given by

$$Q = \int_0^l hPdx\,(t - t_a) = \int_0^l hPdx\,.\,\theta$$

$$= hP \int_0^l \frac{\theta_1 \sinh m(l - x) + \theta_2 \sinh (mx)}{\sinh (ml)} dx$$

(substituting for θ)

$$= \frac{hP}{\sinh (ml)} \left[-\frac{\theta_1 \,.\, \cosh \{m\,(l - x)\}}{m} + \frac{\theta_2 \,.\, \cosh (mx)}{m} \right]_0^l$$

$$= \frac{hP}{\sinh (ml)} \left[-\frac{\theta_1}{m} \{1 - \cosh (ml)\} + \frac{\theta_2}{m} \{\cosh (ml) - 1\} \right]$$

$$= \frac{hP}{m \sinh (ml)} \left[(\theta_1 + \theta_2) \{\cosh (ml) - 1\} \right]$$

But, $m = \sqrt{\dfrac{hP}{kA_{cs}}}$

$\therefore$ $$Q = \sqrt{hPkA_{cs}}\,(\theta_1 + \theta_2) \left[\frac{\cosh (ml) - 1}{\sinh (ml)} \right] \quad ...(2.169)$$

The maximum temperature occurs in the bar where $\dfrac{d\theta}{dx} = 0$, hence differentiating the eqn. (2.168) we have

$$-m\theta_1 \cosh \{m\,(l - x)\} + m\theta_2 \cosh mx = 0$$

or, $$\theta \cosh \{m\,(l - x)\} = \theta_2 \cosh (mx) \quad ...(2.170)$$

The value of x from the above equation gives the position of maximum temperature in the rod.

Example 2.134. *A thin rod of copper (k = 100 W/m°C) 12.5 mm in diameter spans between two parallel plates 150 mm apart. Air flows over the rod providing heat transfer coefficient of 50 W/m²°C. The surface temperature of the plate exceeds the air by 40°C. Determine :*

(i) The excess temperature at the centre of the rod over that of air, and

(ii) The heat lost from the rod in watts. **(M.U.)**

Solution. Refer to Fig. 2.144.

Given : d = 12.5 mm = 0.0125 m; l = 150 mm = 0.15 m; h = 50 W/m²°C; k = 100 W/m°C.

$\theta_1 = \theta_2 = 40°C$ ...Temperature excess for each plate.

(*i*) The excess temperature at the centre of the rod over that of air; θ :

The temperature distribution θ (= $t - t_a$) along the rod is given by

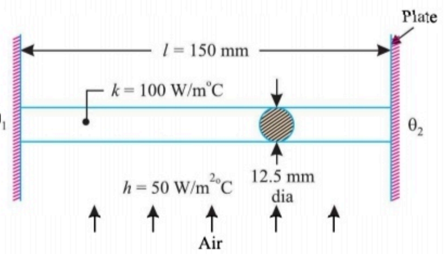


Fig. 2.144.

$$\theta = \frac{\theta_1 \sinh \{m\,(l - x) + \theta_2 \sinh (mx)}{\sinh (ml)} \quad ...[\text{Eqn. (2.168)}]$$

At, $x = \dfrac{l}{2}$, the value of θ is given by

$$\theta\,\Big|_{x = \frac{l}{2}} = 2\theta_1 \left[\frac{\sinh (ml/2)}{\sinh (ml)} \right] \quad ...(i)$$

where, $$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h \times \pi d}{k \times \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 50}{100 \times 0.0125}} = 12.65$$

$\therefore$ $ml = 12.65 \times 0.15 \simeq 1.9$

Substituting the proper values in eqn. (*i*), we have

$$\theta\,\Big|_{x = \frac{l}{2}} = 2 \times 40 \left[\frac{\sinh (1.9/2)}{\sinh (1.9)} \right] = 80 \times \frac{1.0995}{3.2682} = \mathbf{26.9^\circ C} \quad \textbf{(Ans.)}$$

(*ii*) The heat lost from the rod, Q :

$$Q = \sqrt{hPkA_{cs}}\,(\theta_1 + \theta_2) \left[\frac{\cosh (ml) - 1}{\sinh (ml)} \right] \quad ...[\text{Eqn. (2.169)}]$$

$$= \sqrt{h \times (\pi d) k \times \left(\frac{\pi}{4} d^2\right)} \times 2\theta_1 \left[\frac{\cosh (ml) - 1}{\sinh (ml)} \right]$$

$$= \sqrt{hk\pi^2 d^3} \times \theta_1 \times \left[\frac{\cosh (ml) - 1}{\cosh (ml)} \right]$$

$$= \sqrt{50 \times 100 \times \pi^2 \times 0.0125^3} \times 40 \times \left[\frac{\cosh (1.9) - 1}{\sinh (1.9)} \right]$$

$$= 0.31 \times 40 \times \left[\frac{3.4177 - 1}{3.2682} \right] = \mathbf{9.17\ W} \quad \textbf{(Ans.)}$$

Rectangular fin :

Imp. Notes

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad ...(i)$$

or $$[(t - t_a) = C_1 e^{mx} + C_2 e^{-mx}]$$

Case I. *Heat dissipation from an infinitely long fin ($l \to \infty$) :*

$$\theta = \theta_0 e^{-mx} \quad ...(ii)$$

or $$[(t - t_a) = (t_o - t_a) e^{-mx}]$$

$$Q_{fin} = k A_{cs} m (t_o - t_a) \quad ...(iii)$$

[An infinitely long fin is one for which $ml \to \infty$, and this condition may be approached when $ml > 5$]

Case II. *Heat dissipation from a fin insulated at the tip :*

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh [m \{l - x\}]}{\cosh (ml)} \quad ...(iv)$$

$$Q_{fin} = kA_{cs} m (t_o - t_a) \tanh (ml) \quad ...(v)$$

Case III. *Heat dissipation from a fin losing heat at the tip :*

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh \{m (l - x)\} + \frac{h}{km} [\sinh \{m (l - x)\}]}{\cosh (ml) + \frac{h}{km} [\sinh (ml)]} \quad ...(vi)$$

$$Q_{fin} = kA_{cs} m (t_o - t_a) \left[\frac{\tanh (ml) + \frac{h}{km}}{1 + \frac{h}{km} . \tanh (ml)} \right] \quad ...(vii)$$

where $$m = \sqrt{\frac{hP}{kA_{cs}}}$$

[A_{cs} = cross-sectional area ($b \times y$); P = perimeter of the fin {= $2(b + y)$}]

[t_o = temperature at the base of the fin; t_a = temperature of ambient/surrounding fluid].

Efficiency of fin : It is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at the base temperature.

Effectiveness of fin : It is the ratio of the fin heat transfer rate to the heat transfer rate that would exist without a fin.

CHAPTER - 4

Unsteady State Heat Conduction

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Transient (Unsteady) State Heat Conduction

Conduction of heat in unsteady state refers to the transient conditions wherein the heat flow and the temperature distribution at any point of the system / body vary continuously with time (τ).

So, the temp. and rate of heat conduction are dependent both on the time and space co-ordinates, i.e., temp., $t = f(x, y, z, \tau)$

* Transient or Unsteady state of heat conduction occurs in:
  1) Cooling of IC engine cylinder
  2) Cooling and freezing of food
  3) heating and cooling of metal billets (Heat treatment process of metal billets)
  4) Brick burning

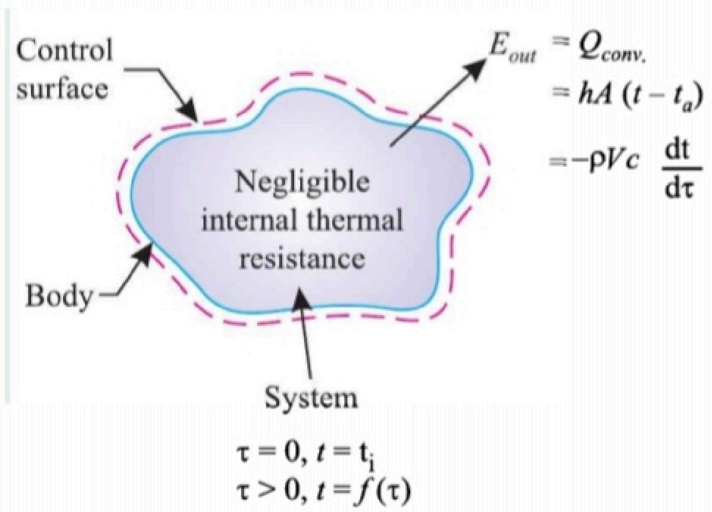
Lumped Heat Analysis

[Transient Conduction In Solids With Infinite Thermal Conductivity ($K \to \infty$)]

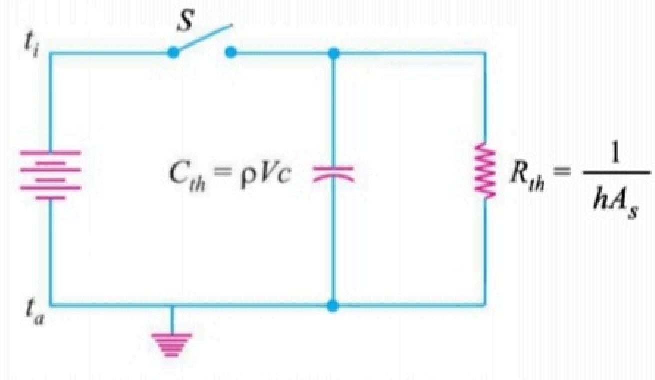
The process in which the internal resistance (or conduction resistance) $\frac{L}{KA}$ is assumed negligible in comparison with its surface resistance (or convective resistance) $\frac{1}{h.A}$, is called Newtonian heating or cooling process. Then the temperature in this process is considered to be uniform (or isothermal) at a given time. Such an analysis is called Lumped heat analysis, because the whole solid whose energy at any time is a function of its temp. (t) and total heat capacity (ρC_p) is termed as one lump.

- Lumped heat analysis presumes that the solid possesses infinitely large thermal conductivity (K).
- This analysis is employed for solids of large conductivity (K) with surface area (A) that are large in proportion to their volume like plates and metallic wires.
- In Lumped heat analysis / capacity / system, the temperature gradient $\left(\frac{dt}{dx}\right)$ within the system / body is negligible and Biot number (Bi) is very small due to high 'K'.

Let us consider a body whose initial temperature is t_i throughout and which is placed suddenly in ambient air or any liquid at a constant temperature t_a as shown in Fig. 4.1(*a*). The transient response of the body can be determined by *relating its rate of change of internal energy with convective exchange at the surface*. That is:



(*a*) General system for unsteady heat conduction



(*b*) Equivalent thermal circuit for lumped capacitance solid

Fig. 4.1. Lumped heat capacity system.

$$Q = -\,\rho Vc\,\frac{dt}{d\tau} = hA_s\,(t - t_a) \qquad ...(4.1)$$

where,
ρ = Density of solid, kg/m³,
V = Volume of the body, m³,
c = Specific heat of body, J/kg°C,
h = Unit surface conductance, W/m²°C,
t = Temperature of the body at any time, °C,
A_s = Surface area of the body, m²,
t_a = Ambient temperature, °C, and
τ = Time, s.

After rearranging the eqn. (4.1), and integrating, we get

$$\int \frac{dt}{(t - t_a)} = -\frac{hA_s}{\rho Vc}\int d\tau \qquad ...(4.2)$$

or,
$$ln\,(t - t_a) = -\frac{hA_s}{\rho Vc}\tau + C_1 \qquad ...(4.3)$$

The boundary conditions are:

At $\tau = 0$, $t = t_i$ (initial surface temperature)

$\therefore$ $C_1 = ln\,(t_i - t_a)$ [From eqn. (4.3)]

Hence
$$ln\,(t - t_a) = -\frac{hA_s}{\rho Vc}\tau + ln\,(t_i - t_a)$$ [Substituting the values in eqn. (4.3)]

or,
$$\frac{t - t_a}{t_i - t_a} = \frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \qquad ...(4.4)$$

Instantaneous and total heat flow rate : The instantaneous heat flow rate Q_i may be computed as follows :

$$Q_i = \rho V c \frac{dt}{d\tau}$$

$$= \rho V c \frac{d}{d\tau}\left[t_a + (t_i - t_a)\exp\left(-\frac{hA}{\rho V c}\tau\right)\right]$$

$$= \rho V c\left[(t_i - t_a)\left(-\frac{hA}{\rho V c}\right)\exp\left(-\frac{hA}{\rho V c}\tau\right)\right]$$

$$= -hA(t_i - t_a)\exp\left(-\frac{hA}{\rho V c}\tau\right) \quad ...(6.5)$$

and the total heat flow (loss or gain) is obtained by integrating equation 6.5 over the time interval $\tau = 0$ to $\tau = \tau$.

$$Q_t = \int_0^{\tau} Q_i \, d\tau$$

$$= \int_0^{\tau} -hA(t_i - t_a)\exp\left(-\frac{hA}{\rho V c}\tau\right)d\tau$$

$$= \left[-hA(t_i - t_a)\frac{\exp\left[-(hA/\rho V c)\tau\right]}{-hA/\rho V c}\right]_0^{\tau}$$

$$= \rho V c\,(t_i - t_a)\left[\exp\left(-\frac{hA}{\rho V c}\tau\right)\right]_0^{\tau}$$

$$= \rho V c\,(t_i - t_a)\left[\exp\left(-\frac{hA}{\rho V c}\tau\right) - 1\right] \quad ...(6.6)$$

In terms of non-dimensional Biot and Fourier numbers, we may write :

$$Q_i = -hA(t_i - t_a)\exp[-B_i F_0]$$

and $$Q_t = \rho V c\,(t_i - t_a)\left[\exp(-B_i F_0) - 1\right] \quad ...(6.7)$$

$$* \quad \frac{hA_s}{\rho \cdot V \cdot C} \cdot \tau = \left(\frac{hV}{K \cdot A_s}\right)\left(\frac{A_s^2 \cdot K}{\rho V^2 C} \cdot \tau\right) = \left(\frac{h \cdot L_c}{K}\right)\left(\frac{\alpha \cdot \tau}{L_c^2}\right)$$

$$\Rightarrow \boxed{\frac{h \cdot A_s}{\rho \cdot V \cdot C} \cdot \tau = B_i \times F_o}$$

Biot Number (B_i) ← → Fourier Number (F_o)

Biot Number (B_i): $B_i = \frac{h \cdot L_c}{K_{solid}}$, L_c = Characteristic Length.

It gives an indication of the ratio of internal (or conduction) resistance to the surface (or convection) resistance.

$$\text{Biot Number, } B_i = \frac{\left(\frac{L_c}{K \cdot A}\right)}{\left(\frac{1}{hA}\right)} = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

$$\Rightarrow B_i = \frac{h \cdot L_c}{K} = \frac{h}{(K/L_c)} \cdot \frac{\Delta T}{\Delta T}$$

$$= \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

Unit: Unitless or Dimensionless.

Significance: If $B_i \leq 0.1$, the Lumped heat approach can be used.

Fourier Number (F_o): $F_o = \frac{\alpha \cdot \tau}{L_c^2}$ (α → Thermal diffusivity; τ → time, sec.)

Unit: Dimensionless

Significance: It signifies the degree of penetration of heating or cooling effect through a solid.

Biot Number (B_i): $B_i = \frac{h.L_c}{K_{solid}}$, L_c = Characteristic Length.

It gives an indication of the ratio of internal (or conduction) resistance to the surface (or convection) resistance.

$$\text{Biot Number, } B_i = \frac{\left(\frac{L_c}{K.A}\right)}{\left(\frac{1}{hA}\right)} = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

$$\Rightarrow B_i = \frac{h.L_c}{K} = \frac{h}{(K/L_c)} \cdot \frac{\Delta T}{\Delta T}$$

$$= \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

Unit: Unit less or Dimensionless.

Significance:- If $B_i \leq 0.1$, the Lumped heat approach can be used.

** Biot number (B_i) is a measure of the relative magnitudes of the two heat transfer mechanisms: Convection at the surface and conduction through the solid. A small value of B_i indicates that the inner resistance ~~to the con~~ of the body to the heat conduction is small relative to the surface resistance to convection between the surface and the fluid. As a result the temp. distribution within the solid becomes fairly uniform, and Lumped System analysis becomes applicable.

* **Biot Number (B_i):** The Biot number approaches zero when the conductivity of solid is very large ($K \to \infty$) or the convection coefficient coefficient of heat transfer is very low ($h_c \to 0$), i.e, when the solid is practically isothermal and the temp change is mostly caused in the fluid by convection at the interface. On the contrary, the Biot number approaches infinity when the thermal resistance predominates ($K \to 0$) or the convection resistance is very low ($\frac{1}{h_c} \to 0$)

$$\text{Biot number, } B_i = \frac{(R_{th})_{cond.}}{(R_{th})_{conv.}} = \frac{hL_c}{K} \quad \left(\text{or, } \frac{hr_o}{K} \leftarrow \text{Cylinders}\right)$$

$B_i \to 0$, when $(R_{th})_{cond.} = \frac{r_o}{K} \to 0$ or $(R_{th})_{conv} = \frac{1}{h} \to \infty$

$B_i \to \infty$, when $(R_{th})_{conv.} = \frac{1}{h} \to 0$ or $(R_{th})_{cond.} = \frac{r_o}{K} \to \infty$

Fourier Number (F_o): $F_o = \frac{\alpha . \tau}{L_c^2}$ ← α: Thermal diffusivity; τ: time, sec.

Unit : Dimensionless

Significance:- It signifies the degree of penetration of heating or cooling effect through a solid.

* Fourier Number, $F_o = \frac{\alpha\tau}{L_c^2} = \frac{KL^2(1/L)}{\rho C_p L^3/\tau} \cdot \frac{\Delta t}{\Delta t} = \frac{\text{The rate at which heat is conducted across a body of thickness } L \text{ and normal area } L^2}{\text{The rate at which heat is stored in a body of volume } L^3}$

Therefore, the Fourier number is a measure of heat conducted through a body relative to heat stored.

Thus, a large value of the Fourier number indicates faster propagation of heat through a body.

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Characteristic Length for different Section

Characteristic length for sphere	$l_c = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{R}{3}$
Characteristic length for solid cylinder	$l_c = \dfrac{\pi R^2 L}{2\pi R(L+R)}$ $if\, l >> R, l_c = \dfrac{R}{2}$
Characteristic length for cube	$l_c = \dfrac{L^3}{6L^2} = \dfrac{L}{6}$
Characteristic length for rectangular plate	$l_c = \dfrac{lbt}{2lb} = \dfrac{t}{2}$
Characteristic length for hollow cylinder	$l_c = \dfrac{\pi(r_0^2 - r_i^2)l}{2\pi r_0 l + 2\pi r_i l + 2\pi(r_0^2 - r_i^2)}$

Temperature Measurement By Thermocouple

Measurement of temperature by a thermocouple is an important application of the lumped heat transfer.

* The response of a thermocouple is defined as the time required for the thermocouple to attain the source temperature.

* Time constant of thermocouple, $\tau^* = \frac{\rho . V . C}{h . A_s}$

* The larger the quantity '$\frac{h . A_s}{\rho V C}$', the faster the exponential term $\left(e^{-\frac{h A_s}{\rho V C}} \right)$ will approach ZERO or the more rapid will be the response of the temperature measuring device.

* For faster response of measuring device (Thermocouple), the time constant 'τ_{th}' should be small/less. This can be accomplished either by increasing the value of 'h' or the ratio $\left(\frac{A_s}{V}\right)$, $A_s >> V \Rightarrow$ Thin section (d↓) or by decreasing the density (ρ) of wire, sp. heat (c) of wire, mass of wire ($m = \rho V$) or by decreasing diameter (d) of wire (d) → Thin Wire.

* Hence, a very thin wire is recommended for the use in Thermocouple to ensure a rapid response (especially when the thermocouple are employed for measuring transient temperature.).

Sensitivity or Time Constant Of Thermocouple :-

* The sensitivity of a thermocouple is the time required for a thermocouple to reach 0.638 or 63.8% of the initial temperature difference.
* The time constant (τ^*) is defined as the time required to yield a value of unity for exponent term in the transient relation.

$$B_i F_o = 1$$

Transient Conduction With Given Temp. Distribution

Problem-1 :- The temperature distribution across a large concrete slab 500 mm thick heated from one side as measured by thermocouples approximate to the following relation:

$$t = 120 - 100x + 24x^2 + 40x^3 - 30x^4$$

where 't' is in °c and 'x' is in metres, Considering an area of $4\ m^2$, calculate : Use, $K_{concrete} = 1.20\ W/m°c$, $\alpha = 1.77 \times 10^{-3}\ m^2/h$

(i) the heat entering and leaving the slab in unit time.

We have $t = 120 - 100x + 24x^2 + 40x^3 - 30x^4$

$$\therefore \frac{dt}{dx} = -100 + 48x + 120x^2 - 120x^3$$

$$\Rightarrow \frac{d^2t}{dx^2} = 48x + 240x - 360x^2$$

Heat entering the slab, $Q_{in} = -K \cdot A \left[\frac{dt}{dx}\right]_{x=0}$

$$= -(1.20 \times 4) \times (-100)$$

$$= 600\ W \quad \text{Ans}$$

Heat leaving the slab, $Q_{out} = -K \cdot A \left[\frac{dt}{dx}\right]_{x=0.5}$

$$= -(1.20 \times 4)(-100 + 24 + 30 - 15)$$

$$= 366\ W \quad \text{Ans}$$

(ii) the heat energy stored in unit time :

Rate of heat storage $= Q_{in} - Q_{out}$

$$= 600 - 366 = 234\ W \quad \text{Ans}$$

(iii) the rate of temp change at both sides of the slab

Rate of temp. change is given by, $\frac{dt}{d\tau} = \alpha \cdot \frac{d^2t}{dx^2}$

$$\Rightarrow \frac{dt}{d\tau} = \alpha(48 + 240x - 360x^2)$$

$$\left(\frac{dt}{d\tau}\right)_{x=0} = 1.77 \times 10^{-3} \times 48 = 0.08496\,°c/h \quad \text{Ans}$$

$$\left(\frac{dt}{d\tau}\right)_{x=0.5} = 1.77 \times 10^{-3} \times (48 + 240 \times 0.5 - 360 \times 0.5^2) = 1.3806\,°c/h \quad \text{Ans}$$

(iv) the point where the rate of heating or cooling is maximum:

For the rate of heating or cooling to be maximum,

$$\frac{d}{dx}\left(\frac{dt}{d\tau}\right) = 0$$

$$\Rightarrow \frac{d}{dx}\left[\alpha \cdot \frac{d^2t}{dx^2}\right] = 0$$

$$\Rightarrow \frac{d^3t}{dx^3} = 0$$

$$\Rightarrow 240 - 720x = 0$$

$$\Rightarrow \boxed{x = 0.333 \text{ m}} \quad \text{Ans}$$

Q. For $B_i << 1$, the temp. gradient in the solid is small and $T(x,t) \approx T(t)$ and the solid temp remains nearly uniform (i.e. temp. gradient, $\frac{dt}{dx} = 0$) (True)

For $B_i >> 1$, the temp. difference across the solid is very large. (True)

Q. When both conduction and convection resistances are almost of importante, the Heisler charts are extensively used to determine the temperature distribution. (True/ False)

(Ans : True)

state

Problem-2: A steel ball 100 mm in diameter and initially at 900°C is placed in air at 30°C. Find:

(i) temp. of the ball after 30 seconds;

(ii) the rate of cooling (°C/min) after 30 seconds;

(iii) Instantaneous heat transfer rate 2 minutes after start of cooling

(iv) Total energy transferred from the sphere during the first 2 minutes.

Take: $h = 20\ W/m^2{}^\circ C$, $K(\text{steel}) = 40\ W/m^\circ C$, $\rho(\text{steel}) = 7800\ kg/m^3$

$c(\text{steel}) = 460\ J/kg^\circ C$, $\alpha = 0.043\ m^2/hr$

Solution:– Given data: $d = 100\ mm$, $r = \frac{100}{2} = 50\ mm = 0.050\ m$

$t_i = 900°C$, $t_a = 30°C$, $h = 20\ W/m^2{}^\circ C$, $K = 40\ W/m^\circ C$

$\rho = 7800\ kg/m^3$, $c(\text{steel}) = 460\ J/kg^\circ C$, $\alpha = 0.043\ m^2/hr$.

Characteristic length, $L_c = \frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{0.05}{3} = 0.01667\ m$

Biot number, $B_i = \frac{hL_c}{K} = \frac{20 \times 0.01667}{40} = 0.008335$

Since $B_i < 0.1$, hence Lumped heat analysis can be applied for the solution of the problem.

(i) We know, $\frac{t - t_a}{t_i - t_a} = e^{\left[-\frac{hA_s}{\rho V c}\cdot \tau\right]}$

$$\Rightarrow \frac{t - 30}{900 - 30} = e^{\left[-\frac{20}{7800 \times 460} \times \frac{1}{0.01667} \times 30\right]} = 0.01$$

$$\Rightarrow \boxed{t = 891.3°C} \quad \text{Ans}$$

(ii) The rate of cooling (°C/min) after 30 seconds $\left(\frac{dt}{d\tau}\right)$

$$\frac{d}{d\tau}\left[\frac{t - t_a}{t_i - t_a}\right] = \frac{d}{d\tau}\left\{e^{\left[-\frac{hA_s}{\rho V c}\cdot \tau\right]}\right\}$$

$$\Rightarrow \frac{1}{t_i - t_a} \cdot \frac{dt}{d\tau} = -\frac{hA_s}{\rho V c}\, e^{\left[-\frac{hA_s}{\rho V c}\tau\right]}$$

$$\Rightarrow \frac{1}{900 - 30} \times \frac{dt}{d\tau} = -\frac{20}{7800 \times 460} \times \frac{1}{0.01667} \times 0.01$$

$\Rightarrow \frac{dt}{d\tau} = -0.288 \ ^{\circ}C/sec = -0.288 \times 60$

$\Rightarrow \boxed{\frac{dt}{d\tau} = -17.28^{\circ} C/min}$ ← Ans

↑ (cooling)

(iii) Instantaneous heat transfer rate (Q_i)

$$Q_i = -hA_s (t_i - t_a) e^{\left[-\frac{hA_s}{\rho V c} \cdot \tau\right]}$$

$$= -20 \times 4\pi (0.05)^2 \cdot (900-30) e^{\left[-\frac{20}{7800 \times 460} \times \frac{1}{0.01667} \times (2 \times 60)\right]}$$

$= -$ ______ W.

(iv) Total energy transferred from the sphere

$$Q_{total} = -hA_s (t_i - t_a) \int_0^{(2\times60)} e^{\left[-\frac{hA_s}{\rho V c} \cdot \tau\right]} \cdot d\tau$$

$$= -hA_s (t_i - t_a) \times \frac{(2\times60)}{B_i \cdot F_o} \times \left(1 - e^{-B_i F_o}\right)$$

$$= -20 \times 4\pi (0.05)^2 \times (900-30) \times \frac{120}{\quad}$$

$= -$ ______ J

Example 4.2. *An aluminium alloy plate of 400 mm × 400 mm × 4 mm size at 200°C is suddenly quenched into liquid oxygen at – 183°C. Starting from fundamentals or deriving the necessary expression determine the time required for the plate to reach a temperature of – 70°C. Assume h = 20000 kJ/m²-h-°C, c_p = 0.8 kJ/kg°C, and ρ = 3000 kg/m³.* **(AMIE Winter, 1997)**

Solution. Surface area of the plate, $A_s = 2 \times \frac{400}{1000} \times \frac{400}{1000} = 0.32 \text{ m}^2$

Volume of the plate, $V = \frac{400}{1000} \times \frac{400}{1000} \times \frac{4}{1000} = 0.00064 \text{ m}^3$

Characteristic length, $L_c = \frac{V}{A_s} = \frac{0.00064}{0.32} = 0.002 \text{ m}$

k for aluminium, at low temperatures may be taken as 214 W/m°C or 770.4 kJ/mh°C.

∴ Biot number, $B_i = \frac{hL_c}{k} = \frac{20000 \times 0.002}{770.4} = 0.0519$

Since B_i is less than 0.1, hence lumper capacitance method may be applied for the solution of the problem.

The temperature distribution is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho V c}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

(For derivation of this relation please refer to Article 4.2)

$$\frac{-70 - (-183)}{200 - (-183)} = \exp\left[-\frac{20000 \times 0.32}{3000 \times 0.00064 \times 0.8} \cdot \tau\right]$$

or,
$$0.295 = e^{-4166.67\,\tau} = \frac{1}{e^{4166.67\,\tau}}$$

or,
$$e^{4166.67\,\tau} = \frac{1}{0.295} = 3.389$$

or,
$$4166.67\,\tau = \ln 3.389 = 1.2205$$

∴
$$\tau = \frac{1.2205}{4166.67} \times 3600 = \mathbf{1.054 \text{ s}} \text{ (Ans.)}$$

Example 4.3. *A solid copper sphere of 10 cm diameter [ρ = 8954 kg/m³, c_p = 383 J/kg K, k = 386 W/m K], initially at a uniform temperature t_i = 250°C, is suddenly immersed in a well-stirred fluid which is maintained at a uniform temperature t_a = 50°C. The heat transfer coefficient between the sphere and the fluid is h = 200 W/m² K. Determine the temperature of the copper block at τ = 5 min after the immersion.* **(AMIE Winter, 1998)**

Solution. *Given* : D = 10 cm = 0.1 m; ρ = 8954 kg/m³; c_p = 383 J/kg K; k = 386 W/m K; t_i = 250°C; t_a = 50°C; h = 200 W/m² K; τ = 5 min = 300 s.

Temperature of the copper block, t :

The characteristic length of the sphere is,

$$L_c = \frac{\text{Volume }(V)}{\text{Surface area }(A_s)} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = \frac{0.1}{6} = 0.0167 \text{ m}$$

Biot number,
$$B_i = \frac{hL_c}{k} = \frac{200 \times 0.01667}{386} = 8.64 \times 10^{-3}$$

Since B_i is less than 0.1, hence lump capacitance method (Newtonian heating or cooling) may be applied for the solution of the problem.

The temperature distribution is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho V c} \cdot \tau\right] \quad \text{...[Eqn. (4.4)]}$$

Substituting the value, we get

$$\frac{t - 50}{250 - 50} = \exp\left[-\frac{200}{8954 \times 0.01667 \times 383} \times 300\right] = 0.35$$

$$\left(\because \frac{A_s}{V} = \frac{L}{L_c} = \frac{1}{0.01667}\right)$$

∴
$$t = (250 - 50) \times 0.35 + 50 = \mathbf{120°C} \text{ (Ans.)}$$

Example 4.6. *A steel ball 50 mm in diameter and at 900°C is placed in still atmosphere of 30°C. Calculate the initial rate of cooling of the ball in °C/min.*

Take: $\rho = 7800$ kg/m³, $c = 2$ kJ/kg°C (for steel); $h = 30$ W/m²°C.

Neglect internal thermal resistance. **(M.U.)**

Solution. *Given*: $R = \frac{50}{2} = 25$ mm $= 0.025$ m; $t_i = 900$°C; $t_a = 30$°C, $\rho = 7800$ kg/m³;

$C = 2$ kJ/kg°C; $h = 30$W/m²°C; $\tau = 1$ min $= 60$ s.

The temperature variation in the ball (with respect to time), neglecting internal thermal resistance, is given by:

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

where,
$$\frac{hA_s}{\rho Vc} \cdot \tau = \frac{h \times 4\pi R^2}{\rho \times \frac{4}{3}\pi R^3 \times c}\tau = \frac{3h\tau}{\rho Rc} = \frac{3 \times 30 \times 60}{7800 \times 0.025 \times (2 \times 1000)} = 0.01385$$

Substituting the values in the above equation, we get

$$\frac{t - 30}{900 - 30} = e^{-0.01385} = \frac{1}{e^{0.01385}} = 0.9862$$

or,
$$t = 30 + 0.9862\,(900 - 30) = 888\text{°C}$$

∴ Rate of cooling $= 900 - 888 =$ **12°C/min. (Ans.)**

Example 4.7. *A cylindrical ingot 10 cm diameter and 30 cm long passes through a heat treatment furnace which is 6 m in length. The ingot must reach a temperature of 800°C before it comes out of the furnace. The furnace gas is at 1250°C and ingot initial temperature is 90°C. What is the maximum speed with which the ingot should move in the furnace to attain the required temperature? The combined radiative and convective surface heat transfer coefficient is 100 W/m² °C. Take k (steel) = 40 W/m°C and α (thermal diffusivity of steel) = 1.16 × 10⁻⁵ m²/s.*

Solution. *Given* : $D = 10$ cm $= 0.1$ m; $L = 30$ cm $= 0.3$ m; $t_i = 1250$°C; $t = 800$°C; $t_a = 90$°C; $k = 40$ W/m°C; $h = 100$ W/m²°C; $\alpha = 1.16 \times 10^{-5}$ m²/s.

Characteristic length,
$$L_c = \frac{V \text{ (volume)}}{A_s \text{ (surface area)}} = \frac{\frac{\pi}{4}D^2L}{\left[\pi DL + \frac{\pi}{4}D^2 \times 2\right]} = \frac{DL}{4L + 2D}$$

$$= \frac{0.1 \times 0.3}{4 \times 0.3 + 2 \times 0.1} = 0.02143 \text{ m}$$

Biot number,
$$B_i = \frac{h\,L_c}{k} = \frac{100 \times 0.02143}{40} = 0.0536$$

As $B_i < 0.1$, then internal thermal resistance of the ingot for conduction heat flow can be neglected.

∴ The time versus temperature relation is given as

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right]$$

Now,
$$\frac{hA_s}{\rho Vc} = \frac{k}{k} \cdot \frac{hA_s}{\rho Vc} = \left(\frac{k}{\rho c}\right)\left(\frac{h}{k}\right)\left(\frac{A_s}{V}\right) = \alpha \cdot \frac{h}{k} \cdot \frac{A_s}{V}$$

$$= 1.16 \times 10^{-5} \times \frac{100}{40} \times \frac{1}{0.02143} = 0.001353$$

Substituting the values in the above equation, we get

$$\frac{800 - 90}{12 - 90} = e^{-0.001353\tau} = \frac{1}{e^{0.001353\tau}}$$

or,
$$0.612 = \frac{1}{e^{0.001353\tau}} \text{ or } e^{0.001353\tau} = 1.634$$

or,
$$0.001353\tau = ln\,(1.634) = 0.491$$

∴
$$\tau = \frac{0.491}{0.001353} = 362.9 \text{ s}$$

Velocity of ingot passing through the furnace,

$$v = \frac{\text{Furance length}}{\text{Time}} = \frac{6}{362.9} = \mathbf{0.01653\ m/s}\ \textbf{(Ans.)}$$

The time constant of the thermocouple is the time required by a thermocouple to reach the following value of initial temperature differences :

(a) 63.2%
(b) 65%
(c) 68%
(d) 70.2%

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Ans. (a) : 63.2%

Thank You

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