Strength Of Material

[All Modules]

Semester : 3rd

Branch : Mechanical Engineering

Byomakesh Panda

Dept. Of Mechanical Engg.

9th JULY,
TUESDAY

STRENGTH OF MATERIALS (6-8)

Strength : resistance to failure is called strength. It is a material property.

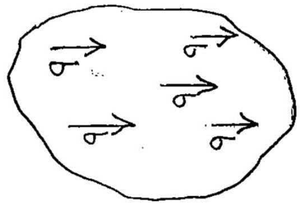
$$\left.\begin{array}{l} M20 \Rightarrow f_{ck} = 20 \text{ MPa} \\ M15 \Rightarrow f_{ck} = 15 \text{ MPa} \end{array}\right\} \text{@ failure, stress developed = streng}$$

Stiffness : resistance against deformation is stiffness. This is a secondary design property. $K\uparrow \ \delta\downarrow$

Assumptions :

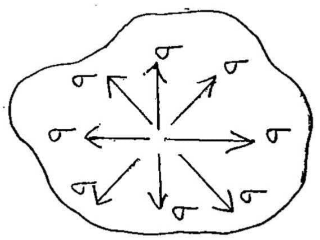
1. Material is continuous. (no voids or no cracks)
2. Material is homogenous and isotropic.

$\underbrace{\text{Homo}}_{\text{same}}\underbrace{\text{genous}}_{\text{origin}}$ – Eg:- wood, iron, gold.
steel, brass, bronze (not homogenous).



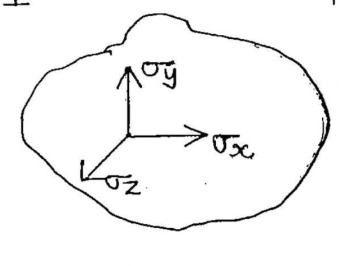
at any point in one direction, same prope

$\underbrace{\text{Iso}}_{\text{same}}\ \underbrace{\text{tropic}}_{\text{directional property}}$ – Eg:- fine grained material (iron, gold, stee
wood (non isotropic).

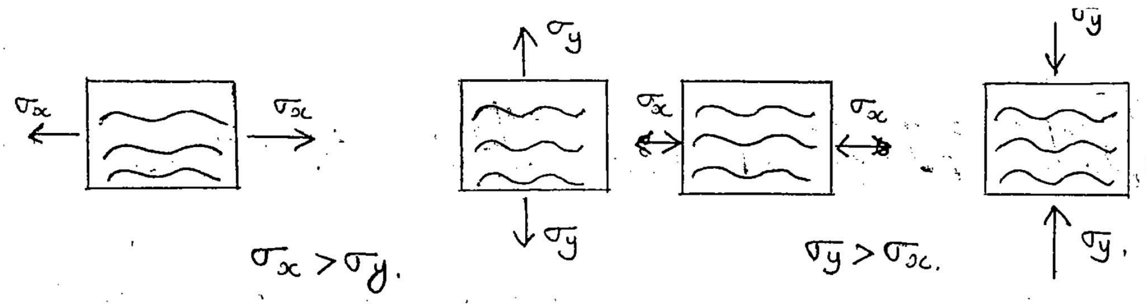


at ~~any~~ one point in any direction, same property.

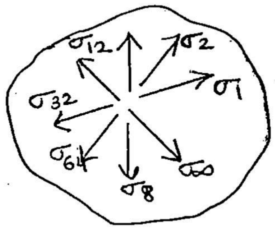
$\underbrace{\text{Ortho}}_{\perp^r}\underbrace{\text{tropic}}_{\text{directional property}}$ – Eg:- Layered material (wood, sedimentary roc
marble, graphite, mica



at one point in $\perp^r$ direction property are different.



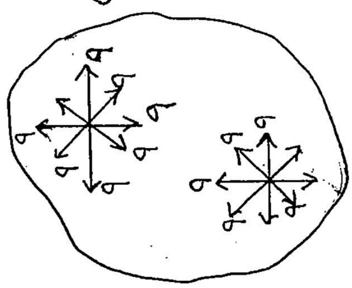
Anisotropic (Non-Isotropic) / Aleotropic



@ one point in different direction property different.

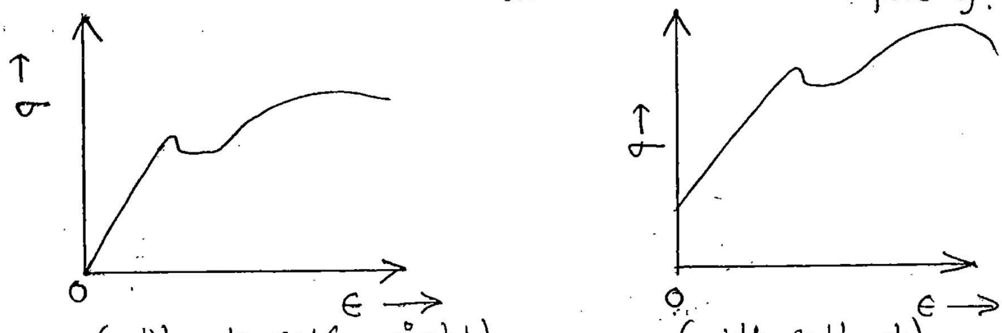
Eg:- Material with cracks and voids

Homogenous + Isotropic – Eg :- Iron, copper, gold.



@ any point in any direction, same property

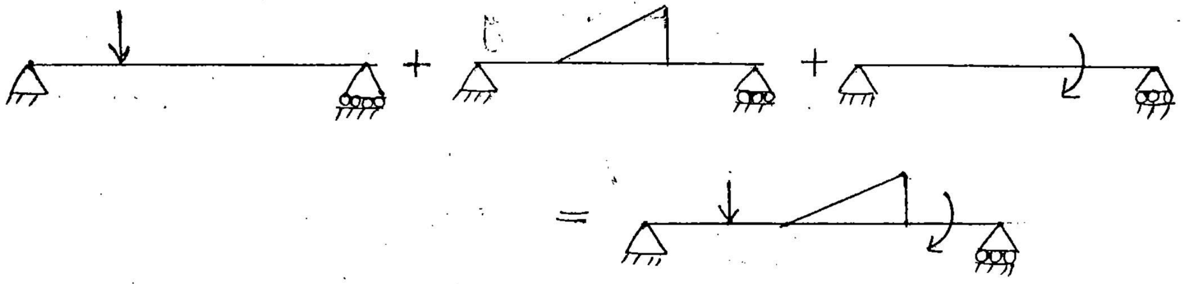
3. Self weight neglected (stress vs strain starts from origin due to this assumption).



(without self weight) (with self wt.)

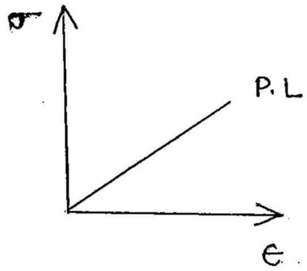
4. Superposition Principle is valid.

Algebraic sum of various effects is equal to the total effect.



Limitations of Super position Principle :

(i) Linear elastic members.



Robert Hooke's law is valid.

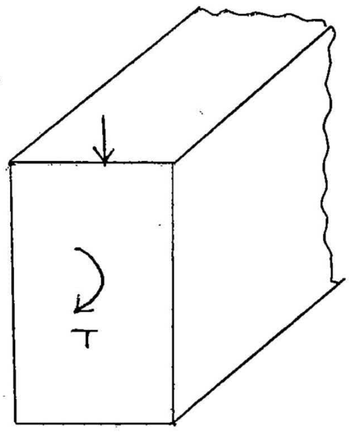
Loads must be upto P.L.

(ii) Deformations are very small.

Not valid for:

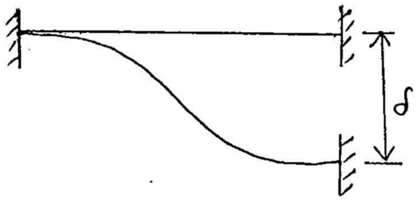
(i) Deep beam.

D > 750 mm.



In deep beams torsion develops due to loading which causes distortion in shape

(ii) Sinking of supports.

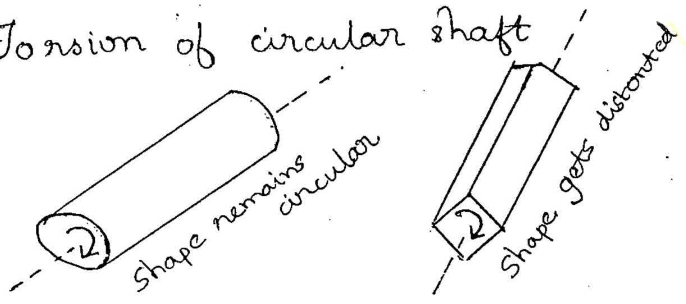


axis gets (curved) distorted.

(iii) Long Columns.

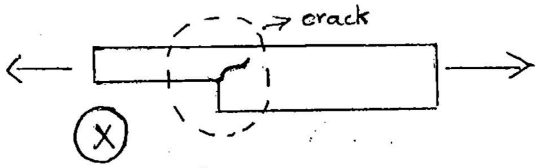
Buckling occurs.

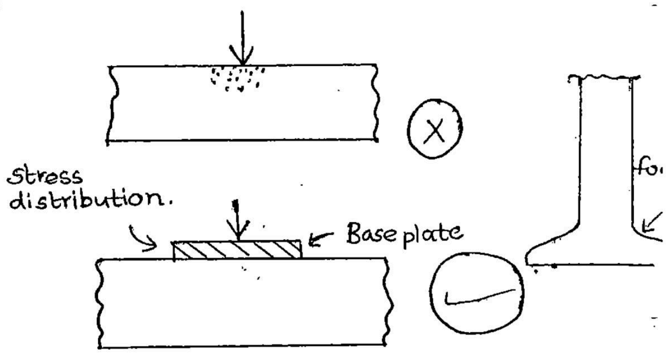
(iv) Torsion of circular shaft

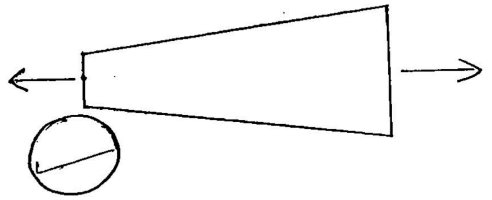


5. St. Venent's Principle is valid.

Sudden change in any parameter causes stress concentra







Stress

The Internal resistance developed against deformation per unit area. is called stress.

$$\sigma = \frac{\text{resisting force}}{\text{Unit area}} \; ;$$

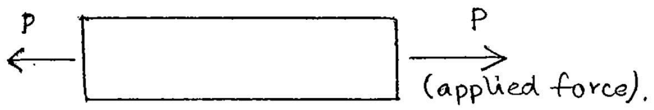
Unit of stress $= N/m^2$

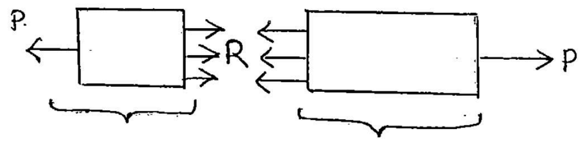
$kPa = kN/m^2$

$* \; MPa = N/mm^2$

$GPa = 10^3 \; N/mm^2$

$= 10^3 \; MPa$





$\Sigma F_x = 0$ $\quad$ $\Sigma F_x = 0$

$P = R$ $\quad$ $P = R$

$$\therefore \; \sigma = \frac{P}{A} = \frac{R}{A}.$$

NOTE: ⊙ A member free to deform without showing reaction or resistance will have zero stress.

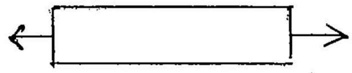
⊙ A member free to move away without any frictional resistance, stress developed is zero.

⊙ A member free to expand or contract due to temperat. change, there will be no stress.

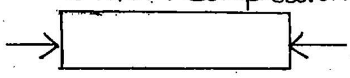
Classification of Stress :

- Classification.
  - Normal
    - Direct
    - Indirect: Bending / flexural stress.
  - Shear.
    - Direct: Tangential force/unit area.
    - Indirect: Torsional Shear stress.

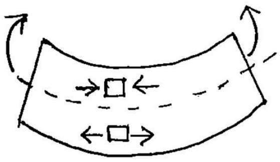
TIE : Tension



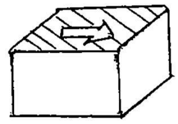
STRUT : Compression



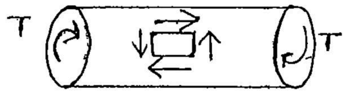
$$\sigma = \frac{R}{A} = \frac{P}{A}$$



$$f = \frac{M}{Z}.$$



$$\tau = \frac{F}{A}.$$



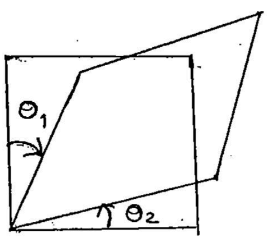
$$\tau = \frac{T}{Z_P}$$

Strains :

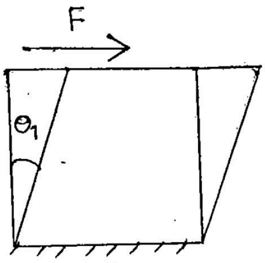
(i) Normal strain (due to normal force),

$$e, \epsilon = \frac{\text{Change in dimension}}{\text{Original dimension.}} \text{; unitless.}$$

(ii) Shear strain (due to shear force) → angular change or distortion b/w any two mutually perpendicular planes in radian is Shear Strain.



$\phi = \theta_1 + \theta_2$



$\theta_2 = 0$

$\phi = \theta_1 + 0$ (angle coming alone, ∴ it should be in radians)

NOTE : As radian is a secondary unit, its dimensionless.

(iii) Volumetric Stress (due to normal force),

$$e_v = \epsilon_v = \frac{\delta V}{V} \quad ; \text{ No unit}$$

NOTE: ⊙ Normal forces can cause change in dimensions as well as volume.

⊙ Shear forces can change the shape without change in volume.

⊙ External force ⟶ Deformation ⟶ Resistance ⟶ Stress

Deformation ⟶ Strain.

Strain is independent & stress depends on strain.

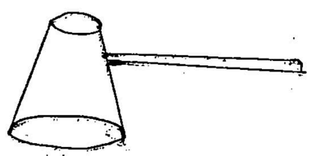
Material Properties:

1. Elasticity → ability to regain shape on removal of external force.

2. Plasticity → member undergoes permanent or plastic deforma at constant load.

3. Ductility ⟶ ~~material~~ can be made into thin wires.
   Eg:- All soft metals (Au, Ag, Al, Cu, steel)
   Ductility is related to tension. Ductile materials are strong in tension and weak in shear. They are moderate in compression.

4. Malleability → pressed into thin sheets.
   Eg: all ductile materials.
   Properties of malleable and ductile are the same.



– mallet. (malleability).
Its related to compression.

5. Brittle ⟶ fails suddenly
   Eg: Cast Iron, concrete, glass.
   All brittle materials are strong in compression and weak in tension. and moderate in shear.

6. Creep – The plastic or permanent deformation due to constant load with time (4)

5

Aug, SATURDAY

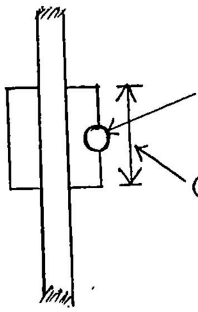
Stress-Strain Curves

* Low Carbon Steels

a) Mild Steel (Fe 250)

Carbon ($\leqslant 0.15\%$) : Carbon is the strength parameter.

Manganese : increases toughness. (resistance to impact loading)



U.T.M (Universal Testing Machine)

[UTM can be used for measuring shear, tension, compression, flexure, torsion etc and ∴ called as Universal.]

Gauge length is independent of length of bar, shape of c/s, rate of loading.

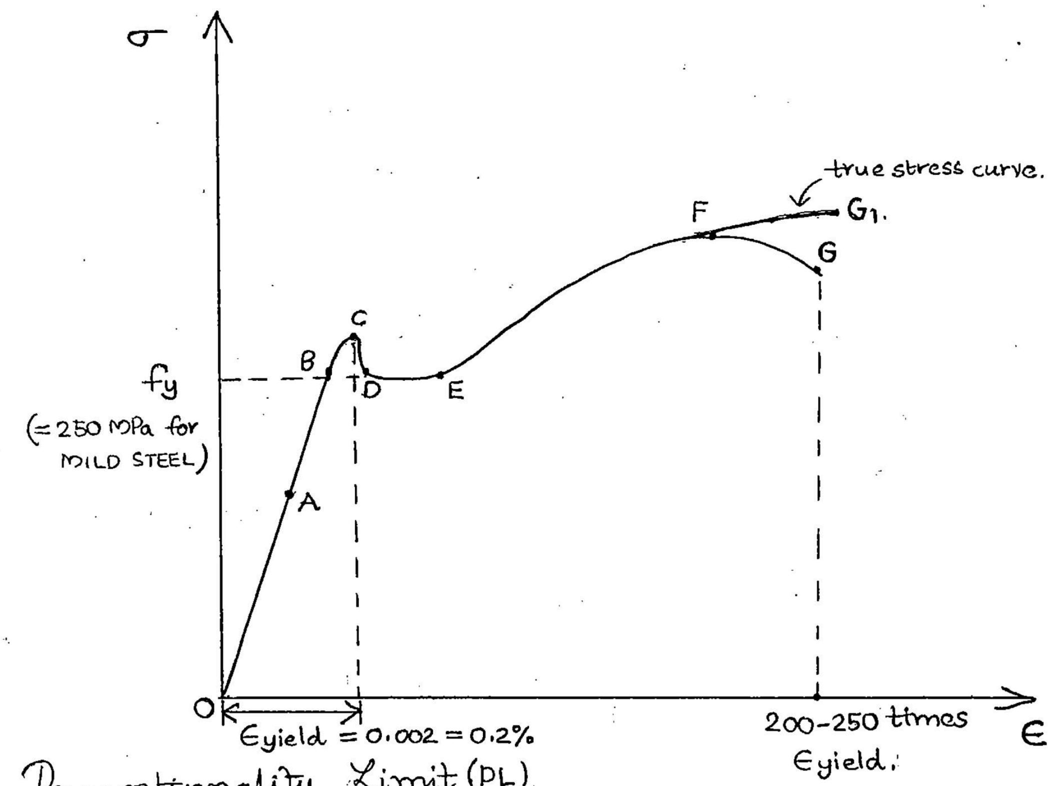
UTM is strain oriented. Resistance offered by the bar is given by Load Dial.

$$\text{Strain} = \frac{\delta(GL)}{GL}$$

$\sigma = \frac{P}{A}$ ← load dial reading ; σ = nominal stress / Initial stress / Engg. stress / Stre

True stress or Instantaneous or Actual stress, $\sigma_o = \frac{P}{A_o}$

$A_o \longrightarrow$ true / instantaneous / actual area.



A : Proportionality Limit (PL)

ie upto A, $\sigma \propto \epsilon$

OA is a straight line.

OA is linear elastic.

Hooke's Law is valid upto PL only.

B : Elastic Limit (EL)

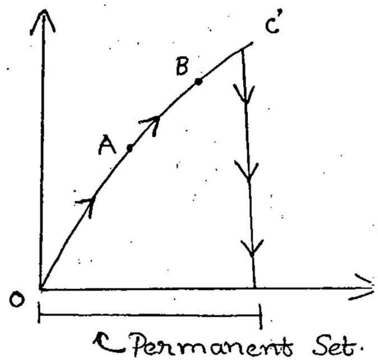
ie upto B, material is elastic.

A to B : graph is slightly curved.

Hooke's Law not valid.

AB : Non linear elastic zone.

NOTE : Loading Beyond Elastic limit causes 'permanent Set' or 'Plastic Deformation' or 'Residual strain' in the material.



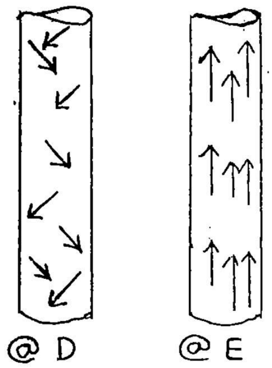
C : Upper Yield Point.

At yield point, resistance of the material suddenly drops down, which occurs at a strain of 0.002 in most of the metals.

$$\boxed{\epsilon_{yield} = 0.002 = 0.2\%}$$

D : lower yield point.

DE : Plastic Zone / Permanent Deformation.

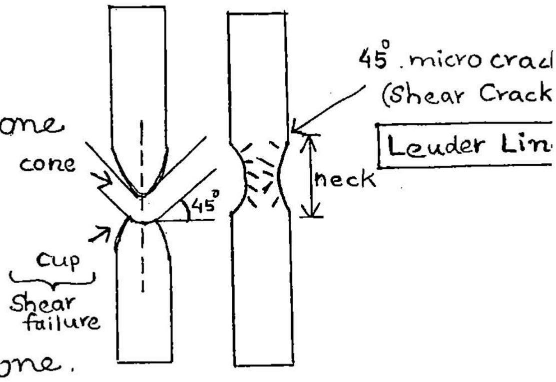


In plastic zone, reorientation of molecules occur. Due to this material becomes nearly homogenous and start resisting the loading

F : Ultimate point (Ultimate stress)

G : Brittle Point (Brittle stress).

Zones :



OA = linear elastic zone

AB = non-linear elastic zone

CD = yield zone.

DE = plastic zone

EF = strain hardening zone.

FG = necking zone / Strain softening zone

In strain hardening zone (EF), material undergoes higher strain to resist little external forces.

Lower yield point (D) is the design stress. in all the designs like Working Stress method, Plastic Theory, Ultimate Load method, Limit State method etc. It is the yield stress corresponding to D. The position of upper yielding point is not stable which may change based on shape and size of specimen used. ∴ lower yield point is preferred in design.

$$\text{Ductility Factor, DF} = \frac{\epsilon_{fail}}{\epsilon_{yield}}$$

For mild steel,

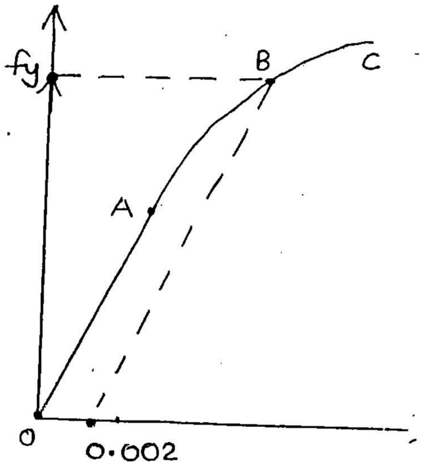
$$DF = 200 \text{ to } 250$$

14th Sept,
SUNDAY

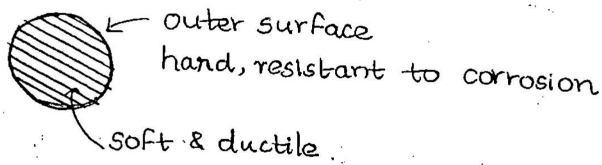
* High Carbon Steel

- Carbon increases strength and hardness but decreases ductility and toughness.

Eg : HYSD Fe 415, Fe 500 (not used nowadays)
TMT Fe 415, Fe 500 (used widely)



TMT – Thermo Mechanically Treated steel.



- Manganese increases toughness.

- Proof Stress or Yield Stress.

It is the stress corresponding to fixed strain (0.2%) is called Proof stress. It is used when exact yield stress is not known. It is obtained by 'Offset method'.

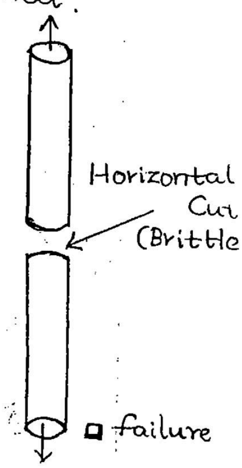
fy → yield or proof stress.

Zones :

OA = linear elastic (Hooke's Law is valid)

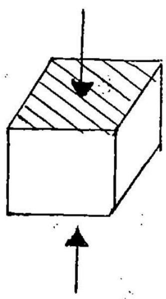
AB = non linear elastic (Hooke's Law is not valid)

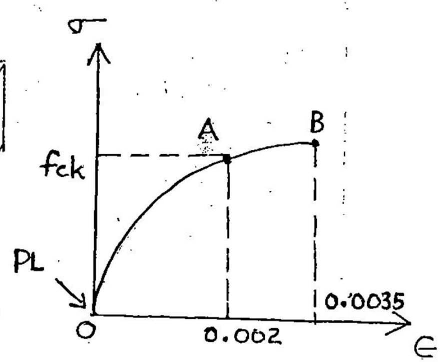
BC = strain hardening zone.



→ Brittle Material.

- Stronger in compression
- moderate in shear
- weak in tension.





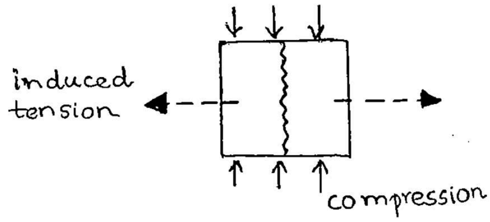
Eg: Concrete, Cast iron, glass.

- Brittle materials are tested in compression whereas ductile materials are tested in tension.

In case of brittle materials, PL will be very close to origin.

A = First cracking point.

B = Failure point.

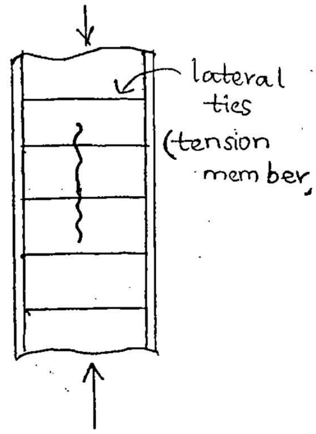


Stress corresponding to A = fck.

fck = first cracking stress (or) ultimate stress.

Stress corresponding to A = Stress corresponding to B.

Crack formation is due to induced tension.



Lateral ties are used for the confinement of concrete.

- Zones:

OA = non linear elastic

AB = strain hardening zone.
(crack widening zone)

- Ductility Factor $= \dfrac{\epsilon_{fail}}{\epsilon_{first\ crack}} = \dfrac{0.0035}{0.002} = 1.75$

- Factor of Safety:

Ductile, $FS = \dfrac{\text{yield stress}}{\text{Working stress}}$

Brittle, $FS = \dfrac{\text{ultimate stress}}{\text{working stress}}$.

- Margin of safety:

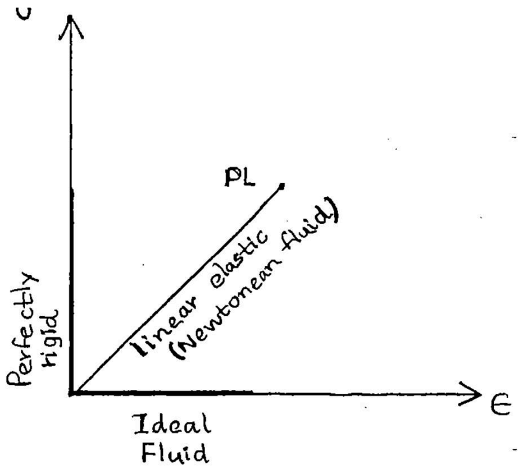
Margin of safety = FS − 1.

used by aerospace engineers where high ductile materials are used in the aeroplane construction. ∵ high ductile materials are used, less FS is required.

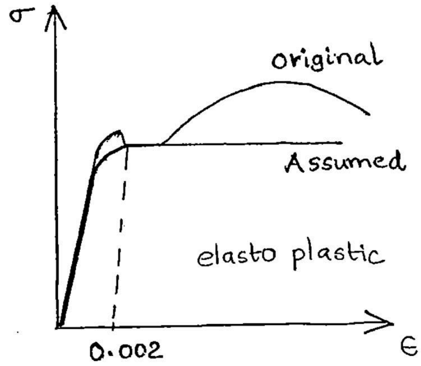
→ Idealised $\sigma - \epsilon$ curves

- assumed
- can be used in designs directly.
- For a perfectly rigid body, there won't be any dimension changes or volumetric changes. ($\delta V = 0$)

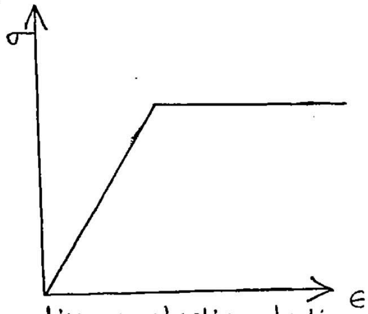
Eg: Diamond, glass.



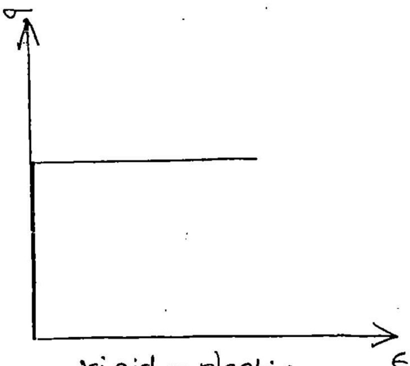
- Ideal Fluid will have dimension changes but no volume changes. as an ideal fluid has no viscosity, no surface tension, incompressible ($\delta V = 0$), irrotational.



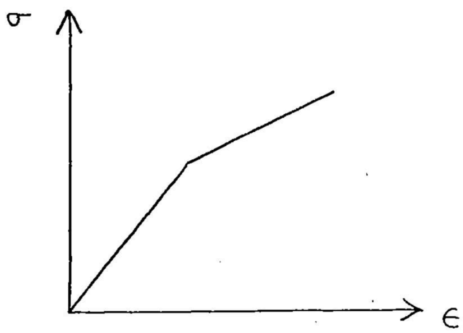
LSM → Idealised $\sigma - \epsilon$ curve for MS.



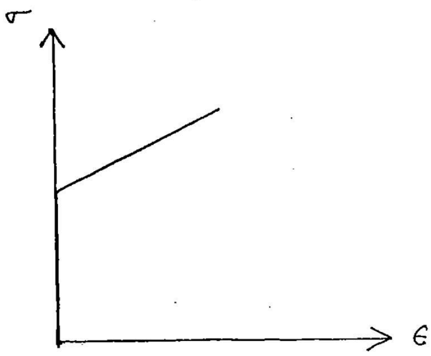
linear elastic-plastic
elasto-plastic.



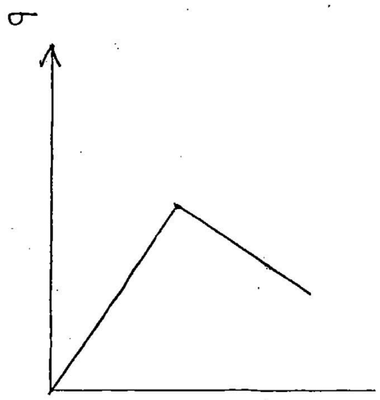
rigid-plastic



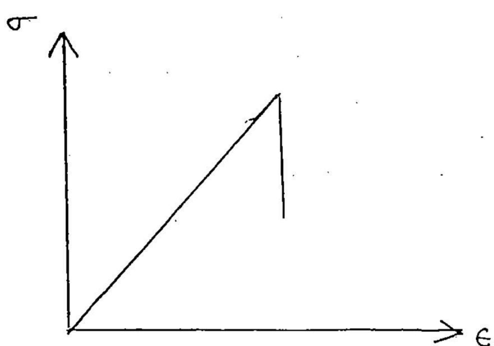
Linear elastic- strain hardening



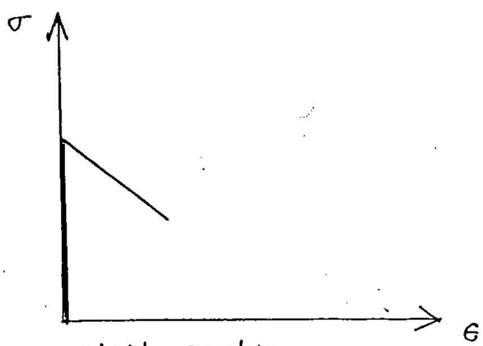
Rigid-strain hardening



linear elastic-
strain softening
(necking)



Linear elastic-yielding



rigid-necking

→ Elastic Constants

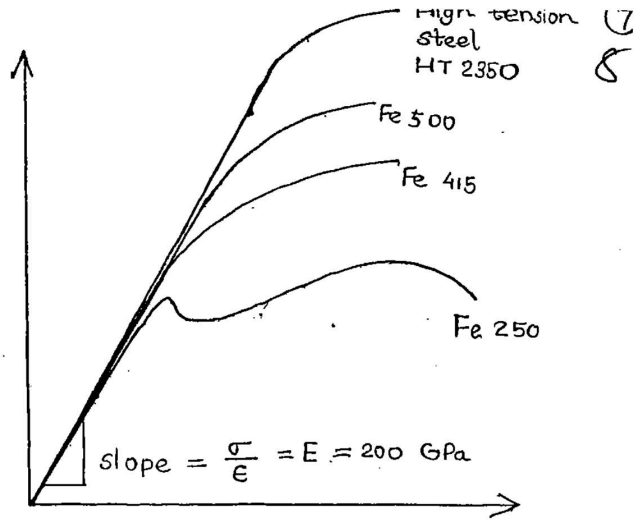
– Within elastic limit

$$\sigma \propto \epsilon$$

– valid exactly upto PL.

$$\sigma = E\epsilon$$

$$\therefore E = \frac{\sigma}{\epsilon}$$

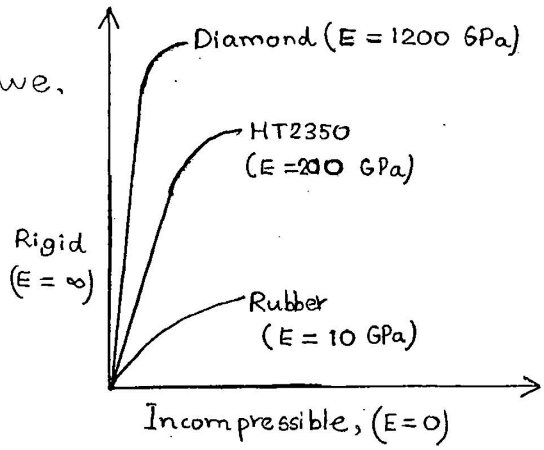


E → Young's modulus (or) Modulus of Elasticity.

It is a non zero positive value and constant for a given material under any conditions.

$$E\ (\text{steel}) = 200\ \text{GPa}$$
$$= 200 \times 10^3\ \text{MPa}$$

For all grades irrespective of carbon.



– E is the slope of $\sigma - \epsilon$ curve.

As slope increases, E also increases.

– Higher the E value, higher will be the elasticity.

– within elastic limit,

Hooke's law in shear stress gives,

$$\tau \propto \gamma \quad \text{(valid upto PL)}$$

$$\tau = G\gamma$$

$$C, N, G = \frac{\tau}{\gamma}$$

G, N, C → shear modulus, (or) rigidity modulus (or) modulus of rigidity

↑ G ⇒ ↓ γ (shear strain)

↓ distortion in shape.

- volumetric stress $\propto$ volumetric strain.

$$\sigma \propto \epsilon_v$$

$\sigma \rightarrow$ Uniform Normal Stress acting all around volume (or) Volumetric stress (or) hydrostatic pressure.

⊙ On a submerged body with hydrostatic pressure, there will be only volumetric changes without change in shape.

$\therefore$ shear stress is zero.

$$\sigma = K . \epsilon_v$$

$$\left.\begin{array}{l}\text{Bulk modulus (or)}\\ \text{Dilation constant}\end{array}\right\} \quad K = \frac{\sigma}{\epsilon_v}$$

Dilation means change in volume.

-K is used only for hydrostatic pressure conditions.

$$\uparrow K \Rightarrow \epsilon_v \downarrow \text{ ie, } \partial v \downarrow \qquad \left\{ \epsilon_v = \frac{\partial v}{V} \right\}$$

$$\downarrow K \Rightarrow \delta v \uparrow$$

$\rightarrow \frac{1}{K}$ = compressibility.

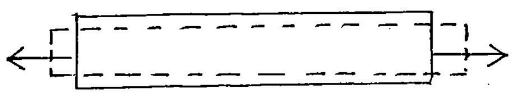
Rigid body $(\delta v = 0)$, $K = \infty$

Incompressible material, $(\delta v = 0)$, $K = \infty$

$\boxed{E > K > G}$; for isotropic material.

→ Poisson's Ratio $(\mu, \gamma, 1/m)$

$$\mu = -\left(\frac{\epsilon_{lat}}{\epsilon_{lin}}\right)$$

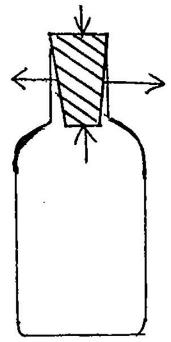


μ has no units.

Range of μ : ~~+ve~~ -ve to 0.5

For genetic material, μ is -ve.

For engg. material, $0 \leq \mu \leq 0.5$



⊙ μ (cork) $= 0$

⊙ $\mu = 0.5$; for incompressible, non dilatant ($dV = 0$)

Eg: Ideal fluids, water.

For rubber, clay, paraffin wax, mercury, μ is nearly 0.5

For $\partial V = 0$, $\mu = 0.5$

⊙ μ (isotropic) $= 0.25$

⊙ μ (soft metals) $\gtrsim 0.25$

More the softness, more the ductility and hence more poisson's ratio

$\mu_{(steel)} = 0.3$; $\mu_{(gold)} = 0.44$.

$\uparrow \mu \Rightarrow \uparrow$ ductility

$\uparrow E \Rightarrow \uparrow$ elasticity

⊙ μ (brittle) < 0.25

μ (concrete) $= 0.15$.

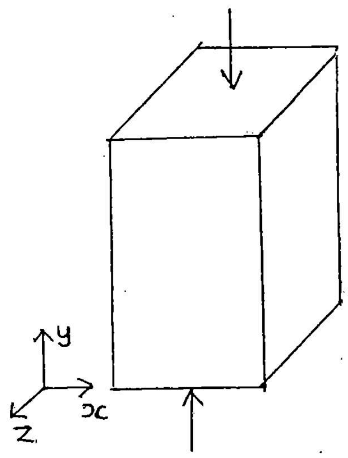
⊙ μ (rigid) $= \dfrac{\epsilon_{lat}}{\epsilon_{lin}} = \dfrac{0}{0}$; not defined.

For incompressible material (ideal),

$\epsilon_{lin} = \epsilon_y = 1$ unit

as no friction b/w molecules,

$\epsilon_{lat} = \epsilon_x = \epsilon_z = \frac{1}{2}$ unit.



$$\mu = \frac{\epsilon_{lat}}{\epsilon_{lin}} = \frac{(1/2)}{1} = 0.5$$

→ Relations b/w E, G, K & μ

$$E = 2G(1+\mu)$$

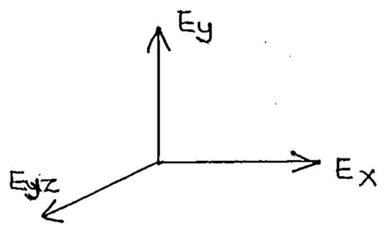
$$E = 3K(1-2\mu)$$

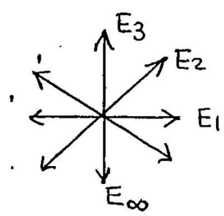
$$\mu = \frac{3K-2G}{6K+2G}$$

$$E = \frac{9KG}{3K+G}$$

Of the four elastic constants, E & μ are independent constar for homogeneous + isotropic materials.

Material	Total EC.	Independent EC
Homogeneous + Isotropic	4	2 (E, μ)
Homogeneous + Orthotropic	12	9
Homogeneous + Anisotropic	∞	21





$E_x \neq E_y \neq E_z$

$G_x \neq G_y \neq G_z$

$K_x \neq K_y \neq K_z$

$\mu_x \neq \mu_y \neq \mu_z$.

P-10

01 $\sigma = \frac{P}{A} = \frac{16000}{4\times4} = 1000 \ kg/cm^2$

$\epsilon = \frac{dl}{l} = \frac{0.1}{200} = 5\times10^{-4}$ $\Rightarrow E = \frac{\sigma}{\epsilon} = \frac{1000}{5\times10^{-4}} = 2\times10^{6}$

$$E = 2G(1+\mu)$$

$$2\times10^{6} = 2G\left(1+\frac{1}{4}\right)$$

$$\therefore\ G = \underline{\underline{0.8\times10^{6}}}\ kg/cm^2$$

5. $$\sigma = \frac{50000}{\frac{\pi}{4}d^2} = 994.718\ kg/cm^2$$

$$\epsilon_{lin} = \frac{\sigma}{E} = \frac{994.718}{10^6} = 9.947\times10^{-4}.$$

$$\mu = \frac{\epsilon_{lat}}{\epsilon_{lin.}}$$

$$\epsilon_{lat} = 0.25\times\epsilon_{lin} = 2.487\times10^{-4}.$$

$$\frac{\partial D}{D} = 2.487\times10^{-4}$$

$$\therefore\ \partial D = 2.487\times10^{-4}\times8 = \underline{\underline{0.002}}\ cm$$

2. $$\epsilon_{lin} = \frac{0.03}{20}$$

$$\epsilon_{lat} = \frac{0.0018}{4} = 4.5\times10^{-4}$$

$$\mu = \frac{4.5\times10^{-4}}{0.03/20} = \underline{\underline{0.3}}$$

3. $$K = \frac{\sigma}{\epsilon_v} = \frac{\sigma}{(\partial V/V)}$$

$$2.5\times10^{5} = \frac{200}{\partial V/20}$$

$$\partial V = \underline{\underline{0.016}}\ m^3$$

4. $$E = 2\times10^{5}\ N/mm^2$$

$$\mu = \frac{1}{4}$$

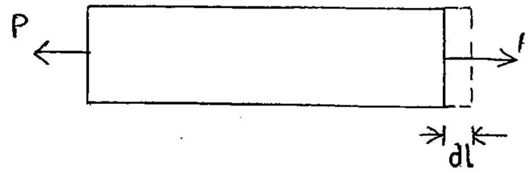
$$E = 2G(1+\mu)$$

$$2\times10^{5} = 2G\left(1+\frac{1}{4}\right) \Rightarrow G = \underline{\underline{0.8\times10^{5}}}\ N/mm^2$$

→ Linear & Volumetric Changes

* Prismatic Bar Subjected to Axial Force

$$\sigma = \frac{P}{A} \; ; \quad \epsilon = \frac{\partial l}{l}$$



$$E = \frac{\sigma}{\epsilon} = \frac{(P/A)}{(\partial l / l)}$$

$$\boxed{\partial l = \frac{Pl}{AE}}$$

— Limitations :-

(i) Prismatic sections only.

(ii) Load upto P.L only

(iii) Gradual loads only (Hooke's Law not valid for impac load

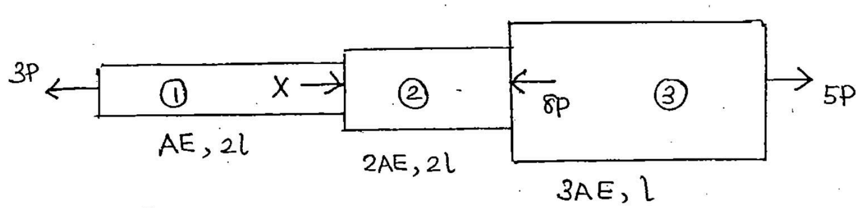
The term 'AE' is called Axial Rigidity.

Unit : $m^2 \cdot \frac{N}{m^2} = \underline{\underline{N}}$

$\uparrow AE \Rightarrow \uparrow$ rigid & stiff bar : $\downarrow \partial l$.

For perfectly rigid bodies, $AE = \infty$

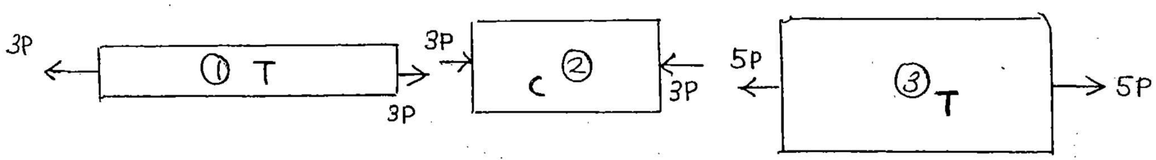
* Composite Bars



$\Sigma F_x = 0$

$\Rightarrow \; 5P - 8P + X - 3P = 0$

$X = +6P$ (assumed direction is correct).

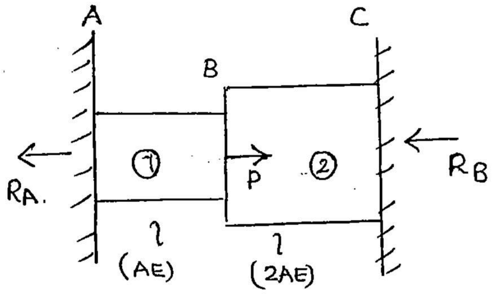


$$\partial l = \partial l_{①} + \partial l_{②} + \partial l_{③} \quad \{\text{use tension as +ve}\}$$

$$= \frac{3P \times 2l}{AE} - \frac{3P \times 2l}{2AE} + \frac{5P \times l}{3AE}$$

$$= +\frac{14Pl}{3AE} \quad \text{(increase in length)}$$

Q

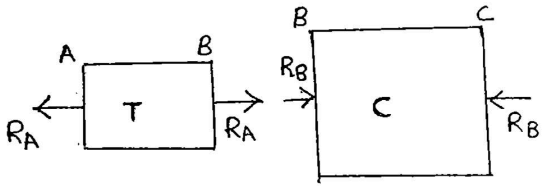


Equilibrium equation, $\Sigma F_x = 0$

$$R_A + R_B = P.$$

Compatibility condition, $\delta_{AC} = 0.$

$$dl_{AB} + dl_{BC} = 0.$$



$$\Rightarrow \frac{R_A l}{AE} + \frac{(-R_B)l}{2AE} = 0.$$

$$R_A + -\frac{R_B}{2} = 0.$$

$$\therefore R_A = \frac{P}{3}$$

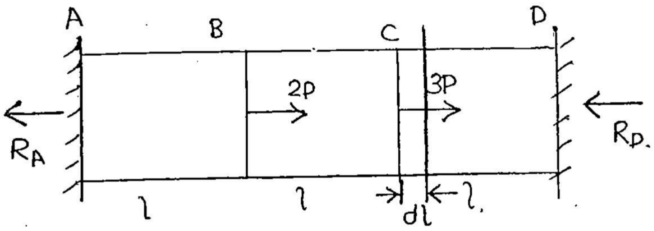
$$R_B = \frac{2P}{3}$$

Stress in $AB = \frac{R_A}{A} = \frac{P}{3A}$

Displacement of $B = dl_{AB}$ or dl_{BC}

$$= \frac{R_A l}{AE} = \frac{Pl}{3AE} \quad \text{(towards right)}$$

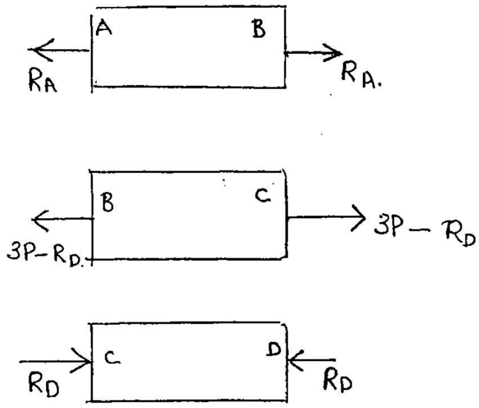
Q.



AE = const.

Find reactions ?

Equilibrium equations : $\left(\Sigma F_x = 0\right)$



$$R_A + R_D = 3P + 2P = 5P$$

Compatibility Conditions : $\left(\delta l_{AD} = 0\right)$

$$\frac{R_A l}{AE} + \frac{(3P - R_D)l}{AE} + \frac{-R_D l}{AE} = 0.$$

$$R_A - 2R_D = -3P.$$

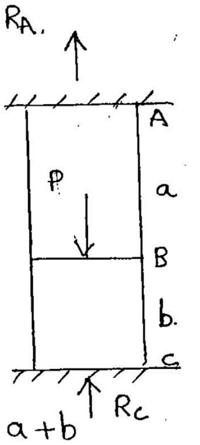
$$R_D = \frac{8P}{3}.$$

$$R_A = \frac{7P}{3}.$$

Displacement of $B = \delta l_{AB} = \dfrac{R_A l}{AE} = \dfrac{7PL}{3AE}$ (towards right).

Displacement of $C = \delta l_{CD} = \dfrac{R_D l}{AE} = \dfrac{8PL}{3AE}$ (towards right)

Q.



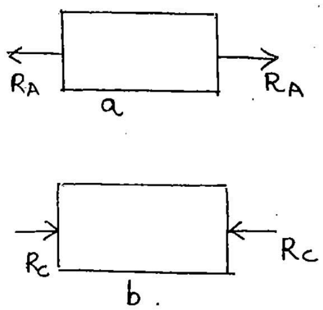
$l = a + b$

AE = constant.

Reactions = ?

$$R_A + R_C = P.$$

$$\frac{R_A \cdot a}{AE} + \frac{-R_C \cdot b}{AE} = 0.$$



$$aR_A - bR_C = 0.$$

$$aR_A = (l - a)R_C$$

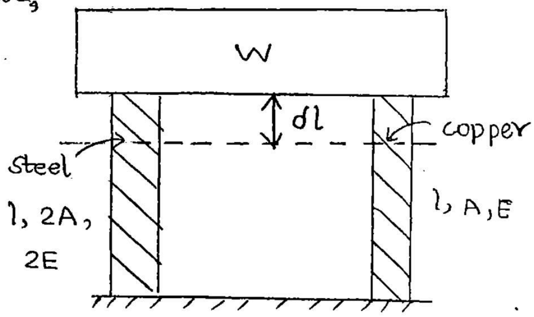
$$R_A = \left(\frac{l-a}{a}\right)R_C.$$

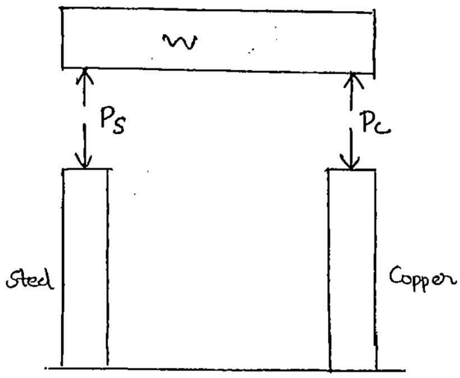
$$\left(\frac{l-a}{a} + 1\right)R_C = P.$$

$$\frac{l}{a}R_C = P \Rightarrow R_C = \frac{Pa}{l}.$$

$$R_A = \frac{Pb}{l}$$

Q. To keep the rigid body horizontal, determine the stress in steel and copper column.





$P_s + P_c = w$ (Eqbm eqn).

Compatibility condition : $dl_s = dl_c$.

$$\frac{P_s l}{2A \cdot 2E} = \frac{P_c l}{AE}$$

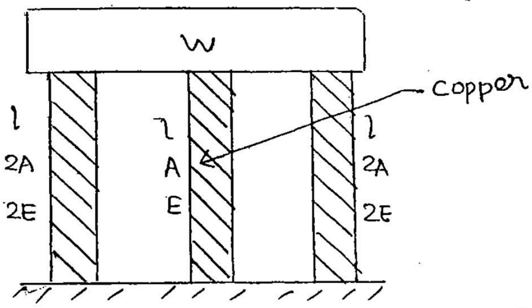
$$P_s = 4 P_c.$$

$$\therefore \quad P_c = \frac{W}{5} \quad \& \quad P_s = \frac{4W}{5}$$

Stress in steel column $= \dfrac{P_s}{A} = \dfrac{4W/5}{2A} = \dfrac{2W}{5A}$ (compression).

Stress in copper column $= \dfrac{P_c}{A} = \dfrac{W/5}{A} = \dfrac{W}{5A}$ (compression).

Q. Two steel bars and a copper bar are supporting a rigid bar of weight W. Calculate stresses.



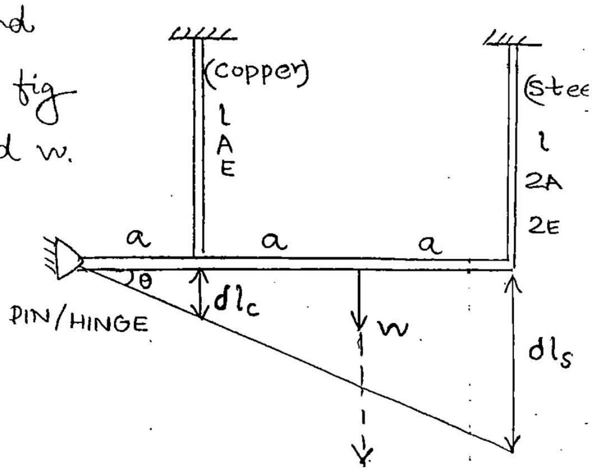
$2P_s + P_c = W.$ $(\Sigma F_x = 0)$

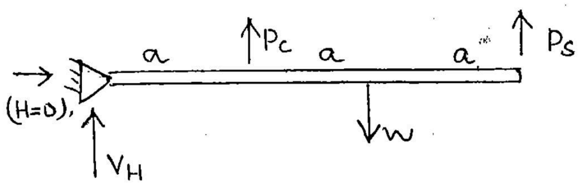
$$\frac{P_s l}{2A \cdot 2E} = \frac{P_c l}{AE}$$

$$P_s = 4P_c$$

$$\therefore P_c = \frac{W}{9} \quad \& \quad P_s = \frac{4W}{9}$$

Q A rigid bar is hinged at one end and supported by two wires as shown in fig. Determine stresses developed due to load w.





Taking moments about hinge,

$$P_c \cdot a + P_s \times 3a = w \times 2a$$

$$P_c + 3P_s = 2w$$

Using similar triangles,

$$\frac{dl_c}{a} = \frac{dl_s}{3a}$$

$$dl_c = \frac{dl_s}{3}$$

$$\frac{P_c \cdot l}{AE} = \frac{P_s \cdot l}{4AE \cdot 3}$$

$$P_s = 12 P_c$$

$$\therefore P_c = \frac{2w}{37} \quad \& \quad P_s = \frac{24w}{37} \quad \text{(tension)}$$

Stress in steel wire, $\sigma_s = \frac{24w}{37 \times 2A} = \frac{12w}{37A}$

Stress in copper wire, $\sigma_c = \frac{2w}{37A}$

$$P_s + P_c = \frac{24w}{37} + \frac{2w}{37} = \frac{26w}{37}$$

$$P_s + P_c + V_H = w$$

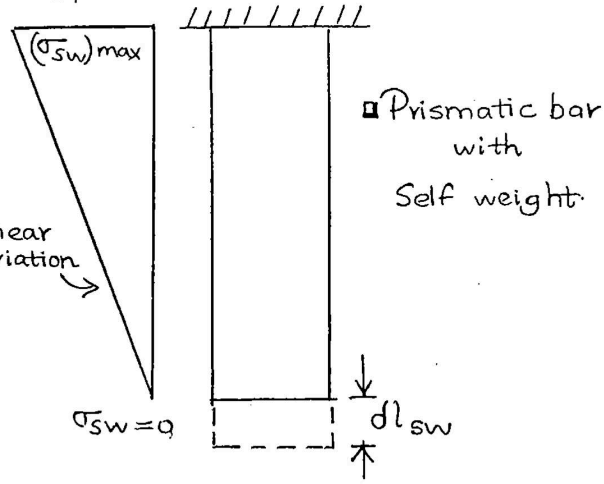
$$\therefore V_H = w - \frac{26w}{37} = \frac{11w}{37}$$

* Self weight Deformation.

$$dl_{sw} = \frac{Wl}{2AE}$$

$$= \frac{(\gamma A l)\,l}{2AE}$$

$$\boxed{(dl)_{sw} = \frac{\gamma l^2}{2E}}$$



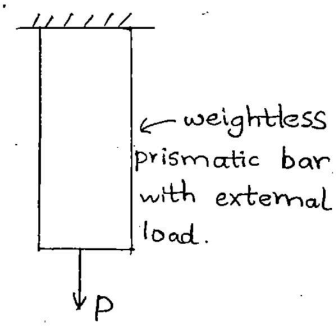
NOTE :

⊙ Self weight deformation is independent of shape and area of c/s, directly proportional to square of length

⊙ Self weight deformation is half that of same self weight attached at the end of a similar weightless bar.

$$P = W$$

$$(dl)_{ext} = \frac{PL}{AE} = \frac{WL}{AE}$$



⊙ Stress due to self weight, $\sigma_{sw} = \frac{W}{A}$

$W \rightarrow$ wt below a c/s, where stress is required.

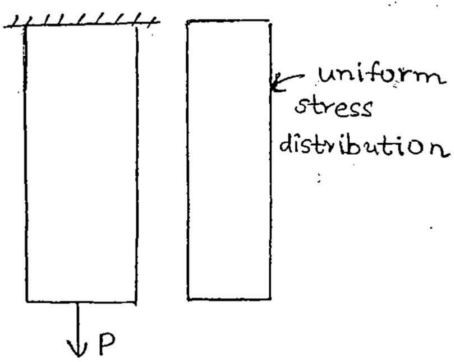
$$(\sigma_{sw})_{free\ end} = 0$$

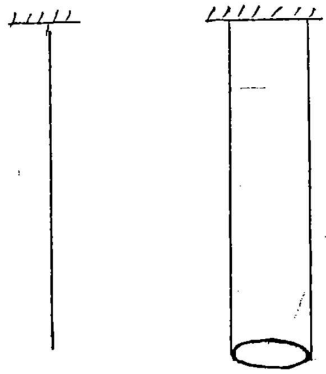
$$(\sigma_{sw})_{fixed\ end} = \frac{W}{A} = \frac{\gamma A l}{A} = \gamma l$$

⊙ Stress due to self weight is also independent of shape and area of c/s, directly proportional to length.

Weightless prismatic bar with external load, $\sigma_{ext} = \frac{P}{A}$

Uniform stress distribution which is independent of length.





l = same

E, γ = same

$(\delta l)_{sw} = \dfrac{\gamma l^2}{2E}$ → same

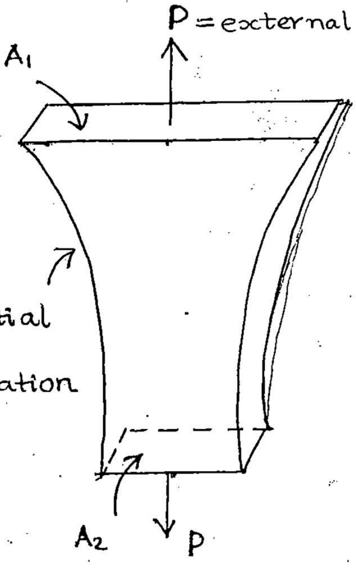
$(\sigma)_{sw} = \gamma l$ → same.

→ Bar of Uniform Strength.

Along the length of a bar, if stress developed is constant then it is bar of uniform strength.

Eg:- weightless prismatic bar subjected to external loading. In practise weightless members are not possible. Self weight will be acting along with external load. In such a case, prismatic members cannot be bar of uniform strength

* Bar of Uniform Strength with Self wt + External load.



$$\frac{A_1}{A_2} = e^{(\gamma l/\sigma)}$$

$$\ln\left(\frac{A_1}{A_2}\right) = \frac{\gamma l}{\sigma}$$

γ → wt. density.

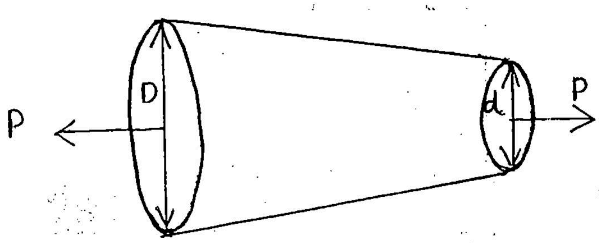
l → length of bar.

σ → const. / uniform stress along the length of bar.

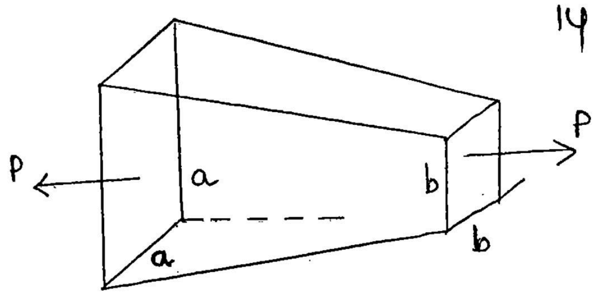
19th Sept, FRIDAY

→ Tapering Bars

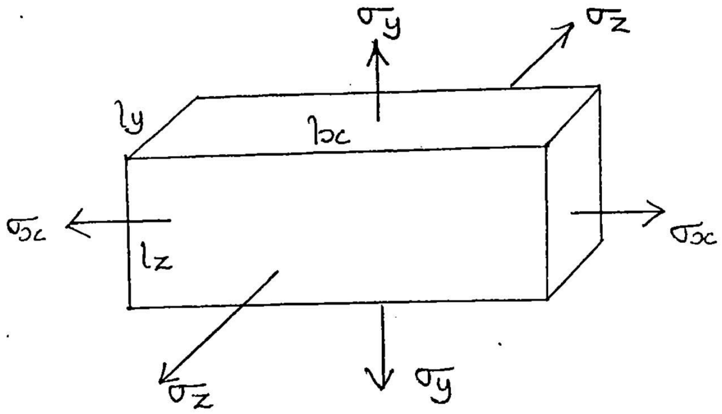
* $$\delta l = \frac{Pl}{\frac{\pi}{4}(Dd)E}$$



$$* \; \partial l = \frac{Pl}{(a.b)\,E}$$



→ Volumetric Strain.

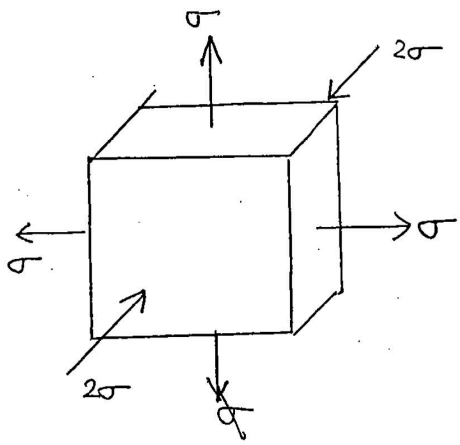


$$\frac{\partial l_x}{l_x} = \epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

Q. Find ∂v for the cube shown...?



$$\boxed{\frac{\partial v}{v} = \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z.}$$

Put $\sigma_x = +\sigma$, $\sigma_y = \sigma$, $\sigma_z = -2\sigma$.

$$\epsilon_x = \frac{\sigma}{E} - \frac{\mu\sigma}{E} + \mu\frac{2\sigma}{E} = \frac{\sigma}{E} + \mu\frac{\sigma}{E}$$

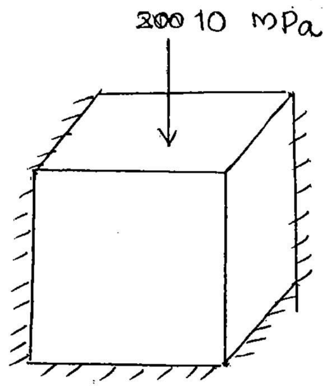
$$\epsilon_y = \frac{\sigma}{E} - \mu\frac{\sigma}{E} + \frac{\mu 2\sigma}{E} = \frac{\sigma}{E} + \mu\frac{\sigma}{E}.$$

$$\epsilon_z = -2\cancel{\mu}\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = -\frac{2\sigma}{E} - 2\mu\frac{\sigma}{E}$$

$$\frac{\partial v}{v} = \epsilon_x + \epsilon_y + \epsilon_y = \underline{\underline{0}}$$

Q A cube of size 'a' is restrained in all directions and free at the top. A compressive stress of 10 MPa is applied in y direction as shown in fig. Determine ① Uniform stress developed in x & z directions. ② strain in y direction

$\sigma_x = ?$, $\sigma_y = -10$ MPa, $\sigma_z = ?$



E = 200 GPa.

$\mu = 0.3$.

$$\frac{\partial V}{V} = \epsilon_x + \epsilon_y + \epsilon_z.$$

$$\epsilon_x = +\frac{\mu\sigma_y}{E} + \frac{\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\epsilon_z = +\frac{\mu\sigma_y}{E} - \frac{\mu\sigma_x}{E} + \frac{\sigma_z}{E}$$

But $\epsilon_x = \epsilon_z = 0.$

$$0 = \frac{0.3 \times 10}{2 \times 10^5} + \frac{\sigma_x}{E} - \frac{0.3\sigma_z}{E}$$

$$\Rightarrow \sigma_x - 0.3\sigma_z + 3 = 0.$$

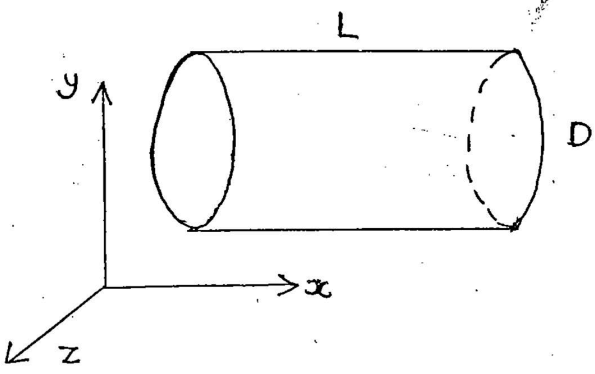
$$\sigma_x = \sigma_z = \sigma$$

$\therefore \sigma_x = \sigma_y = -4.29$ MPa (compressive).

$$\epsilon_y = -\frac{10}{E} - \frac{0.3 \times -4.29}{E} - \frac{0.3 \times -4.29}{E}$$

$= -3.713 \times 10^{-5}$ ~~mm~~ ~~= 0.0037 mm~~ (−ve mean ↓ in dimension

→ Cylinder



$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z.$$

$$= \frac{\partial l}{l} + \frac{\partial D}{D} + \frac{\partial D}{D}.$$

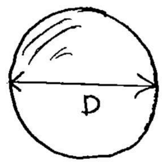
$$= \epsilon_l + \epsilon_h + \epsilon_h$$

$$\boxed{\epsilon_v = \epsilon_l + 2\epsilon_h}$$

$\epsilon_l \rightarrow$ linear / axial / longitudinal strain.

$\epsilon_h \rightarrow$ hoop / circumferential strain

$\rightarrow$ Sphere



$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\partial D}{D} + \frac{\partial D}{D} + \frac{\partial D}{D}.$$

$$\boxed{\epsilon_v = 3\epsilon_h}$$

P-4

Q1. Scalar : Magnitude + No direction. Eg: distance, speed.

Vector : Magnitude + One direction. Eg: displacement, velocity.

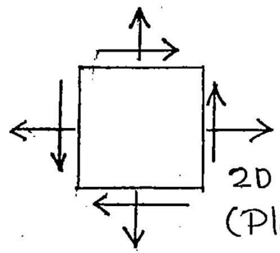
Tensor : Magnitude + more than one direction.

Eg:- stress, strain, MI

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}_{3\times3}$$

component of stresses (spatial)

Tensors can be expressed in Matrix form for computer applicatio



2D tensor (Plane stress)

Q-6

Q6. Visco-elastic $\rightarrow$ Elasto plastic.

Q7. Tenacity — maximum tensile strength.

P-9

3. $E = 2G(1+\mu)$.

when $\mu = 0$, $\frac{G}{E} = 0.5$

when $\mu = 0.5$, $\frac{G}{E} = 0.33$

$\Rightarrow G = (0.33 \text{ to } 0.5)E$

→ Temperature Stresses :

- Indirect stress.
- external loads are direct stresses.

$\alpha \rightarrow$ coeffecient of linear (thermal) expansion.

It is the strain developed per unit change in temperatu
'α' is a material property and is constant for given material.

$$\alpha_{steel} = \alpha_{concrete} = 12 \times 10^{-6} \ /^\circ C.$$

$\uparrow \alpha$: $\uparrow$ active for temperature.

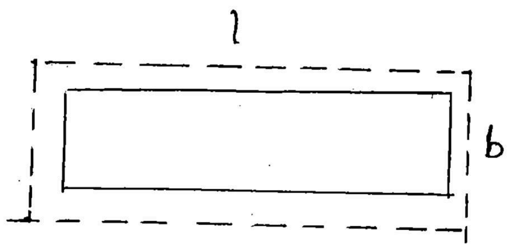
$$\epsilon_t = \alpha t.$$

$$\frac{\partial l}{l} = \epsilon_t = \alpha_t.$$

$$\Rightarrow \partial l = l \alpha t$$

$$\frac{\partial b}{b} = \epsilon_t = \alpha t.$$

$$\Rightarrow \partial b = b(\alpha t)$$



Prismatic steel bar ($\uparrow t$).

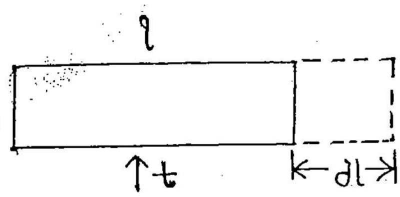
⊙ As temperature increases due to uniform heating, all the dimensions increase. Due to uniform cooling, all the dimensi decrease.

(i) Prismatic bar free to expand or contract.

$$\epsilon_t = \alpha t.$$

$$\frac{\partial l}{l} = \alpha t$$

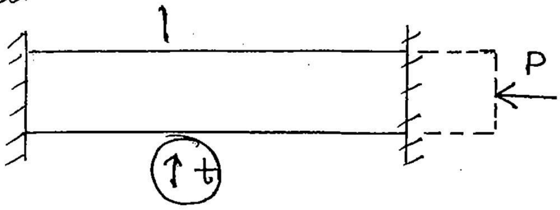
$\Rightarrow$ Free expansion along length, $\partial l = l \alpha t$



Member is free to expand or contract, therefore no stress will be induced.

(ii) Fixed Rigidly (along length)

Free expansion = Expansion prevented.



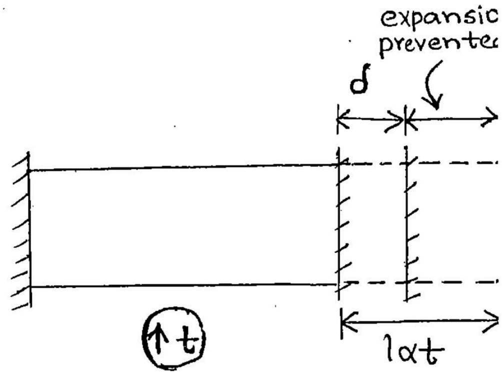
$$l\alpha t = \frac{Pl}{AE}$$

$$\boxed{\sigma_t = (\alpha t)E}$$

$\uparrow t$: compression

$\downarrow t$: tension.

(iii) Yielding Supports.



If $l\alpha t \leq \delta$; no stress developed.

If $l\alpha t > \delta$; stress developed.

$$\text{Expansion prevented,} = \frac{Pl}{AE}$$

$$l\alpha t - \delta = \frac{Pl}{AE}$$

$$\sigma_t = \frac{(l\alpha t - \delta)E}{l}$$

Due to ~~yielding~~ heating supports move outward, come closer due to cooling

Q. A steel bar of 5m length is at a room temp of 30°c. The bar is uniformly heated to 90°c. Determine temperature stress developed if bar is:

(i) free to expand.

(ii) expansion prevented along length.

(iii) Supports yield by 1.5 mm along length.

(iv) Supports yield by 5 mm along length.

use $E = 200$ GPa, $\mu = 0.3$.

Ans: (i) zero.

(ii) $\sigma_t = \alpha t E = 12\times10^{-6} \times (90-30) \times 200\times10^{3}$ MPa.

$= 144$ MPa.

(iii) $\delta l = l\alpha t = 5\times12\times10^{-6}\times 60 = 3.6$ mm.

$$\sigma_t = \frac{(l\alpha t - \delta)E}{l} = \frac{(3.6-1.5)}{5000}\times 2\times10^{5}$$

$$= \underline{\underline{84 \text{ MPa}}}$$

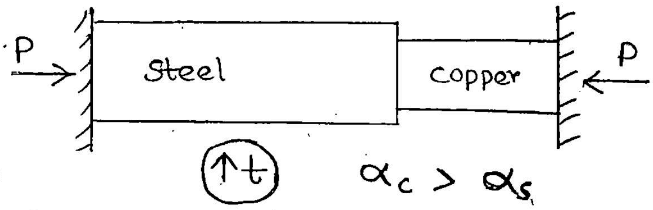
(iv) $\delta l < \delta \Rightarrow$ <u>no stress</u>.

→ Composite Bars

– made of different materials.

* Series :

Free expansion of both bars = expansion prevented by both bars



$$(l\alpha t)_s + (l\alpha t)_c = \left(\frac{Pl}{AE}\right)_s + \left(\frac{Pl}{AE}\right)_c$$

$$P_s = P_c = P$$

$$\sigma_s = \frac{P}{A_s} \quad ; \quad \sigma_c = \frac{P}{A_c}$$

For rigid supports, ↑t : compression.

↓t : tension.

P-18

Q.6. $L_s = L_a = 1$ m; $\alpha_s = 11\times10^{-6}/°c$; $\alpha_a = 24\times10^{-6}/°c$

$E_s = 200$ GPa , $E_a = 70$ GPa ; $A_s = 100$ mm^2, $A_a = 200$ mm^2

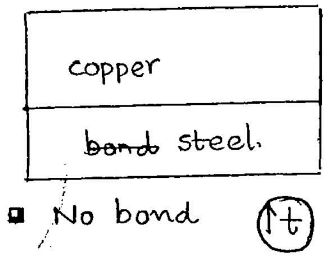
$\Delta t = 58° - 38° = 20°$.

$$1\times11\times10^{-6}\times20 + 20\times1\times24\times10^{-6} = \frac{P\times1}{100\times10^{-6}\times200\times10^{3}} + \frac{P\times1}{200\times70\times10^{3}}$$

$$P = \underline{\underline{5.76 \text{ kN}}}$$

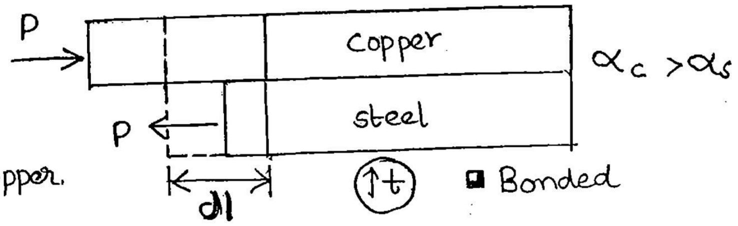
* Parallel.

- Uniform heating: no warping



As there is no bond and no supports, both copper and steel will expand individually upon heating and ∴ no stresses are induced.

- Net change in length of steel = net change in length of copper.

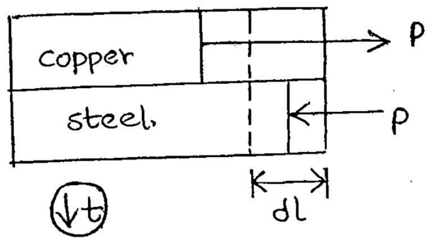


$$(l\alpha t)_s + \left(\frac{Pl}{AE}\right)_s = (l\alpha t)_c - \left(\frac{Pl}{AE}\right)_c \;;\ \text{(compatibility condition)}$$

$(\alpha\downarrow)$ $(\alpha\uparrow)$

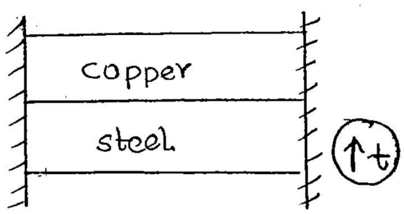
$$P_s = P_c = P$$

Same compatibility equation can be used for both increase and decreased in temperature, the nature of stresses should be changed accordingly.



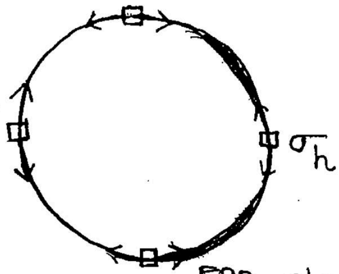
- For ideal composite material, α must be nearly equal.
Eg: Concrete & Steel

- Both in compression if between rigid supports.

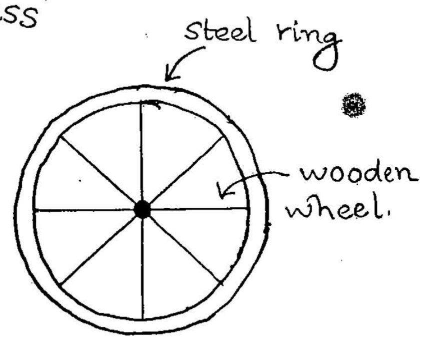


Rigid Supports

→ Hoop Stress (or) Circumferential Stress



FBD steel ring



d → initial diameter of steel ring

D → diameter of rigid wooden wheel.

D → final diameter of steel ring

- Hoop strain $= \epsilon_h = \dfrac{\pi D - \pi d}{\pi d}$.

- Hoop stress, $\sigma_h = \epsilon_h E$.

$$= \left(\frac{D-d}{d}\right) E.$$

∴ tension in steel ring & compression in wooden wheel.

- Min increase in temperature for fixing,

$$\epsilon_h = \epsilon_t.$$

$$\frac{D-d}{d} = \alpha t.$$

$$\Rightarrow \boxed{t = \frac{D-d}{\alpha d}}$$

Q. A steel ring of 499 mm ϕ is to be fitted over a wooden wheel 500 mm ϕ. E of steel = 200 GPa, $\alpha_s = 12 \times 10^{-6}$ /°C. Determine (i) hoop stress developed.

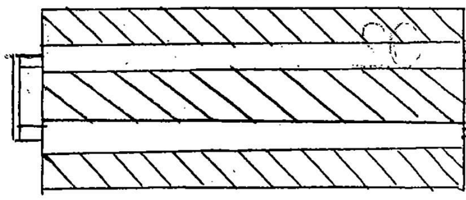
(ii) min increase in temp. for fixing.

(i) $\sigma_h = \left(\dfrac{D-d}{d}\right) E = \dfrac{(500-499)}{499} \times 2 \times 10^5 = 400.8$ MPa

(ii) Min. $t = \dfrac{D-d}{d\alpha} = \dfrac{500-499}{499 \times 12 \times 10^{-6}} = \underline{\underline{167^\circ C}}$

Q.08 Parallel ($\alpha_g > \alpha_s$)

($\downarrow t = 200°F$).



$$(l\alpha t)_g - \left(\frac{Pl}{AE}\right)_g = (l\alpha t)_s + \left(\frac{Pl}{AE}\right)_s$$

$$10\times10^{-6}\times 200 - \frac{P}{200\times100\times10^3} = 6\times10^{-6}\times 200 + \frac{P}{100\times200\times10^3}$$

$$\underline{\underline{P = 8\ kN}}$$

Q.09. $\sigma_s = \frac{P}{A_s} = \frac{8000}{100} = \underline{\underline{80\ MPa}}$

$\sigma_{gm} = \frac{P}{A_g} = \frac{8000}{200} = \underline{\underline{40\ MPa}}$

Q.05 $(\alpha t)_a - \left(\frac{Pl}{AE}\right)_a = (\alpha t)_s + \left(\frac{Pl}{AE}\right)_s.$

$25\times10^{-6}\times80 - \frac{P}{\ }$

24th sept,
WEDNESDAY

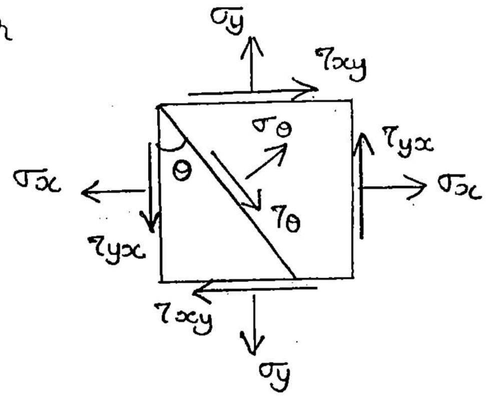
02 COMPLEX STRESS & STRAINS

→ 2D (or) Biaxial (or) Plane Stress system

All the stresses will be developing in one perpendicu plane only.

Eg: Beams, shafts, any thin member

2D Stress Tensor :- $\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}_{2\times2}$



⊙ In a member (or element) normal stresses are balanced b(force equilibrium, shear stresses are balanced by moment equilibrium.

For moment equilibrium, $\tau_{xy} = \tau_{yx}$.

∴ for a 2D stress tensor, there will be a total of 4 stress components available. Among them, 3 are independent components.

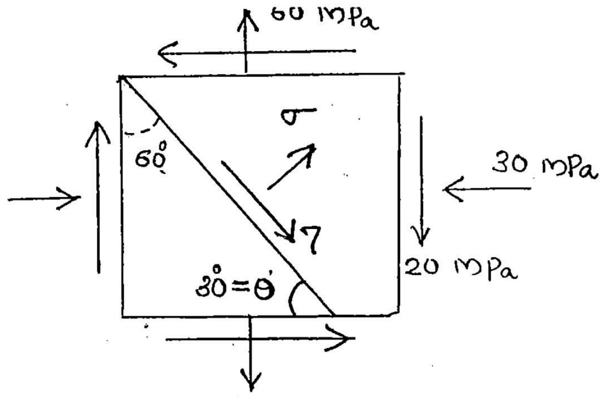
If horizontal shear stress is due to external loads, a vertical shear stress of opposite nature develops for balancing called complementary shear stress.

Stress components on Inclined Plane :

$$\boxed{\begin{aligned} \sigma_\theta &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_\theta &= \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \end{aligned}}$$

NOTE : Above formulas are valid only for the given basic element.

Q.



$\sigma_x = -30$ MPa

$\sigma_y = 60$ MPa

$\tau_{xy} = -20$ MPa.

$\theta = 60°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta.$$

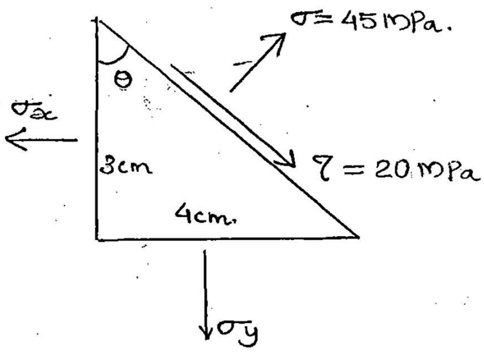
$$= \frac{-30+60}{2} + \frac{-30-60}{2}\cos 2(60°) + -20\sin 2(60) = \underline{20.18} \text{ MPa}$$

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta.$$

$$= \frac{-30-60}{2}\sin(2\times 60) - -20\cos 2(60).$$

$= -\underline{48.97}$ MPa (−ve means shear should be opp. direction)

Q.



$\tan\theta = \frac{4}{3}$

$\theta = \underline{53.13°}$

$\tau_{xy} = 0$

$$45 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\times 53.13) + 0.$$

$$90 = \sigma_x + \sigma_y + (\sigma_x - \sigma_y)\times -0.28$$

$$= 0.72\,\sigma_x + 1.28\,\sigma_y. \longrightarrow (1)$$

$$20 = \frac{\sigma_x - \sigma_y}{2}\sin(2\times 53.13) - 0.$$

$$40 = 0.96\,\sigma_x - 0.96\,\sigma_y. \longrightarrow (2)$$

$\sigma_x = 71.66$ MPa

$\sigma_y = 30$ MPa

→ Principal Stresses.

$$\left.\begin{array}{l}\text{Major, } \sigma_1 \\ \text{Minor, } \sigma_2\end{array}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The normal stress across the principal plane is principal str

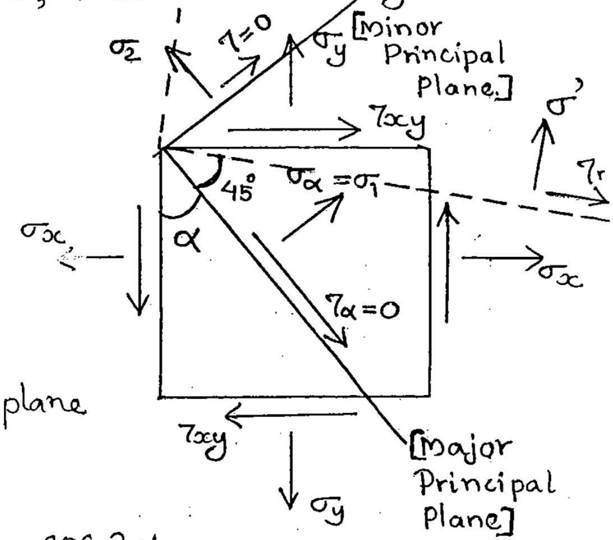
→ Principal Planes.

– The plane on which only principal (normal) stress will be acting.

– On principal plane, shear stress is zero.

– If shear stress is zero on a plane, on the perpendicu' plane also shear stress is zero.

– In 2D system, there will be two mutually perpendicul principal planes. On both the planes, shear stress is zero.



* To locate principal plane :

Assume principal plane is making an angle α as shown.

Shear stress on that plane must be zero if its a principal plane

$$\tau_\alpha = 0 = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha.$$

$$\boxed{\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

α → angle of major principal plane

$(\alpha + 90)$ → angle of minor principal plane.

* Max Shear Stress :

$$\boxed{\tau_{max} = \pm\left[\frac{\sigma_1 - \sigma_2}{2}\right] = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

– In 2D system, there'll be two max. shear stresses of equal magnitude but opposite in nature

* Maximum Shear Stress Planes.

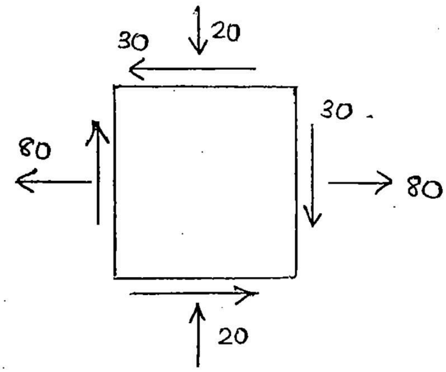
– The plane on which maximum shear stress is acting. In 2D system, there will be ~~be~~ two τ_{max} planes seperated by 90°.

– The angle b/w any principal plane and the nearest τ_{max} plane is 45°.

– On the τ_{max} plane, there may be normal stress which is equal to σ' or $\sigma_{avg} = \dfrac{\sigma_1+\sigma_2}{2} = \dfrac{\sigma_x+\sigma_y}{2}$

– If $\sigma'=0$, then its called 'Pure shear Stress'. (On τ_{max} plane, only shear stress alone will be acting.)

Q. Calculate $\sigma_1, \sigma_2, \tau_m, \sigma'$



$\tau_{xy} = -30$

$\sigma_x = +80$

$\sigma_y = -20$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80+(-20)}{2} + \sqrt{\left(\frac{80+20}{2}\right)^2 + (-30)^2}$$

$$= 30 + 58.309 = 88.31 \text{ kPa}.$$

$$\sigma_2 = 30 - 58.309 = -28.309 \text{ kPa}$$

$$\tau_m = \frac{\sigma_1-\sigma_2}{2} = \frac{88.31-(-28.309)}{2} = 58.309$$

$$\sigma' = \frac{\sigma_1+\sigma_2}{2} = \frac{88.31+-28.31}{2} = \underline{\underline{30}}$$

3rd Oct, Friday

→ Mohr's Circle

- Graphical method given by Otto Mohr
- Basically developed for 2D (plane) stress system.
- Centre of Mohr Circle lies on x-axis where normal stress is represented. The distance of centre of Mohr circle from origin is OC = σ' or σ_{avg}

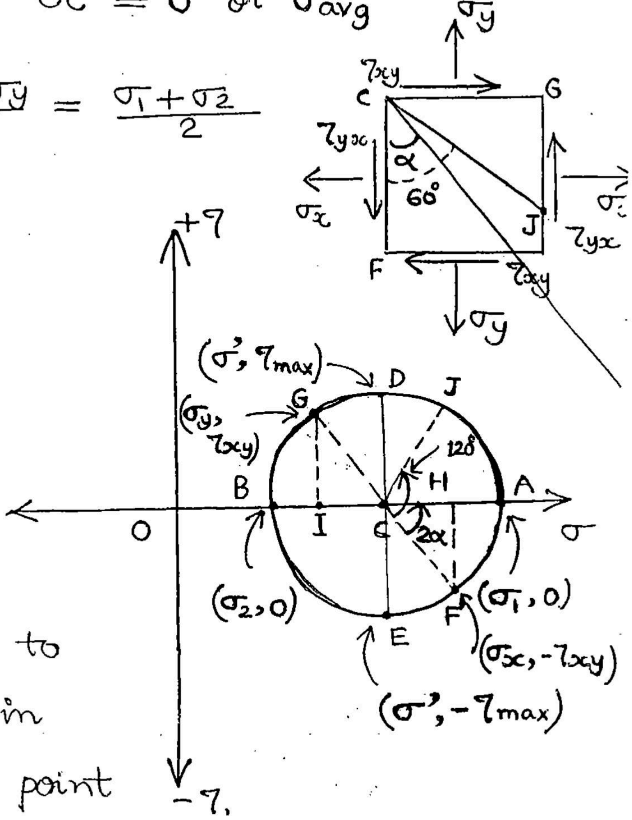
$$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

- Radius of Mohr circle,

$$R = \tau_{max}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{\sigma_1 - \sigma_2}{2}$$



- Each radial line drawn to the Mohr circle is a plane in the material or element. The point on the circle corresponding to the radial line gives the co-ordinates of normal and shear stresses on the plane

CA : Major Principal Plane

CB : Minor Principal Plane

CD & CE : τ_{max} Plane.

- All the angles at the centre of Mohr Circle are twice of actual

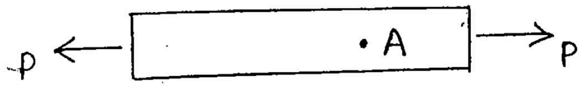
OH = σ_x & HF = $-\tau_{xy}$ (anti-cw)

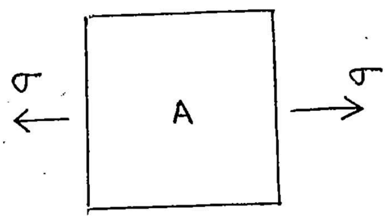
OI = σ_y & IG = $+\tau_{xy}$ (clock-wise).

* Special Cases :

(i) 1 D

Eg: Tie, strut.

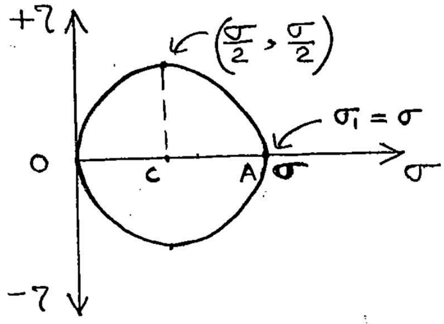




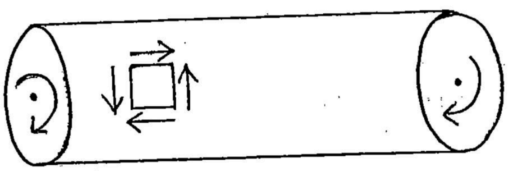
$\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = 0$.

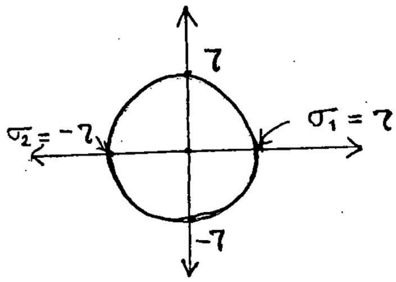
$OC = \dfrac{\sigma_x + \sigma_y}{2} = \underline{\underline{\dfrac{\sigma}{2}}}$; $\text{Radius} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \underline{\underline{\dfrac{\sigma}{2}}}$



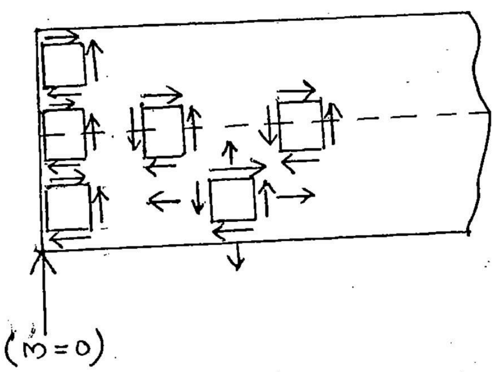
(ii) Pure Shear





If $\sigma' = 0$ on τ_{max} plane, it is Pure Shear condition.

- any element on the axis of a beam
- element on surface of shaft.
- any element at the support of a beam.



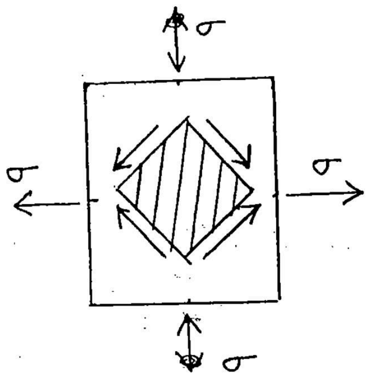
$\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau$

$OC = \sigma' = 0$.

Radius, $\tau_{max} = \tau$.

* If centre of Mohr circle coincides with origin, it is a Pure Shear condition

(iii)



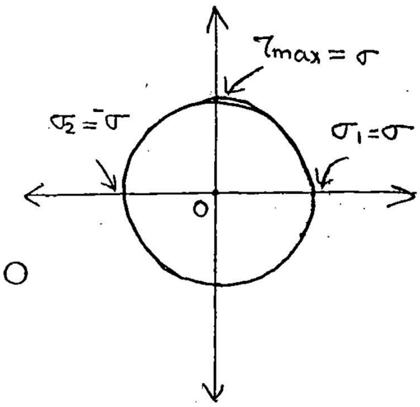
$\sigma_x = \sigma$

$\sigma_y = -\sigma$

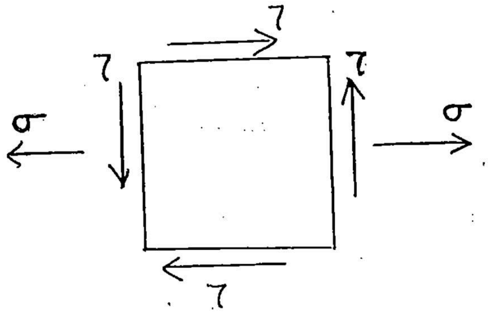
$\tau_{xy} = 0$

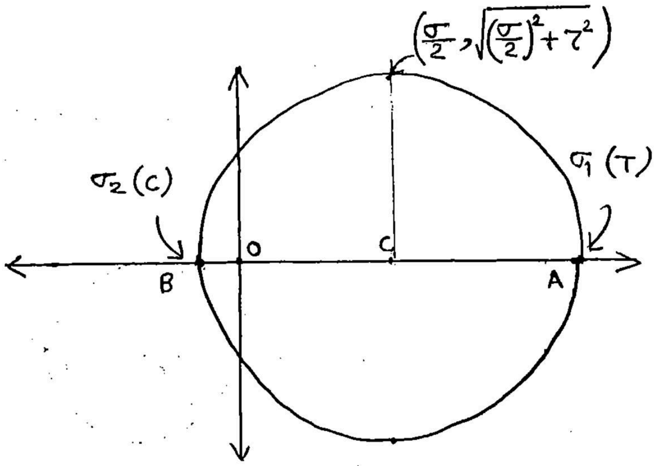
$OC = \sigma' = 0$

$\tau_{max} = \sigma$



(iv) Beams.





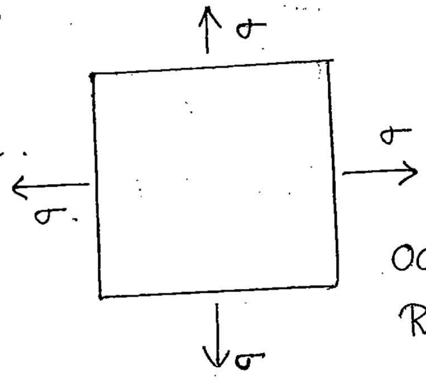
Even though transverse load is applied on the beam, which is normal to the axis of beams, the shear stress will develop b/w layers and tension or compression will act along the axis of the beam. The normal stress in the direction of load is always zero in beams.

$\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = \tau$.

$OC = \sigma' = \frac{\sigma}{2}$ & Radius, $\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

* In beams, Principal stress will be opposite in nature. because of bending, one face of beam is under tension and the other face is under compression

(v). Isotropic Condition.



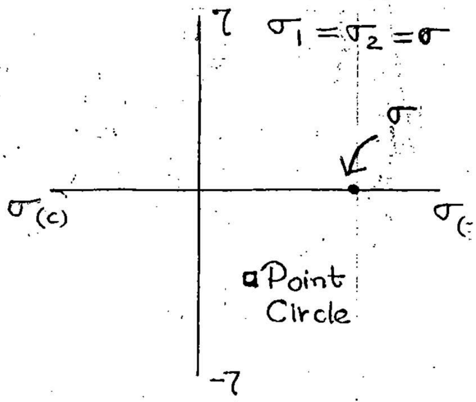
$\sigma_x = +\sigma$

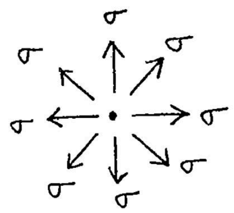
$\sigma_y = +\sigma$

$\tau_{xy} = 0$.

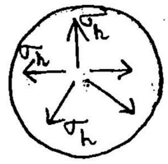
$OC = \sigma$

$\text{Radius} = 0$



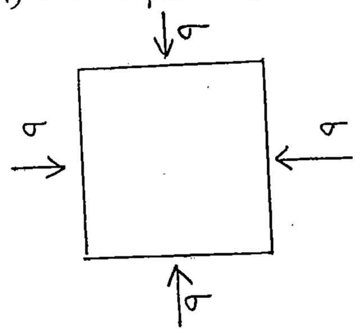


Isotropic condition.



– On the surface of a thin sphere, at a point in all the directions, only hoop tension will be acting without shear stress. called Isotropic condition.

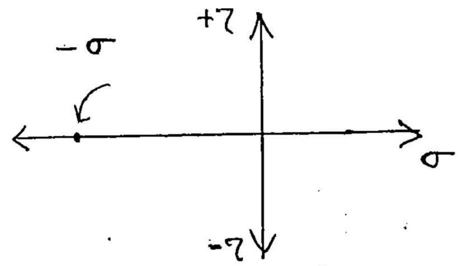
(vi) Isotropic condition.

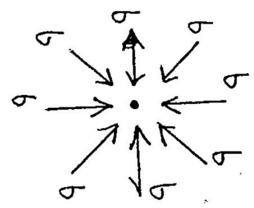


$\sigma_x = -\sigma$

$\sigma_y = -\sigma$

$\tau_{xy} = 0.$

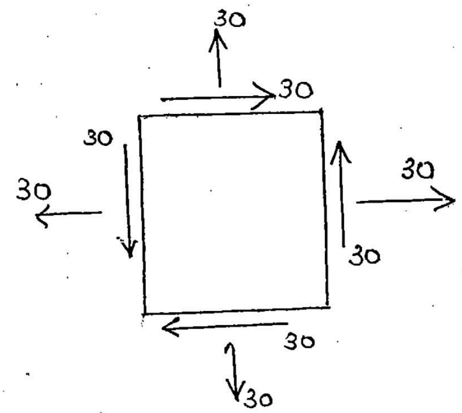




Hydrostatic pressure condition

– On a submerged body under hydrostatic pressure condition shear stress is zero. There will be only change in volume without distortion in shape..

(vii)



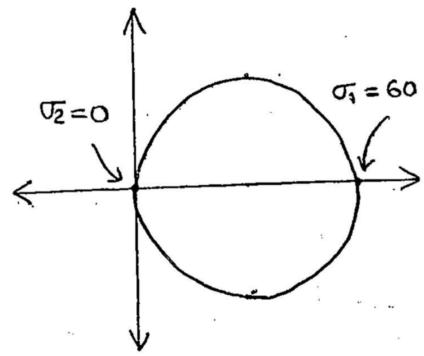
$\sigma_x = 30$

$\sigma_y = 30$

$\tau_{xy} = 30$

$\sigma = 30$

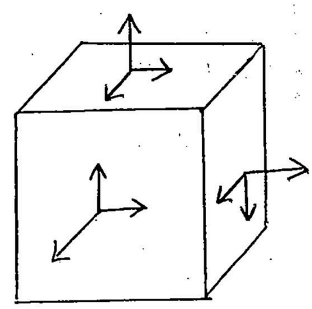
Radius = 30.



THURSDAY
October

→ 3D Stress System

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}_{3\times3}$$



For symmetry of stress tensor:

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{xz} = \tau_{zx}$$

$$\tau_{zy} = \tau_{yz}$$

	3D	2D	1D
Total stress components	9	4	1
Independent components.	6	3	1

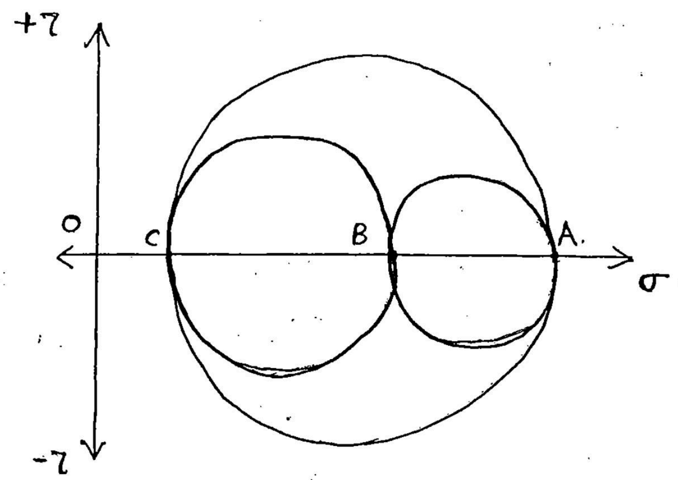
2D (Plane Stress)

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}_{2\times 2}$$

1D (uni-axial)

$$\begin{bmatrix} \sigma \end{bmatrix}_{1\times 1}$$

* 3D Mohr Circle:



$\sigma_1 \rightarrow$ major (OA)

$\sigma_2 \rightarrow$ intermediate (OB).

$\sigma_3 \rightarrow$ minor (OC)

τ_{max} in 3D = max. radius

$$= \frac{AC}{2}$$

$$= \frac{OA - OC}{2}$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}}$$

Eg 1: Principal stresses 40, 20, 10 MPa.

$$\tau_{max\,(3D)} = \frac{40-10}{2} = \underline{\underline{15 \text{ MPa}}}$$

Eg 2: Principal stresses 30 MPa, 50 MPa.

τ_{max} in 2D (Plane stress system) $= \frac{50-30}{2} = 10$ MPa.

$\tau_{max} = \frac{50-0}{2} = 25$ MPa

- In a problem, if only τ_{max} is asked to calculate, it should be based on 3D only. If only two principal stresses are given in the problem consider the third principal stress (σ_3) as zero

Eg: 3 Principal stresses :- 50 MPa & -20 MPa.

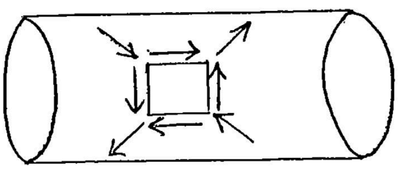
τ_{max} in 2D $= \frac{50-(-20)}{2} = 35$ MPa.

$\tau_{max} = \frac{50+20}{2} = \underline{\underline{35 \text{ MPa}}}$

$\sigma_1 = 50$ MPa
$\sigma_2 = 0$
$\sigma_3 = -20$ MPa.

- If principal stresses are opposite in nature (one tensile & the other compressive), $\tau_{max(2D)} = \tau_{max(3D)}$

Such a case will arise in beams, shafts or any member subjected to bending except thin cylinders and spheres.



$\left.\begin{matrix}\sigma_1 = +\tau \\ \sigma_3 = -\tau\end{matrix}\right\}$ 2D

$\left.\begin{matrix}\sigma_1 = +\tau \\ \sigma_2 = 0 \\ \sigma_3 = -\tau\end{matrix}\right\}$ 3D

Eg 4: Principal stresses -30 MPa, -80 MPa.

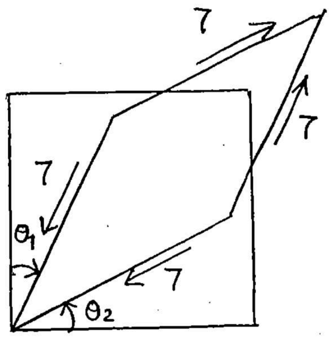
τ_{max} in 2D $= \frac{-30-(-80)}{2} = \underline{\underline{25 \text{ MPa}}}$

$\tau_{max} = \frac{0-(-80)}{2} = \underline{\underline{40 \text{ MPa}}}$

$\left.\begin{matrix}\sigma_1 = 0 \\ \sigma_2 = -30 \\ \sigma_3 = -80\end{matrix}\right\}$ 3D.

→ Strain Analysis (2D)

Stresses	σ_x	σ_y	τ_{xy}
Strain	ϵ_x	ϵ_y	$\phi_{xy}/2$



Shear strain is the angular deformation b/w two mutually ⊥r planes in radians.

$$\phi = \theta_1 + \theta_2$$

For square elements (due to symmetry)

$$\theta_1 = \theta_2$$

$$\Rightarrow \phi = \theta_1 + \theta_1 = 2\theta_1 = 2\theta_2$$

$$\therefore \boxed{\theta_1 = \frac{\phi}{2} \quad \& \quad \theta_2 = \frac{\phi}{2}}$$

→ Strain on Inclined Plane :

$$\epsilon_\theta = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta .$$

$$\frac{\phi_\theta}{2} = \frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta - \frac{\phi_{xy}}{2}\cos 2\theta .$$

→ Principal Strains :

$$\left.\begin{matrix}\epsilon_1 \\ \epsilon_2\end{matrix}\right\} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

- The Plane on which shear stress and the corresponding shear strain is zero. On the same planes, both Principal stresses and corresponding Principal strains will be acting.

* $\tan(2\alpha) = \dfrac{2\left(\frac{\phi_{xy}}{2}\right)}{\epsilon_x - \epsilon_y}$

* Maximum shear strain (ϕ_{max})

$$\frac{\phi_{max}}{2} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\Rightarrow \boxed{\phi_{max} = \epsilon_1 - \epsilon_2}$$

→ Strain Gauges

No: of strain gauges required:

$$\left.\begin{array}{l} 1D \rightarrow 1 \text{ no.} \\ 2D \rightarrow 3 \text{ no.} \\ 3D \rightarrow 6 \text{ no.} \end{array}\right\} \text{no. of independent stress components.}$$

* Types:

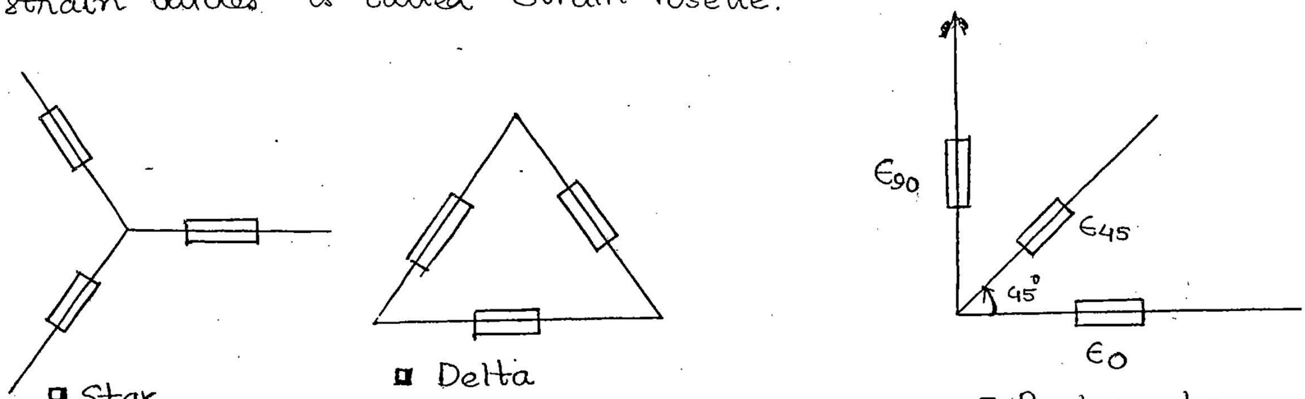
(i) Mechanical.

(ii) Electrical.

(iii) Digital.

* Strain Rosette.

The arrangement of strain gauges to obtain relevant strain values is called Strain rosette.



▪ Star ▪ Delta ▪ Rectangular

Step 1 : Read 3 strain gauge values

Step 2 : Calculate ϵ_x, ϵ_y, ϕ_{xy}

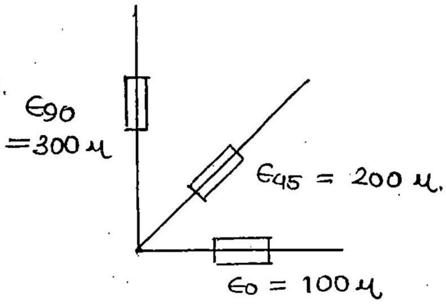
Step 3: P- strains ϵ_1 & ϵ_2

Step 4: P- stresses using E & μ

Step 5: $\sigma_1 \ngtr$ permissible stress.

Strain values on a rectangular strain rosette are shown in fig. Determine principal stresses, if $E = 2\times10^5$ MPa and $\mu = 0.3$. Also check the safety of the member if permissible stress in the material is 200 MPa.

$$\epsilon_\theta = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta.$$



Use $\theta = 0$, $\epsilon_0 = 100\mu$

$$100\mu = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2} + 0.$$

Use $\theta = 90$, $\epsilon_{90} = 300\mu$.

$$300\mu = \frac{\epsilon_x+\epsilon_y}{2} + \left(\frac{\epsilon_x-\epsilon_y}{2}\right)\times -1 + 0.$$

$$\boxed{\because\ \epsilon_{45} = \frac{\epsilon_0+\epsilon_{90}}{2} \quad \Rightarrow\ \epsilon_1 = \epsilon_{90},\ \ \epsilon_2 = \epsilon_0}$$

$\Rightarrow$ $\epsilon_x = 100\mu$ & $\epsilon_y = 300\mu$.

Use $\theta = 45$, $\epsilon_{45} = 200\mu$.

$$\epsilon,\ 200\mu = \frac{\epsilon_x+\epsilon_y}{2} + 0 + \frac{\phi_{xy}}{2}$$

$$\Rightarrow \phi_{xy} = 0$$

$$\epsilon_1 = \frac{\epsilon_x+\epsilon_y}{2} + \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= 200\mu + \frac{100-300}{2}$$

$$= 100\mu.$$

$$\epsilon_2 = 200\mu - \frac{100-300}{2} = 300\mu.$$

$\therefore$ $\epsilon_1 = 300\mu$ & $\epsilon_2 = 100\mu$

- If $\phi_{xy} = 0$, then ϵ_x & ϵ_y are directly the values of ϵ_1 & ϵ_2.

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \Rightarrow 300\mu = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \Rightarrow 100\mu = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\epsilon_1 + \mu\epsilon_2 = \frac{\sigma_1}{E}(1-\mu^2).$$

$$\therefore \sigma_1 = \frac{E(\epsilon_1 + \mu\epsilon_2)}{1-\mu^2} = \frac{2\times10^5(300\mu + 0.3\times100\mu)}{1-0.3^2}$$

$$= \underline{\underline{72.527}} \text{ MPa.}$$

$$\sigma_2 = \frac{E(\epsilon_2 + \mu\epsilon_1)}{1-\mu^2} = \frac{2\times10^5(100\mu + 0.3\times300\mu)}{1-0.3^2}$$

$$= \underline{\underline{41.76}} \text{ MPa}$$

$\Rightarrow \sigma_1 \not>$ Permissible stress ($=200$ MPa)

$\therefore$ Safe

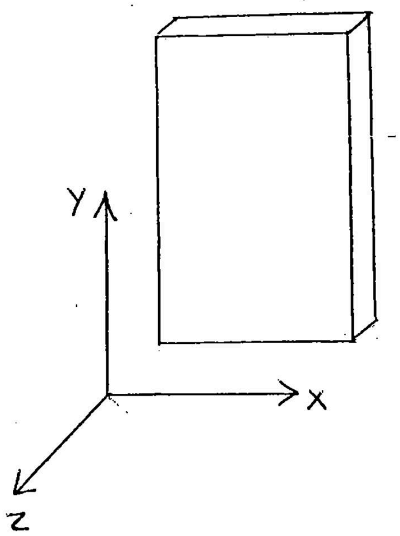
→ Plane Stress System (2D)

Eg: Beams, shafts, thin members

$\sigma_x \neq 0$ $\boxed{\sigma_z = 0 \quad \epsilon_z \neq 0}$

$\sigma_y \neq 0$ $\tau_{xz} = 0$

$\tau_{xy} \neq 0$ $\tau_{yz} = 0.$

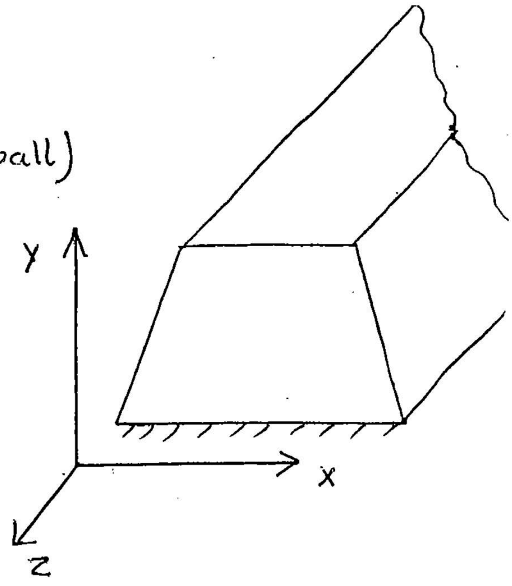


$z \rightarrow$ direction along thickness.

→ Plane Strain System.

Eg: Long members (dam, retaining wall)

$\epsilon_x \neq 0$	$\boxed{\epsilon_z = 0 \qquad \sigma_z \neq 0}$
$\epsilon_y \neq 0$	$\phi_{xz} = 0$
$\phi_{xy} \neq 0$	$\phi_{yz} = 0$



P-25

Q.08. $\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$

$0 = \sigma_z - 0.3 \times 150 - 0.3 \times -300.$

$\therefore \sigma_z = \underline{\underline{-45 \text{ MPa}}}$

09. $\sigma_x = 65 \text{ N/mm}^2$, $\sigma_y = -13 \text{ N/mm}^2$, $\tau_{xy} = 20 \text{ N/mm}^2$.

$\sigma_1 = \frac{65-13}{2} + \sqrt{\left(\frac{65+13}{2}\right)^2 + 20^2}$

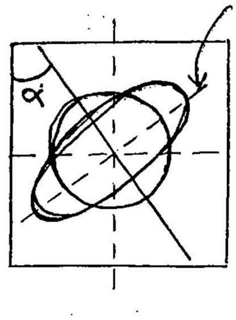
$= 26 + 43.83 = 69.83 \text{ N/mm}^2.$

$\sigma_2 = 26 - 43.83 = \underline{\underline{-17.83}} \text{ N/mm}^2$

10. Major axis of ellipse will develop in the direction of σ_1 which will be $\perp^r$ to major principal plane.

$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 20}{65 - (-13)}.$

$\alpha = 13.57^\circ$ (with vertical).



Angle of major axis of ellipse (along which σ_1 is acting)

$= \alpha + 90 = \underline{\underline{103.5^\circ}}$

11. Length of major axis:

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\frac{\partial D}{D} = \frac{70}{2 \times 10^5} - 0.3 \frac{(-18)}{2 \times 10^5}$$

$$\delta D = 0.113 \text{ mm.}$$

Major axis length $= 300 + 0.113 = \underline{\underline{300.113}}$ mm

Length of minor axis:

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\frac{\partial D}{D} = \frac{-18}{2 \times 10^5} - 0.3 \times \frac{70}{2 \times 10^5}$$

$$\partial D = -0.0585 \text{ mm.}$$

Minor axis length $= 300 - 0.0585 = \underline{\underline{299.94}}$ mm

9th Oct,
THURSDAY

03. SHEAR FORCE & BENDING MOMENT

→ Equilibrium Equations.

(i) 1 D

$\Sigma F_{\text{along axis}} = 0.$

(ii) 2D (Plane Stress).
Eg: Beams, shafts.

$\Sigma F_y = 0$; $\Sigma F_x = 0$; $M_z = 0.$

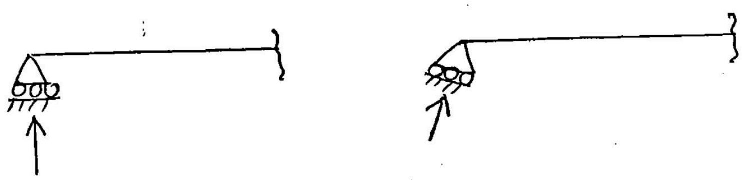
(iii) 3D (spatial)

$\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0.$

$\Sigma M_x = 0$; $\Sigma M_y = 0$; $\Sigma M_z = 0$

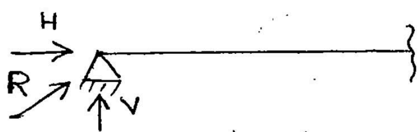
→ Types of Support.

(i) Roller Support.



Eg: Old bridges.

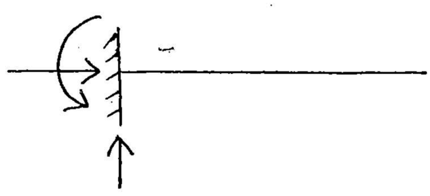
(ii) Hinged Support. (Pinned)



(2 reaction components)

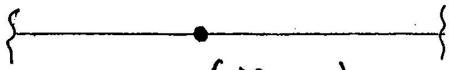
Eg: Old bridge.

(iii) Fixed Support.



(3 reaction components).

(iv) Internal Hinge.

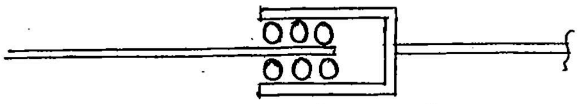


Eg: Elbow joint

(M = 0)

(v) Shear Hinges

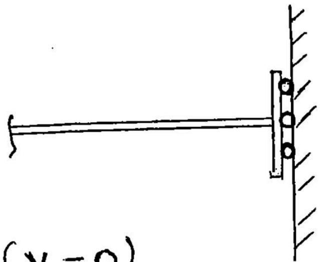
a) Axial Shear Hinge



Eg: Shock absorber.

(H = 0)

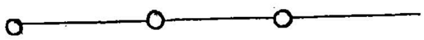
b). Transverse. ($\perp^r$ to axis).



Eg: Rolling Shutters.

(V = 0)

vi) Links.

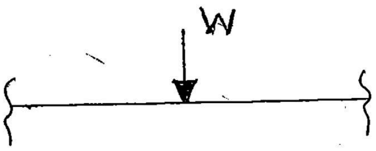


Eg: Truss.

Transfer axial forces

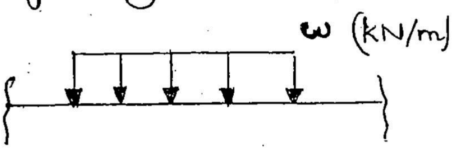
→ Types of Loads.

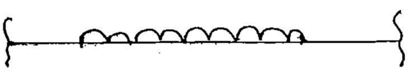
1. Concentrated or Point Load.



(more stress concentration)

2. Uniformly Distributed load (udl).



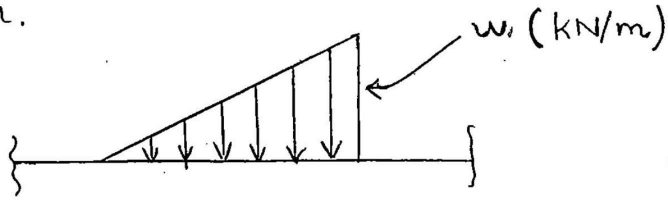


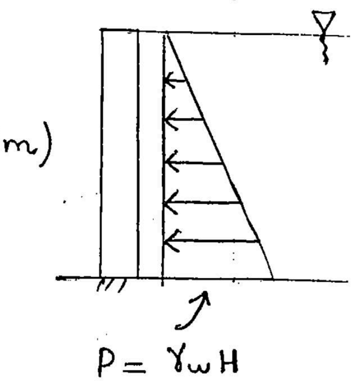
As per IS 875, (Part 1) DL, (Part 2) LL, (Part 3) WL, (Part 4) SL, acts as udl.

But as per IS 1893, earthquake load is a random load.

3. Uniformly Varying Load (UVL).

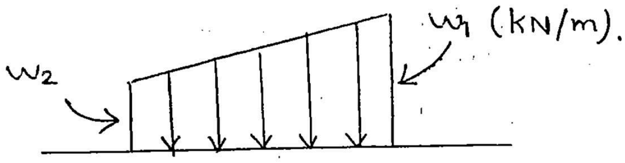
a) Triangular.



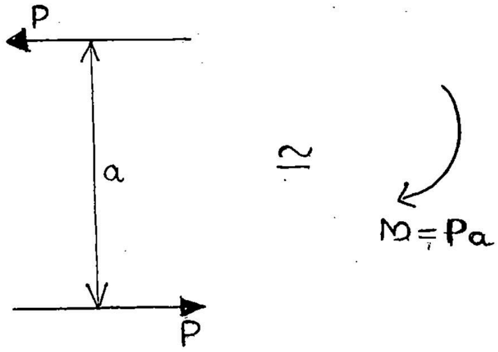


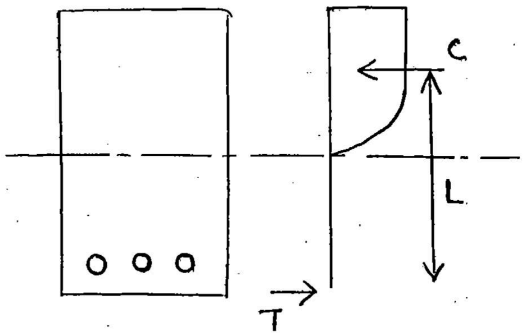
Retaining Wall.

b) Trapezoidal.



4. Couple

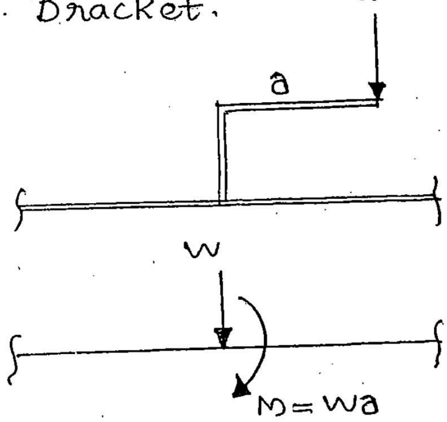




$M = CL$ or TL

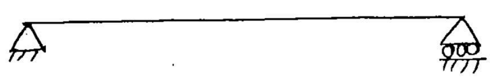
$L \rightarrow$ lever arm.

5. Bracket.



→ Types of Beams.

(i) Simply Supported Beam



(ii) Propped (Supported) Cantilever.

→ Shear Force Diagram. & Bending Moment Diagram.

Diagram showing variation of SF along a structure.

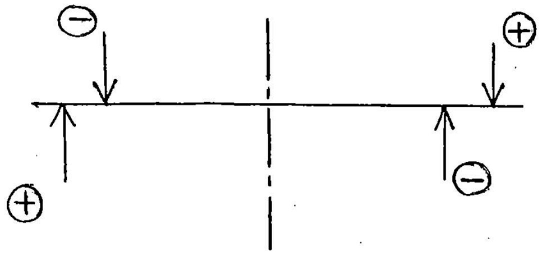
* SF at a point (or) SF @ a section.

Algebraic sum of vertical (or) transverse forces either to the left or to the right of a section

Sign convention:

Clockwise shear — +ve

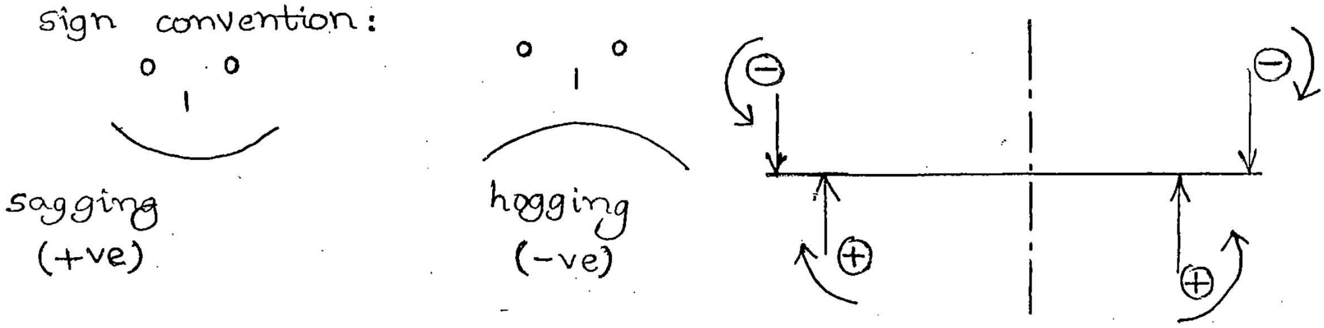
Anti-clockwise shear — -ve.



* BM at a Section (or) BM at a Point.

Algebraic sum of moments either to the left or to the right of a section

Sign convention:



→ Relation b/w rate of loading, SF & BM

w → rate of loading (kN/m)

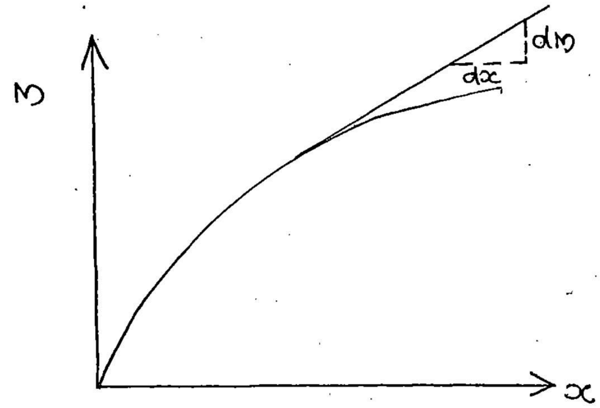
F → SF (kN).

M → BM (kN·m)

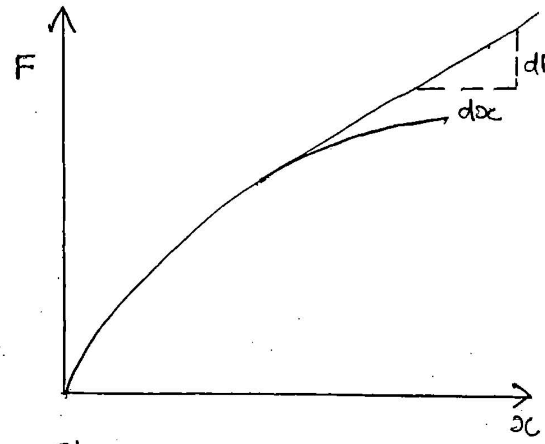
$$F = \frac{dM}{dx} \quad \text{--->} ①$$

$$w = \frac{dF}{dx} \quad \text{--->} ②$$

Rate of change of BM gives SF; and rate of change of SF is rate of loading.



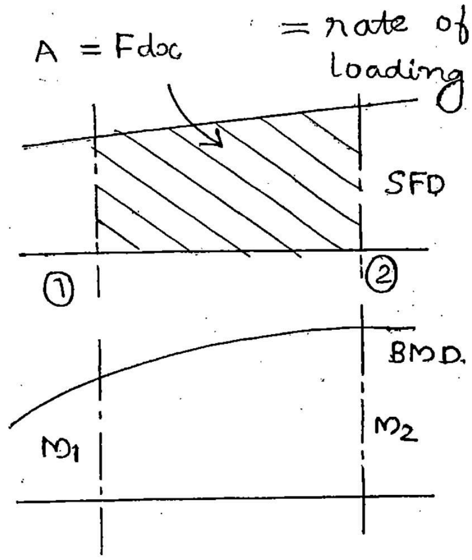
Slope to BMD $= \frac{dM}{dx} = SF$



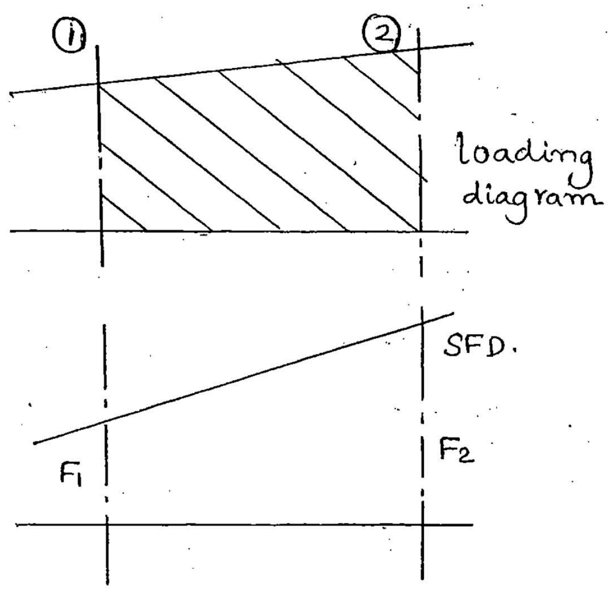
Slope of SFD $= \frac{dF}{dx}$ = rate of loading

From ①, $dM = F dx$.

$|M_2 - M_1|$ = area of SFD b/w 1 & 2.



From ②, $dF = w.dx$



$|F_2 - F_1|$ = area of loading diagram b/w 1 & 2.

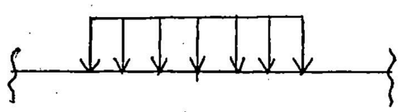
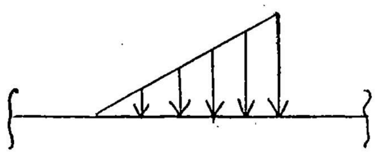
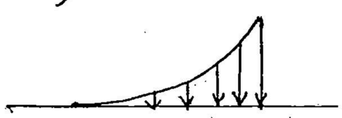
* For M to be maximum

$$\frac{dM}{dx} = 0 \Rightarrow \boxed{F = 0}$$

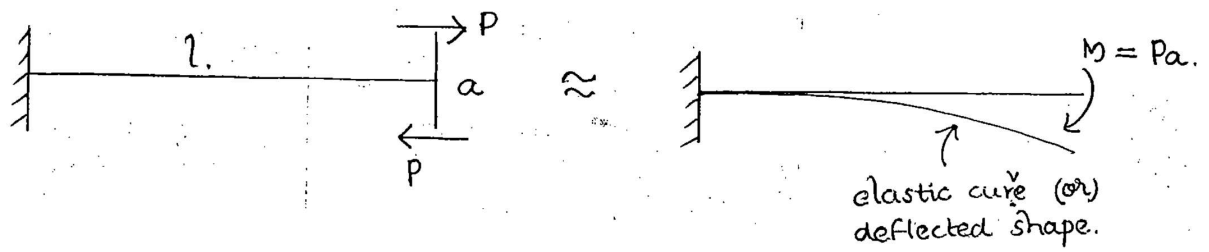
At the point of maximum magnitude of BM, shear force must be zero. At the point of maximum magnitude of SF, BM need not be zero.

- In a beam, if more than one zero SF point is acting, at all the points BM need not be maximum. (at the point of max BM, SF is zero)
- The above condition is valid only for transverse or vertical or gravity loads, only; not applicable for concentrat moments.

th Oct, WEDNESDAY

Loading	SFD (kN)	BMD (kN-m)
 No variation of load.	Uniform / Constant / Horizontal st. line (x^0)	Linear / Inclined straight line. (x^1)
 uniformly distri. load (udl) (x^0)	(x^1)	2° parabola / Square parabola. (x^2)
 (x^1)	(x^2)	3° parabola / Cubic parabola (x^3)
 Parabolic load (x^2)	(x^3)	(x^4)

Q. Draw SFD & BMD :

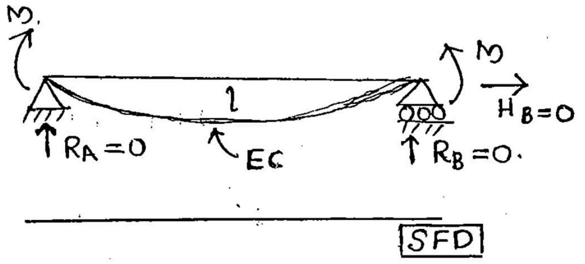


This is a case of pure bending.

For pure bending, SF = 0

BMD = non zero constant

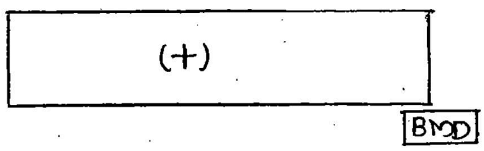
Q



$\Sigma M_A = 0$

$\Rightarrow R_B \times l - M + M = 0$

$\therefore R_B = 0.$



$\Sigma F_y = 0$

$\Rightarrow R_A + R_B = 0$

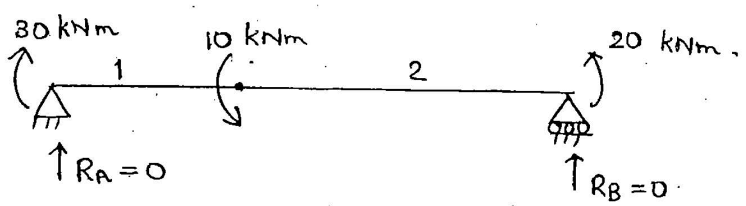
$\therefore R_A = 0.$

This is a pure bending criterion.

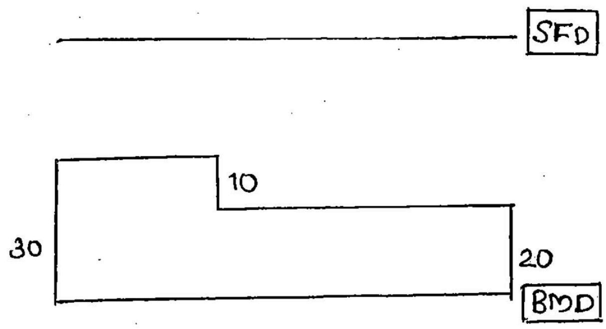
- In real beams, self wt. causes shear force. Therefore pure bending is not possible in practise.

Elastic Curve : It is the deflected shape. For pure bending, it is arc of a circle (R = const), otherwise it is parabola.

Q.

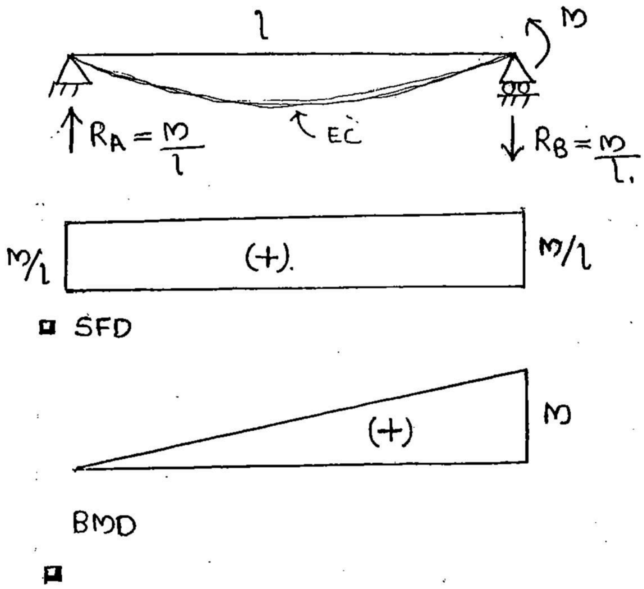


Net moment acting on beam = 30 − 10 − 20 = <u>0</u>



Whenever a concentrated moment acts on the beam, a jump happens in BMD.

Q.



$\Sigma M_A = 0$

$\Rightarrow -R_B \times l - M = 0$

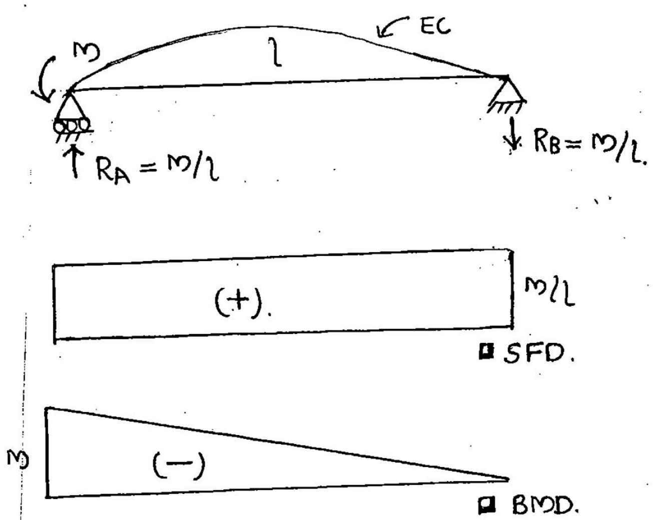
$\therefore R_B = -\frac{M}{l}$

$\Sigma F_y = 0$

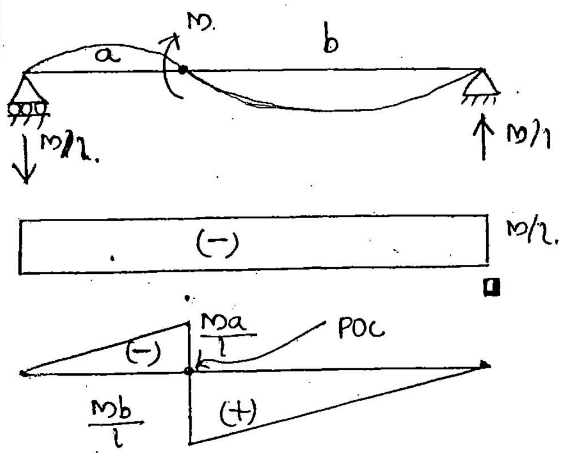
$\Rightarrow R_A + R_B = 0$

$\therefore R_A = \frac{M}{l}$

Q.



Q.



Here $b > a$

$\therefore$ Design BM $= \frac{Mb}{l}$

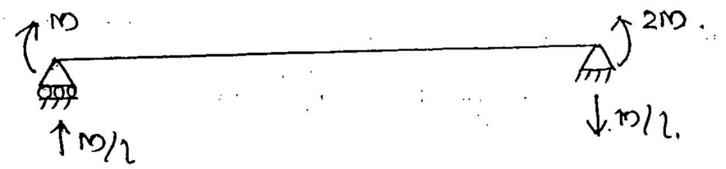
Point of Contraflexure: Point where bending moment changes sign, or curvature of the beam reverses its direction.

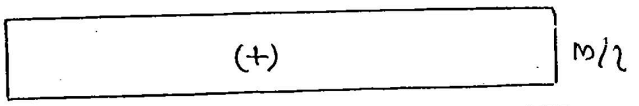
- BMD is always drawn on the tension side. So point of contraflexure determines the portion at which reinforcement is provided. (top or bottom of beam)

* **Design BM (or) Absolute BM:**

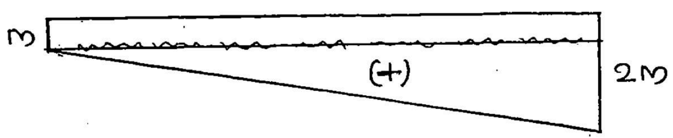
Maximum magnitude of BM over a beam.

Q.





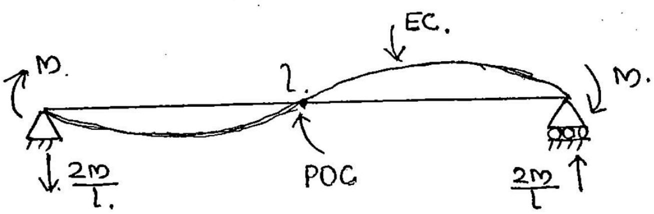
▪ SFD

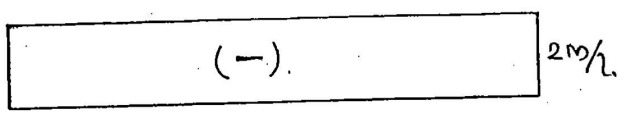


Design BM = 2M.

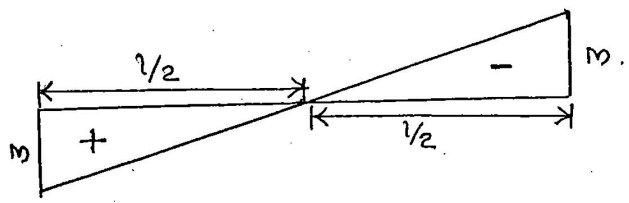
▪ BMD

Q.

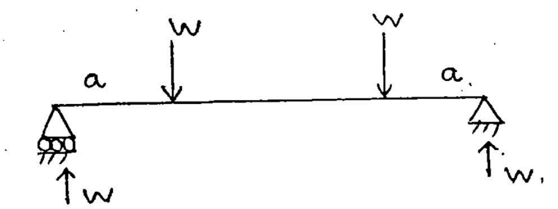


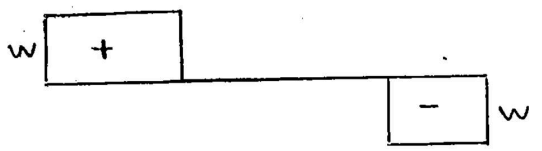


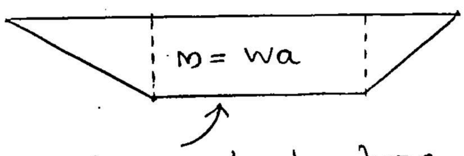
Design BM = M.



Q.



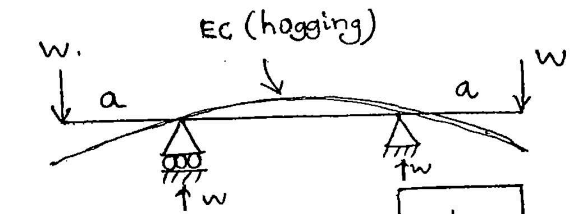


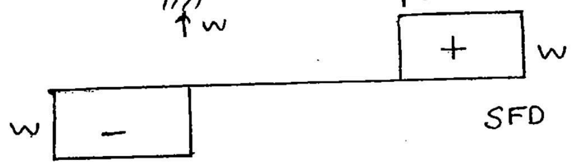


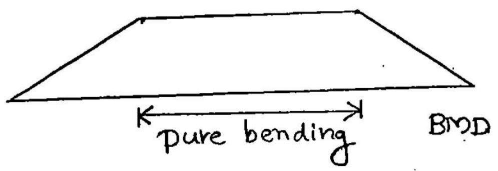
BM is constant where SF is zero (Pure bending).

In laboratories, we apply two-point load systems. It is done to eliminate shear and obtain pure bending criterion. Cracks formed will be due to bending – <u>flexural crack</u>

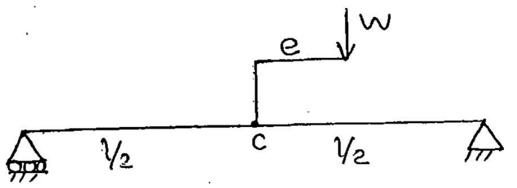
Q.



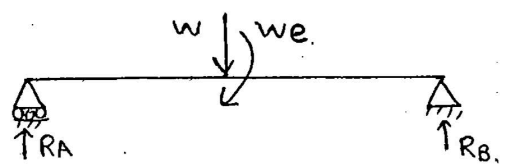




Q.



≈



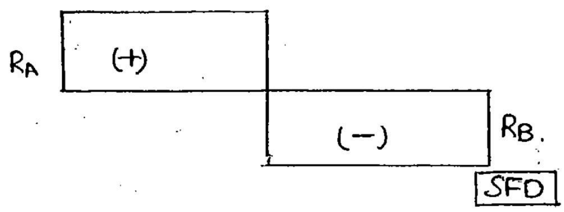
$R_B \times l = We + \frac{Wl}{2}$

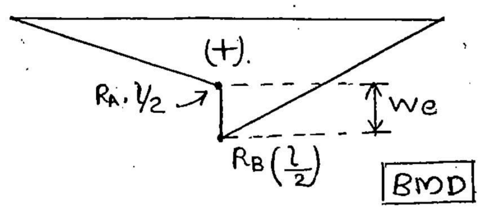
$R_B = \underline{\underline{\frac{We}{l} + \frac{W}{2}}}$

$R_A + R_B = W$

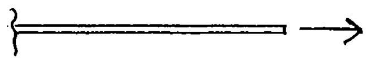
$R_A = W - \left(\frac{We}{l} + \frac{W}{2}\right)$

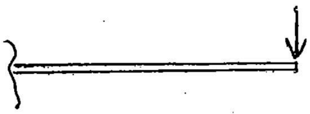
$= \underline{\underline{\frac{W}{2} - \frac{We}{l}}}$

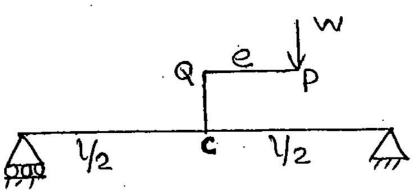


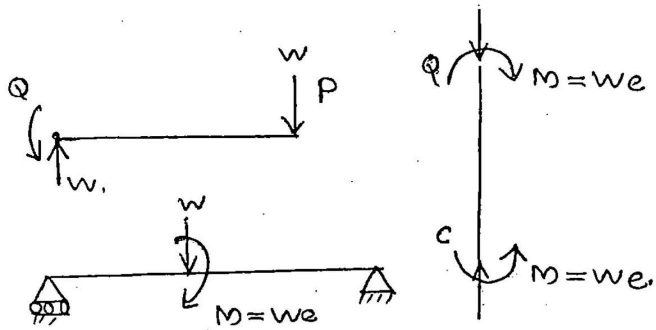


* Design Forces :







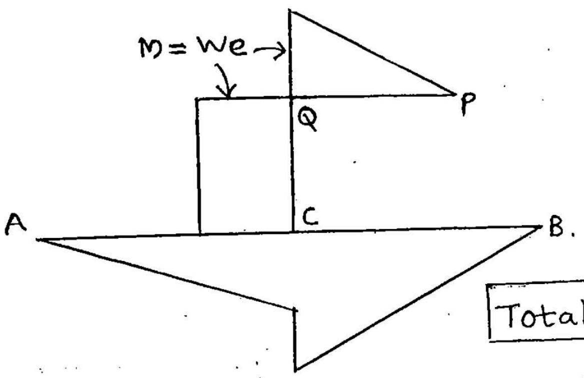


PQ → SF, BM.

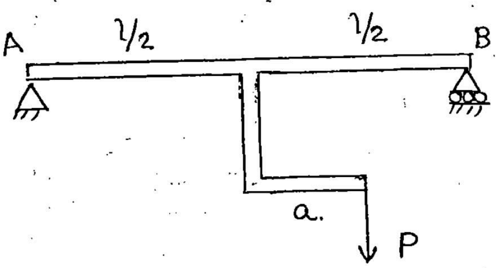
QC → AF(comp), BM

AB → SF, BM.

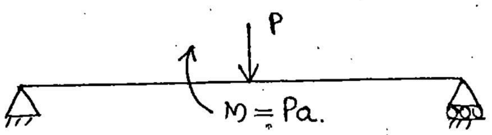
$\boxed{\text{Vertical jump in SFD indicates conc. load or reaction.}}$



Total BMD.

Q. – 

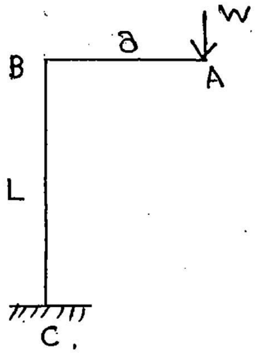
Find design BM on beam AB.

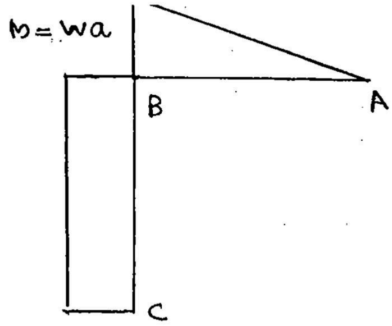


⇒ Now its same as previous.

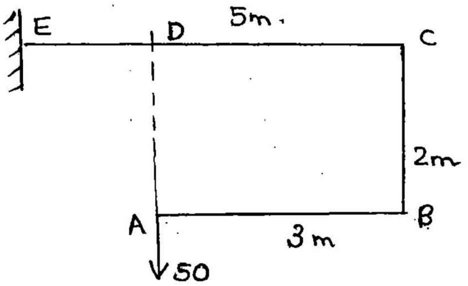
$$\text{Design BM} = R_B\left(\frac{l}{2}\right) = \left(\frac{Pa}{l} + \frac{P}{2}\right)\frac{l}{2} = \underline{\underline{\frac{Pa}{2}}} + \frac{Pl}{4}$$

Q Draw BMD.





Q.

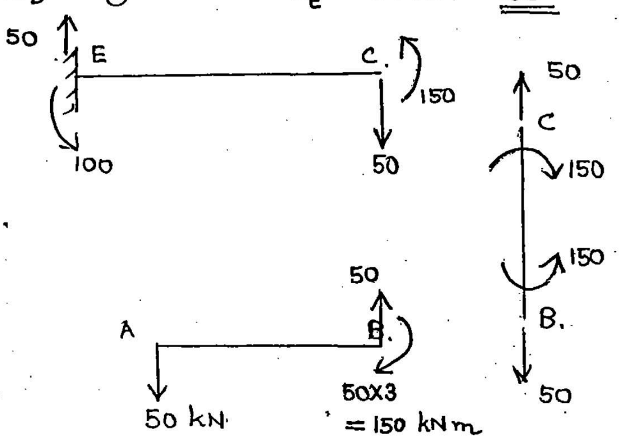


$M_A = 0$

$M_B = 50 \times 3 = 150$

$M_C = 50 \times 3 = 150$

$M_D = \text{zero}$ & $M_E = 50 \times 2 = \underline{\underline{100}}$



Design Forces:

AB $\longrightarrow$ SF, BM.

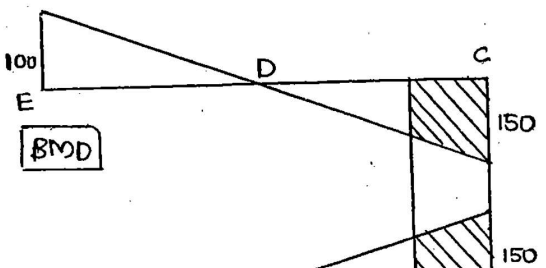
BC $\longrightarrow$ Pure BM, AF (tension).

CE $\longrightarrow$ SF, BM

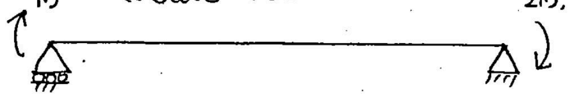
@ E:

$M_E = +150 - 50 \times 5$

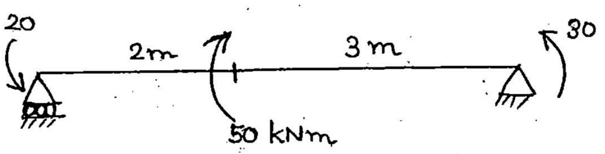
$= -100$ kNm (hogging).

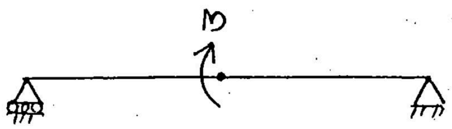


locate POC

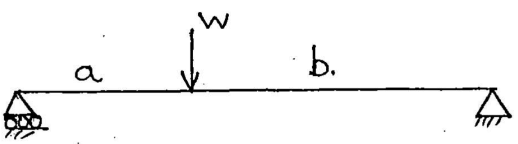


Q.

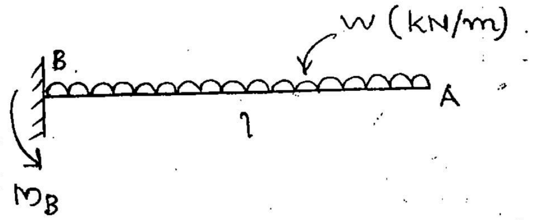


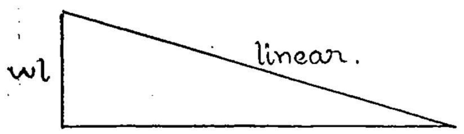


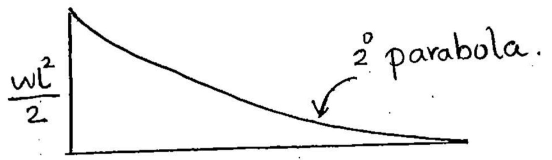
Q.



Q.





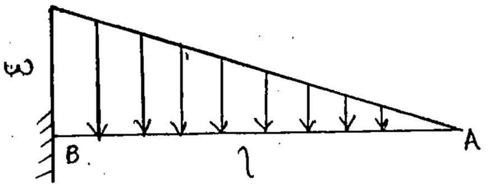


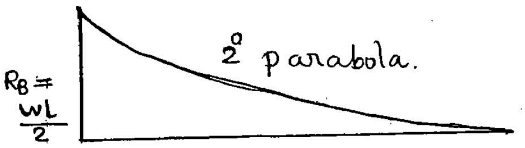
Shear Force, $F = \frac{dM}{dx}.$

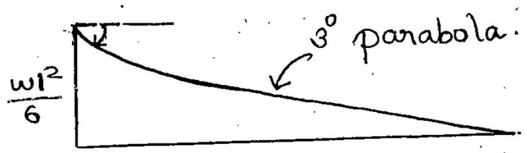
where $\frac{dM}{dx}$ is slope of BMD

So, shape of BMD is (positive slope) concave. and not convex.

Q.







$(SF)_A = 0.$

$(SF)_B = \frac{1}{2} \times w \times l = \frac{wl}{2}$

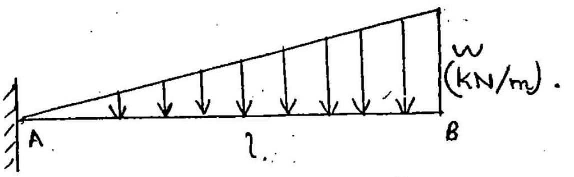
$w = \frac{dF}{dx}$

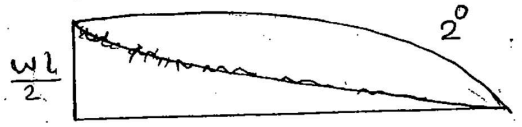
rate of loading

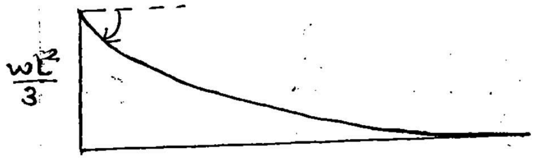
slope of SFD.

$M_B = -\frac{1}{2} wl \times \left(\frac{1}{3} l\right) = -\frac{wl^2}{6}$ (hog)

Q.







Rate of loading max at B.

$\therefore \frac{dF}{dx}$ = slope max at B.

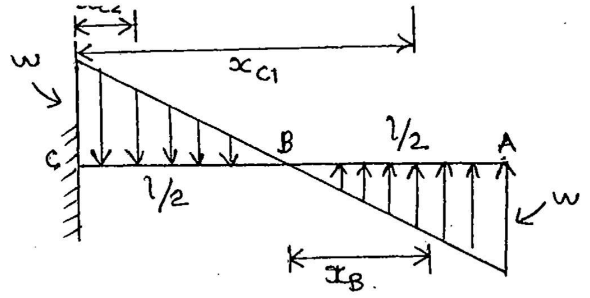
Similarly min. rate of loading at A. $\therefore$ slope = zero at A.

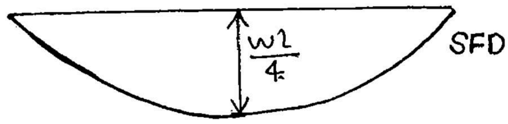
$M_B = -\frac{1}{2} \times wl \times \frac{2}{3} \times l = \frac{wl^2}{3}.$

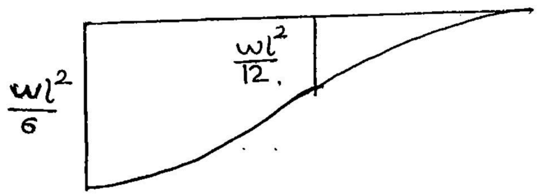
$(SF)_A$ = max. $\Rightarrow \frac{dM}{dx}$ = max.

$(SF)_B = 0 \Rightarrow \frac{dM}{dx} = 0.$

Q.







$(SF)_A = 0$

$(SF)_B = -\frac{1}{2}\times\frac{l}{2}\times w = -\frac{wl^2}{4}$

$(SF)_C = 0.$

$w = \frac{dF}{dx}$

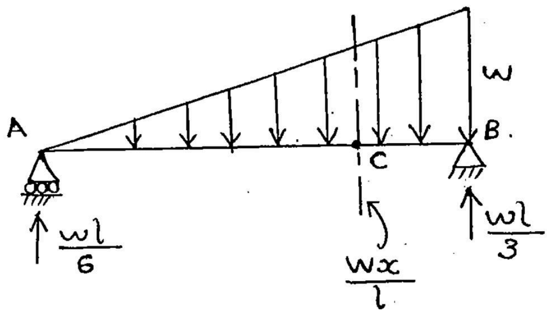
$M_A = 0.$

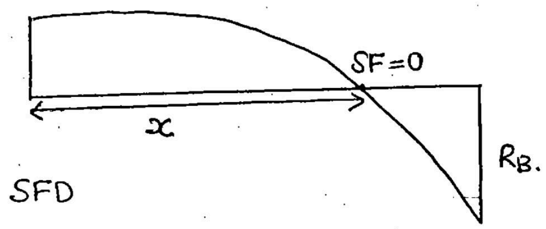
$M_B = \frac{wl}{4}\times\frac{2}{3}\times\frac{l}{2}$

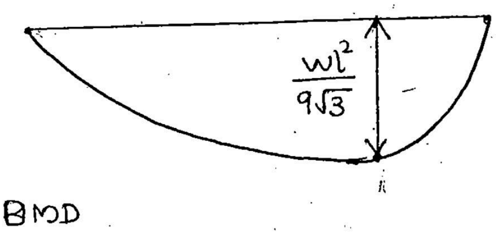
$= \frac{wl^2}{12}$ (sagging)

$M_C = \frac{1}{2}\times\frac{l}{2}\times w\left(\frac{2}{3}\cdot\frac{l}{2}+\frac{l}{2}\right) - \frac{1}{2}\times\frac{l}{2}\,w\left(\frac{1}{3}\times\frac{l}{2}\right) = \underline{\underline{\frac{wl^2}{6}}}$

Q.







$R_A + R_B = \frac{wl}{2}.$

$\Sigma M_A = 0$

$R_B \times l = \frac{wl}{2}\left(\frac{2}{3}\times l\right).$

$R_B = \frac{wl}{3}.$

$(SF)_C = R_A -$ hatched area of Δ^{le}

$0 = \frac{wl}{6} - \frac{1}{2}x\left(\frac{wx}{l}\right)$

$\Rightarrow x = l/\sqrt{3}$ (from A).

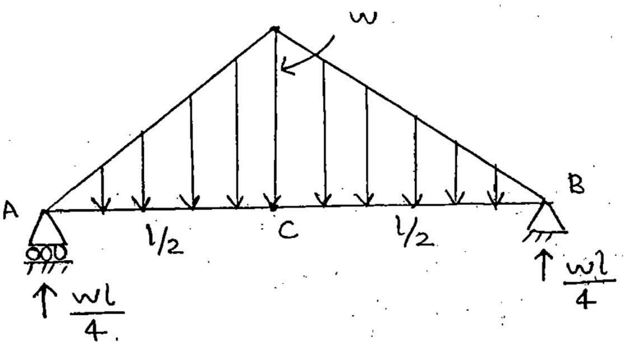
$M_A = M_B = 0.$

Max BM @ zero SF point $\Big\}$ $M_C = R_A \cdot x -$ hatched area $\times \frac{x}{3}.$

$M_C = \frac{wl}{6}x - \frac{1}{2}x\left(\frac{wx}{l}\right)\frac{x}{3}.$

$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}} = \underline{\underline{\frac{wl^2}{9\sqrt{3}}}}$ $\quad \frac{l^3}{3\sqrt{3}}$

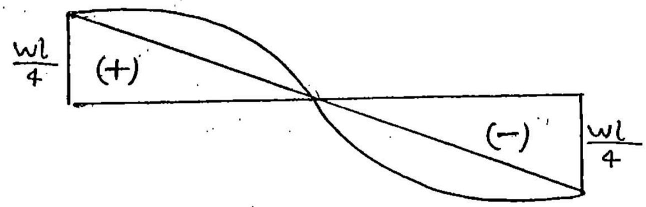
Q.



$$R_A + R_B = \frac{wl}{2}$$

$$R_B \times l = \frac{wl}{2} \times \frac{l}{2}$$

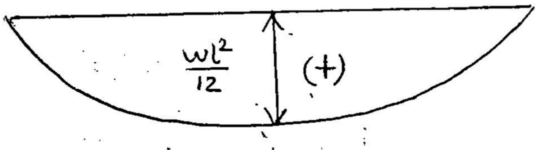
$$R_B = \frac{wl}{4} = R_A.$$



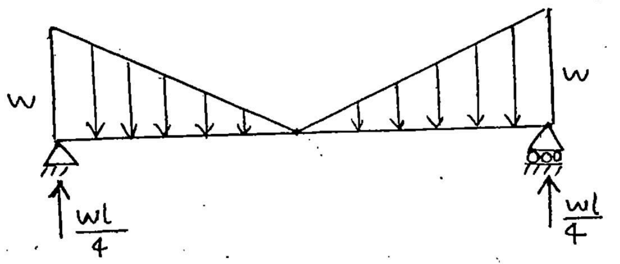
$$M_C = R_A \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{1}{2} \times \frac{l/2}{3}.$$

$$= \frac{wl}{4} \times \frac{l}{2} - \frac{wl^2}{24}.$$

$$= \underline{\underline{\frac{wl^2}{12}}}$$

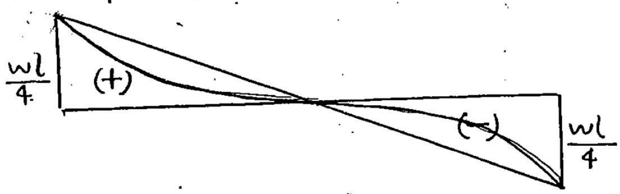


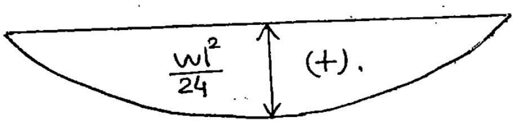
Q.



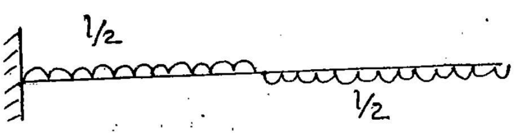
$$M_C = \frac{wl}{4} \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{l}{2}$$

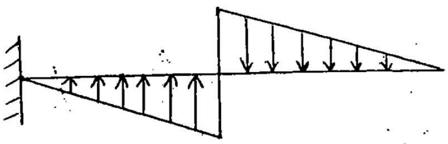
$$= \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}.$$

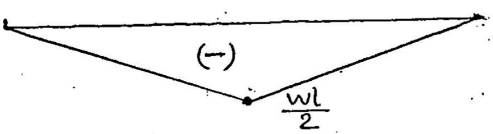


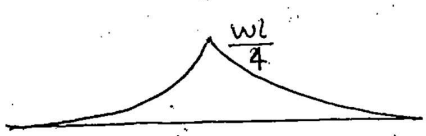


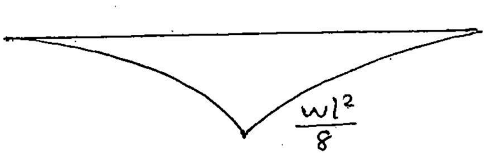
Q.

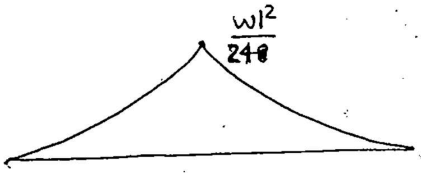








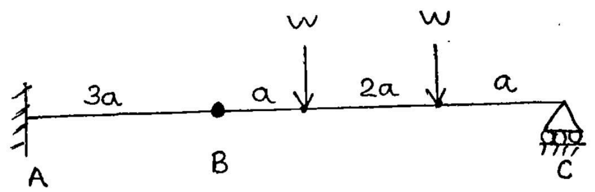


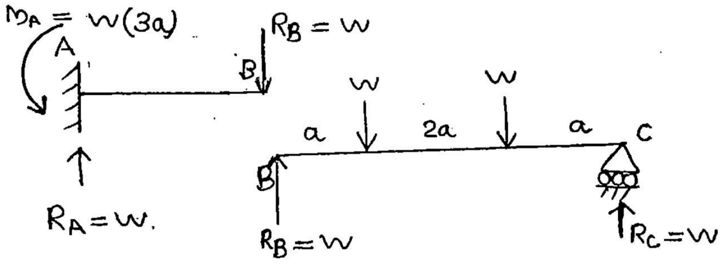


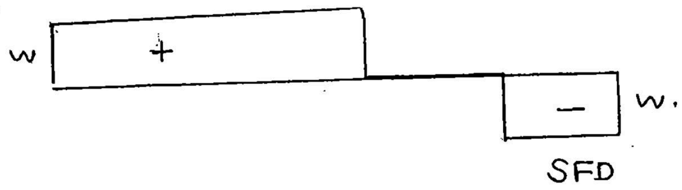
→ Beams with Internal Hinges :

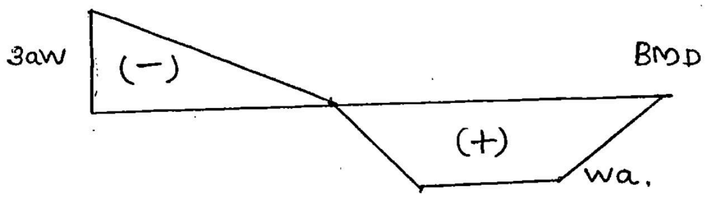
Internal hinge is also called as 'Moment Hinge' ($M=0$).

Q.

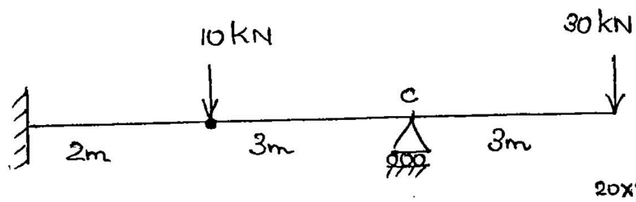


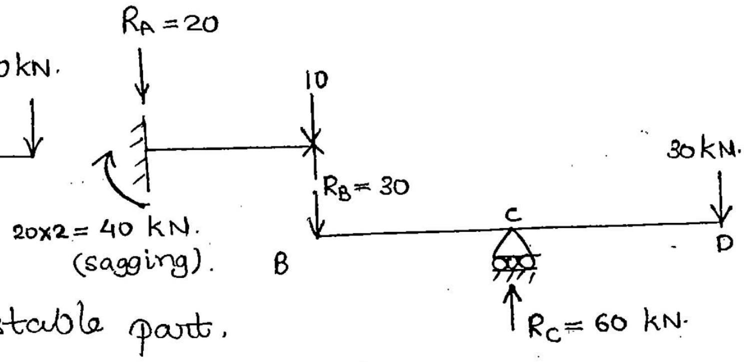






Q.

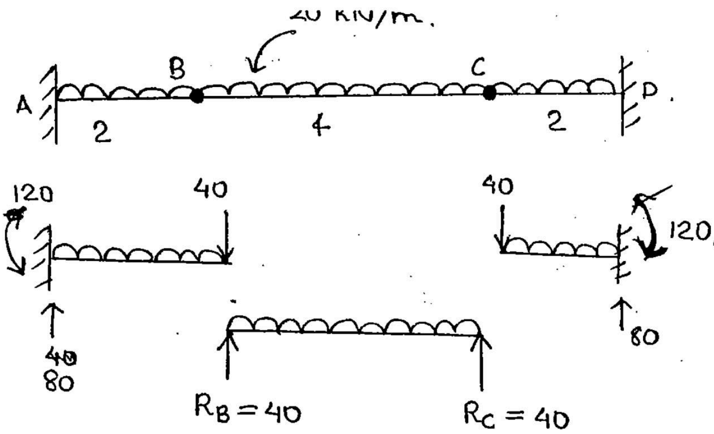




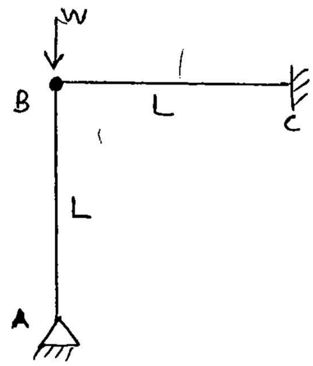
Cantilever portion is the stable part.

Apply 10 kN in the stable part.

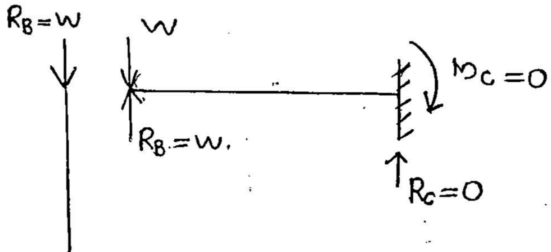
Q.



Q.

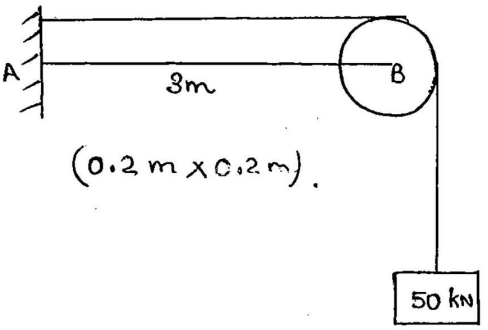


Find R_c ?

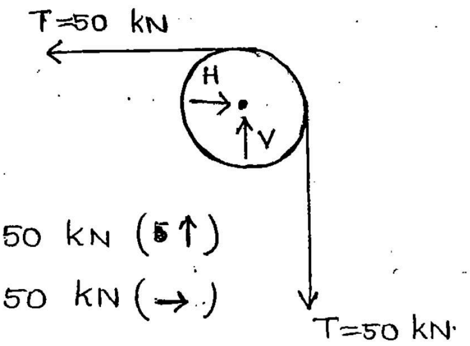


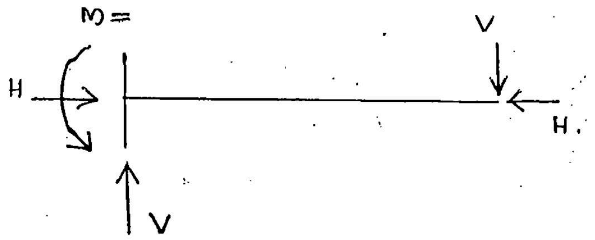
$\Rightarrow R_c = 0.$

Q.



Find σ, M, SF/ V at A.





V = 50 kN (↑)

H = 50 kN (→)

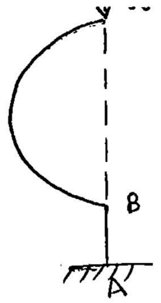
Axial force = H = 50 kN.

$M = 50 \times 3 = 150$ kNm. (hogging).

SF = V = 50 kN.

$$\sigma = \frac{AF}{\text{c/s area of beam}} = \frac{50}{0.2 \times 0.2} = 1250 \text{ kN/m}^2$$

Q



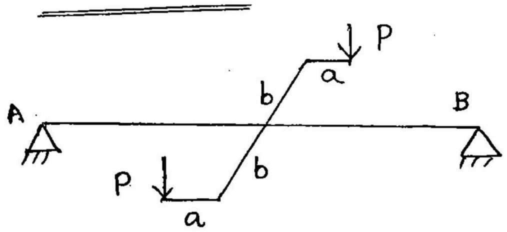
Line of action of w passes through A.

$\therefore M_A = 0$

Design force in AB = Axial force (compression only)

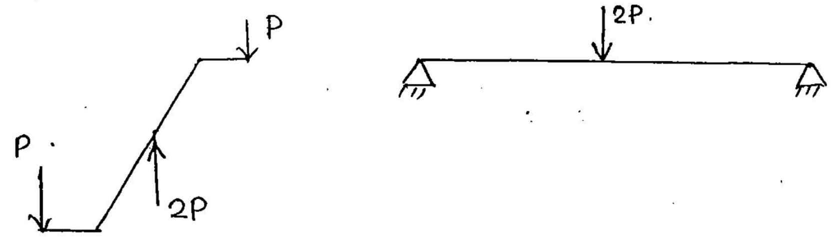
No BM, No SF

Q.

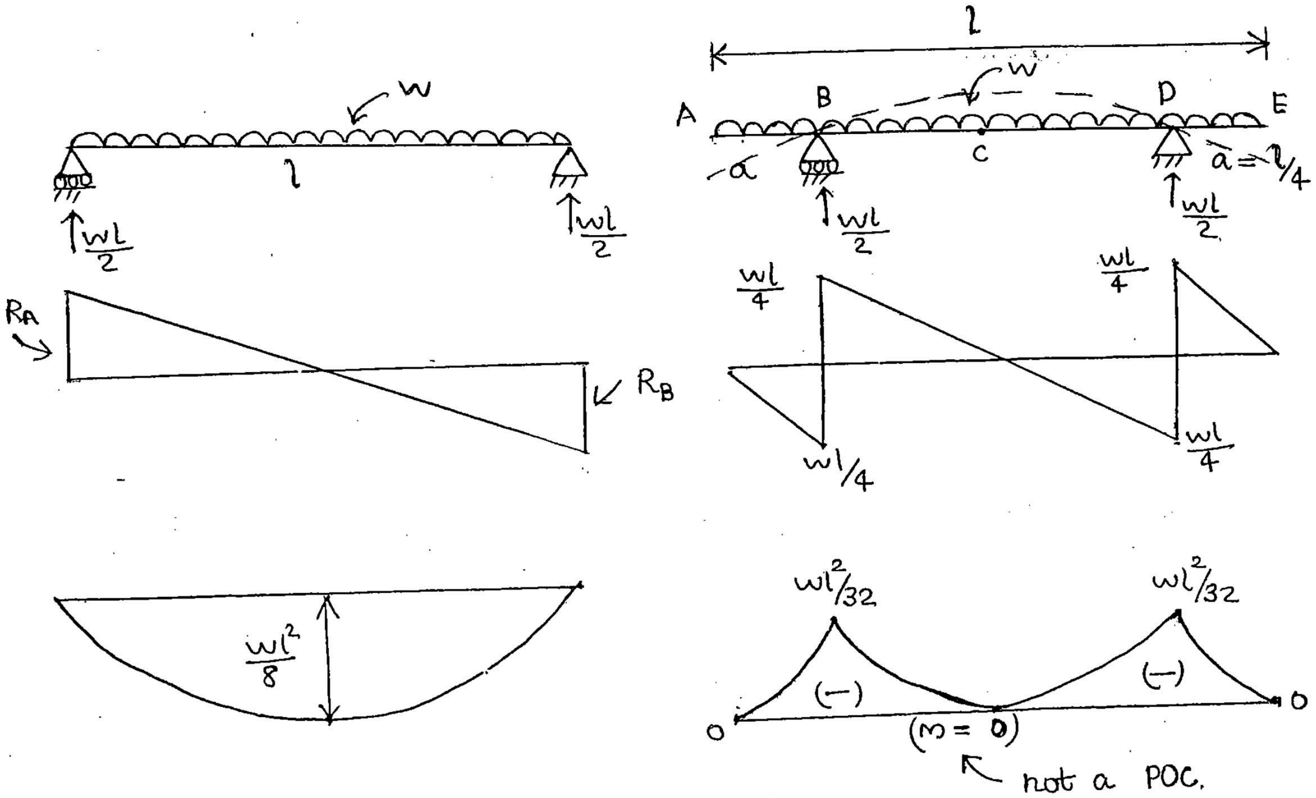


. Calculate design force in AB

Design force in AB = SF & BM



Q.



So providing a overhang, $(a = l/4)$,

the design BM can be reduced for SSB with udl.

$(SF)_A = 0$

$(SF)_{B,\text{left}} = -wa = -w\frac{l}{4}$

$(SF)_{B,\text{right}} = -wa + \frac{wl}{2} = -\frac{wl}{4} + \frac{wl}{2} = \frac{wl}{4}$

$(SF)_C = \frac{wl}{2} - \frac{wl}{2} = 0.$

$M_A = 0.$

$M_B = -wa \times \frac{a}{2} = -w\frac{l}{4} \times \frac{l}{8} = \underline{\underline{-\frac{wl^2}{32}}}$

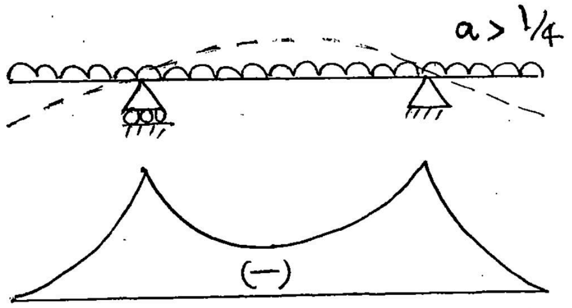
$M_C = \frac{wl}{2} \times \frac{l}{4} - \frac{wl}{2} \times \frac{l}{4} = 0$

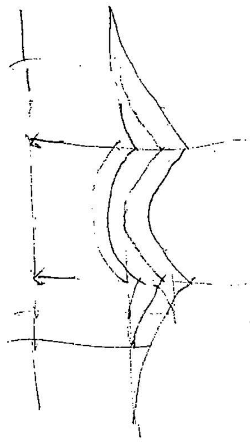
⊙ Point of Inflection : The point where BM just becomes zero.

All POCs are POIs; the converse may not be true.

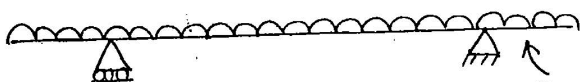
⊙ Compared to simply supported beam, BM <u>decreases by 4 times</u> for a beam with overhang $(= l/4)$.

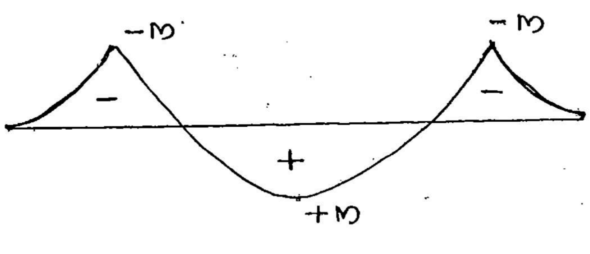
Q.





Q.





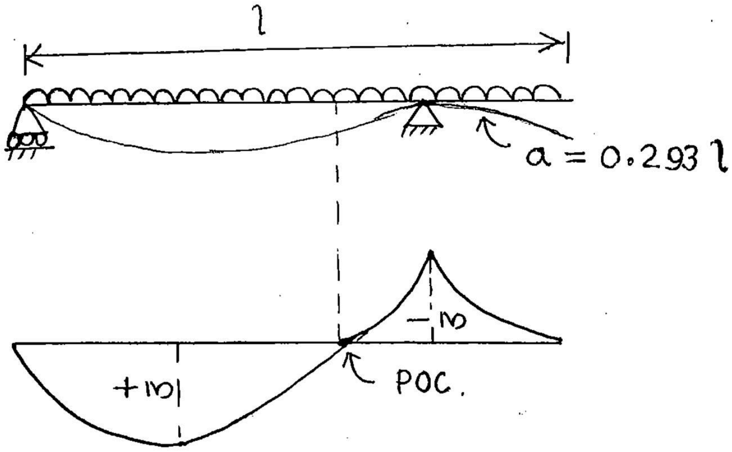
$+M = -M$

$\Rightarrow$ Sagging BM = Hogging BM.

$M = wa\frac{a}{2} = \underline{\underline{\frac{wl^2}{46.67}}}$

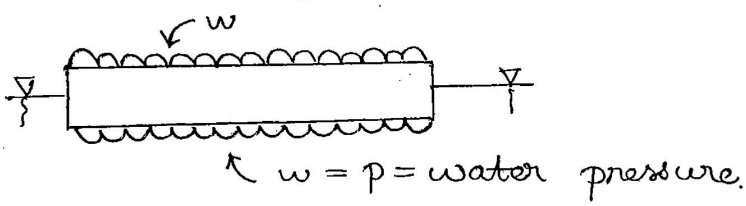
Compared to SSB, BM decreases by $\frac{46.67}{8} = 5.8$ times.
So this is the least design BM when overhang provided on both sides.

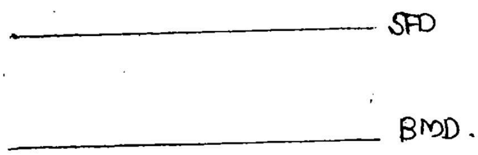
Q.



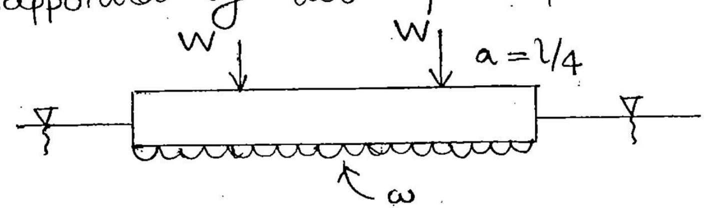
Design criteria:
Sagging BM = hogging BM $= wa\left(\frac{a}{2}\right) = \frac{wl^2}{23.3}$
Compared to SSB, BM decreases by $\frac{23.3}{8} = 2.9$ times

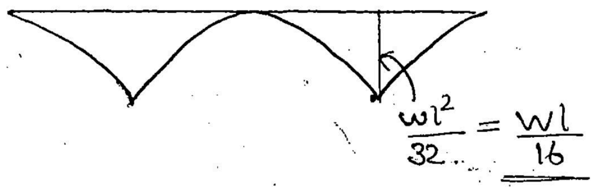
Q. A wooden log of uniform c/s is floating on water with self weight. Draw SFD & BMD.





Q. A wooden log floats on water as shown in fig and supported by two equal point loads. Draw BMD

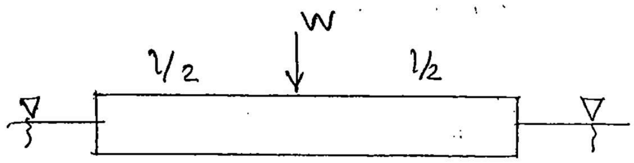




$$wl = 2w$$

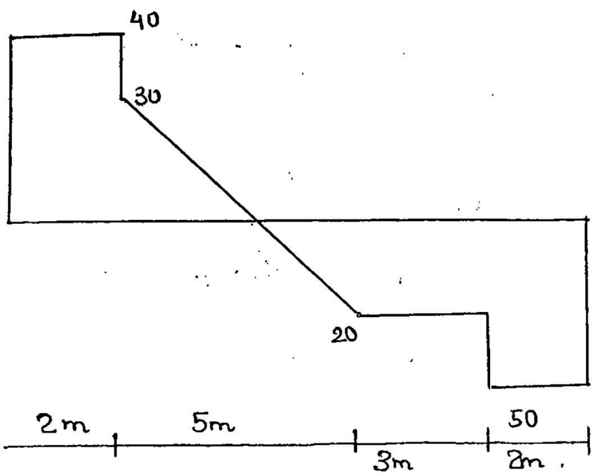
$$\frac{wl^2}{32} = \frac{2wl}{32} = \underline{\underline{\frac{Wl}{16}}}$$

Q. A wooden log is floating on water with central load W. Draw SFD & BMD.



→ Convertion of SFD to Loading

Q.

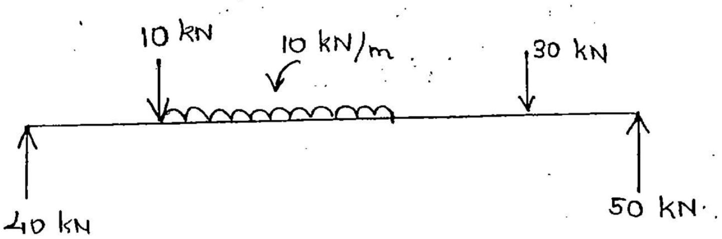


Intensity of loading,

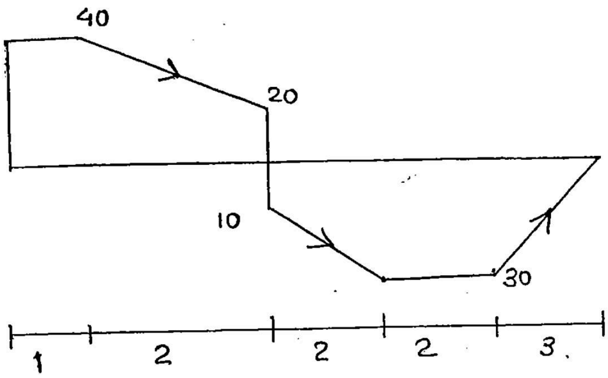
$$w = \frac{dF}{dx}$$

$$= \frac{30-(-20)}{5}$$

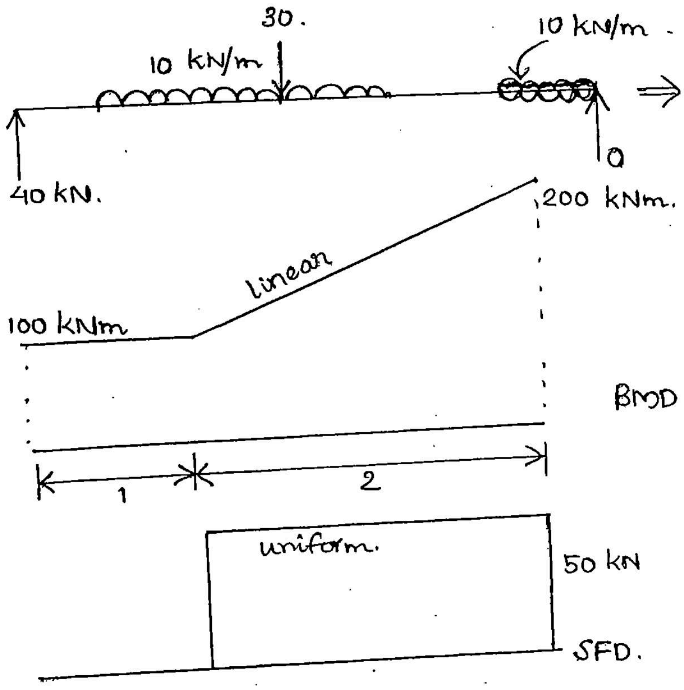
$$= 10 \text{ kN/m}.$$



Q.



Q



$\Rightarrow \Sigma F_y = 0$

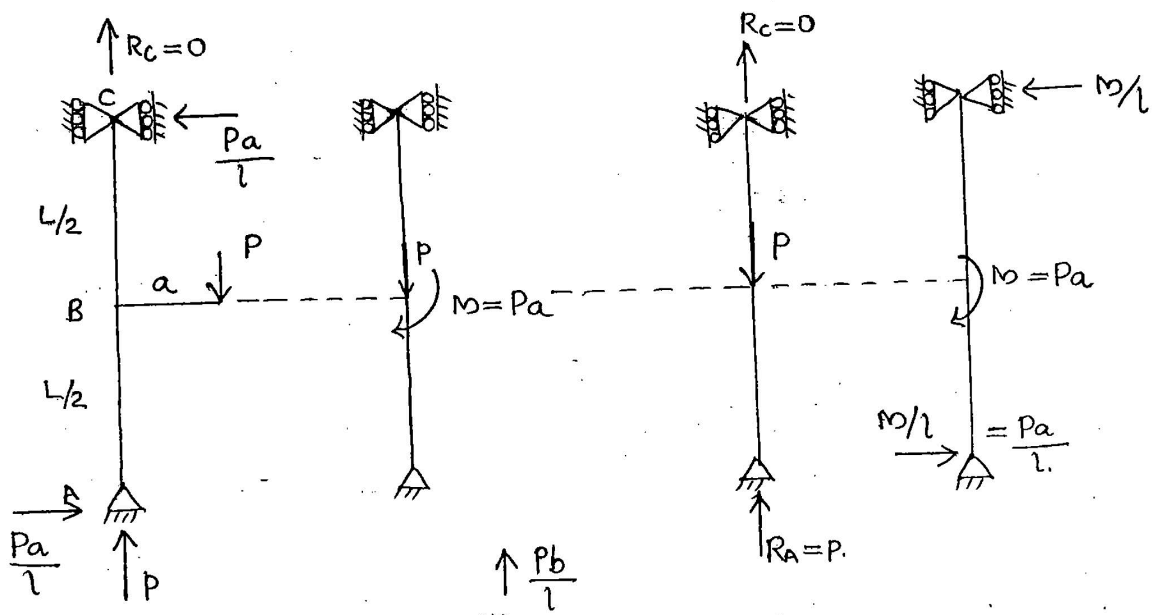
$$F = \frac{dM}{dx}$$

Slope of BMD $= 0$

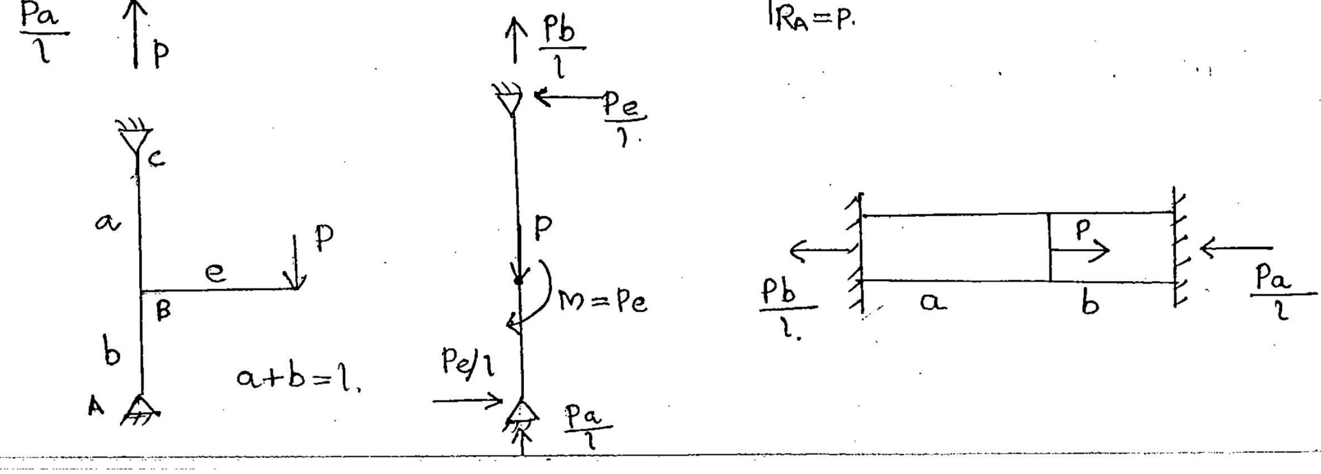
$$F = \frac{\text{Change in BM.}}{\text{length of beam.}}$$

$$= \frac{200-100}{2} = \underline{\underline{50\ kN}}$$

Q.



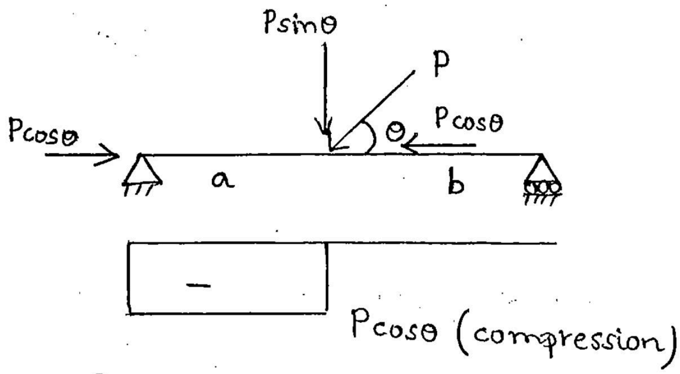
Q



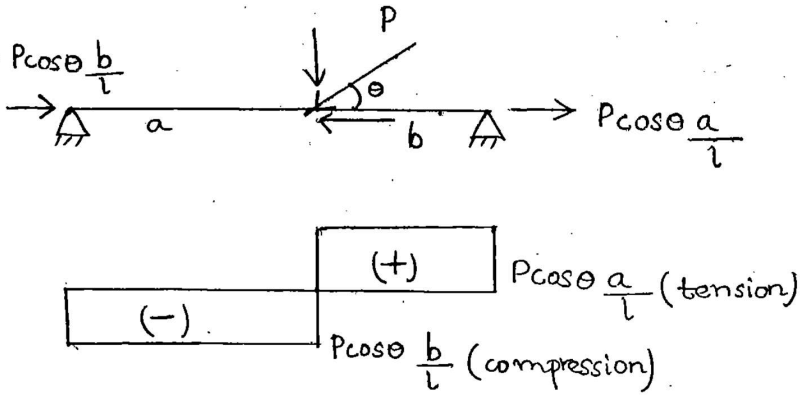
→ Axial Force Diagram.

- due to axial loads.
- inclined loads.

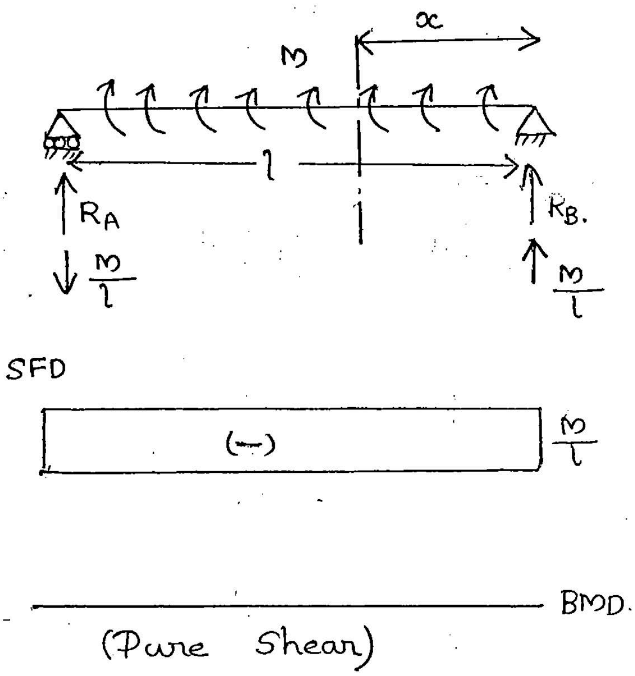
Q



Q.



Q.



Total distributed moment $= M$

$l \longrightarrow M$

$x \longrightarrow \frac{Mx}{l}$

$$M_x = R_B x - \frac{Mx}{l}$$

$$= \frac{M}{l}x - \frac{Mx}{l} = 0$$

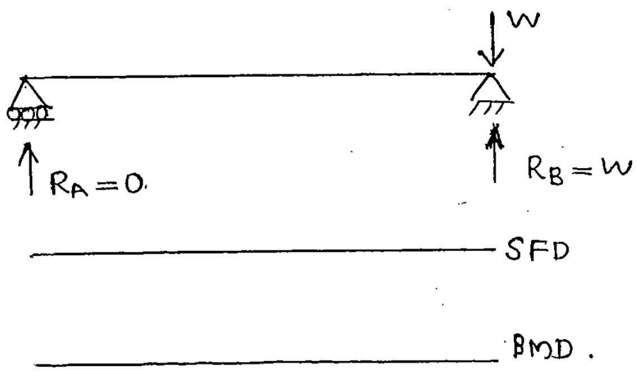
- **Pure Shear :-**

SF → non zero constant & max.

BM = 0.

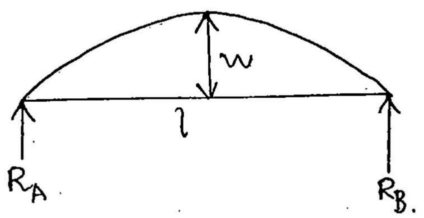
- Only example of <u>Pure Shear Condition</u>.

Q.



Q-29

11.

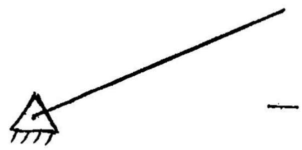


Max SF = Max reaction.

$$R_A = R_B = \frac{\text{Total load}}{2} = \frac{\text{area}}{2}$$

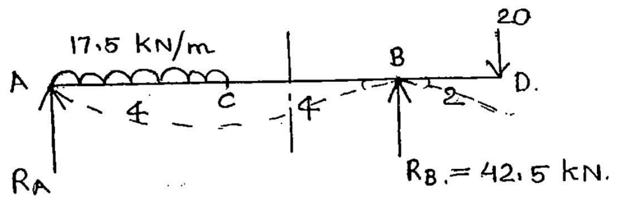
$$= \frac{\frac{2}{3} lw}{2} = \underline{\underline{\frac{wl}{3}}}$$

12.



— purely axial load.

06.

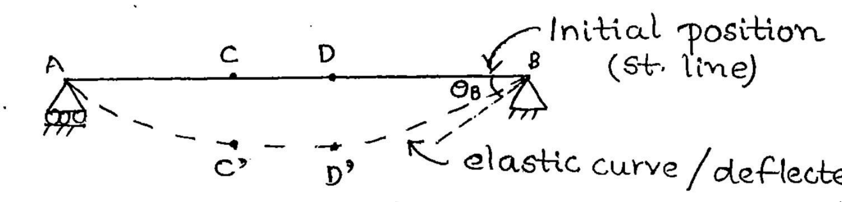


$$M_x = -20x + R_B(x-2)$$

$$0 = -20x + 42.5(x-2) \Rightarrow x = \underline{\underline{3.78\ \text{m}}}$$

7th Oct,
FRIDAY.

8. SLOPES & DEFLECTIONS



$$\theta_B = \left(\frac{dy}{dx}\right)_B.$$

Loading	Shape of EC
No load.	Straight line.
Pure bending (SF=0 & BM=const)	Arc of a circle (R = const.)
SF + BM	Parabola.

Deflection :

Displacement of a point from initial position to final position on elastic curve is called deflection.

$y_C = CC'$; $y_D = DD'$; $y_A = 0$; $y_B = 0$.

Slope :

The angle between tangents drawn to the initial point and final point on elastic curve in radia is called slope.

1. Macaulay's Double Integration Method

Euler's curvature equation:

$$\frac{d^2y}{dx^2} = \rho = \frac{1}{\text{radius of curvature}}$$

Bending Equation:

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow \frac{1}{R} = \frac{M}{EI}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\boxed{EI\frac{d^2y}{dx^2} = M_x}$$

$$\Rightarrow EI\frac{dy}{dx} = \int M + C_1 \text{ ; slope equation.}$$

$$EI\, y = \iint M + C_1 x + C_2 \text{ ; differential equation.}$$

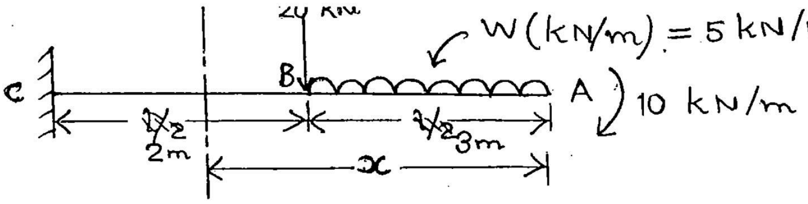
where EI is assumed as constant.

C_1 & C_2 are integration constants calculated by boundary conditions

NOTE:

- While substituting x values if any term in the bracket is $-ve$, treat the value as zero.

Q.



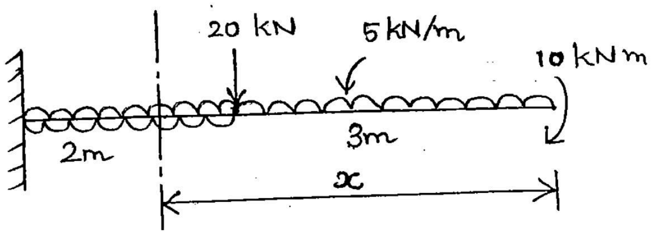
Rule 1: While taking a section for M_x, it should start from any one of the ends, in cantilever better from a free end.

Rule 2: The section should cross all the zones of the beam.

Rule 3: If udl is acting over a beam the section should cut the udl; otherwise extend the udl with compensation

Rule 4: If conc. moment is acting over a beam, consider a distance term with power zero.

Find the max slope & deflection at the free end. (EI = const)



$$EI\frac{d^2y}{dx^2} = M_x = -10x^0 - \frac{5}{2}x^2 + \frac{5}{2}(x-3)^2 - 20(x-3).$$

$$EI\frac{dy}{dx} = -10x - \frac{5}{6}x^3 + \frac{5}{6}(x-3)^3 - \frac{20}{2}(x-3)^2 + C_1$$

$$EI\,y = -5x^2 - \frac{5x^4}{24} + \frac{5}{24}(x-3)^4 - \frac{20}{6}(x-3)^3 + C_1x + C_2$$

At C; $x = 5m$, $\frac{dy}{dx} = 0$ & $y = 0$ (fixed end).

$$EI \times 0 = -50 + \frac{-5\times5^3}{6} + \frac{5}{6}\times 8 - 10(2)^2 + C_1$$

$$\underline{\underline{C_1 = 187.5}}$$

$$EI \times 0 = -5\times5^2 - \frac{5\times5^4}{24} + \frac{5}{24}\times2^4 - \frac{20}{6}\times2^3 + 187.5\times5 + C_2$$

$$C_2 = \underline{\underline{-658.96}}$$

Max slope (at free end A) $(x=0)$ $= C_1 = \dfrac{187.5}{EI}$

Max deflection (at free end A) $(x=0)$. $= C_2 = \dfrac{-658.96}{EI.}$

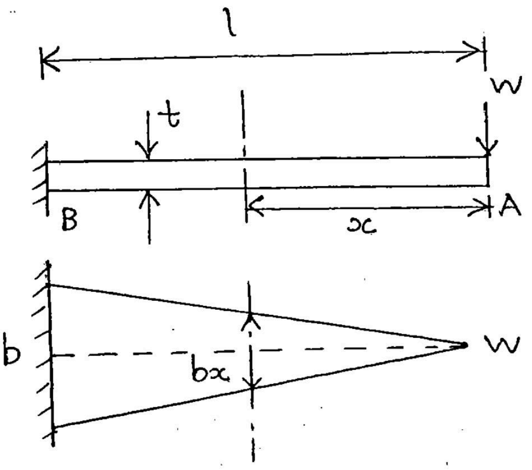
$$\therefore \; \theta_{max} = \underline{\underline{\frac{187.5}{EI}}} \quad \& \quad y_{max} = \underline{\underline{\frac{-659}{EI}}} \text{ (downward deflection)}$$

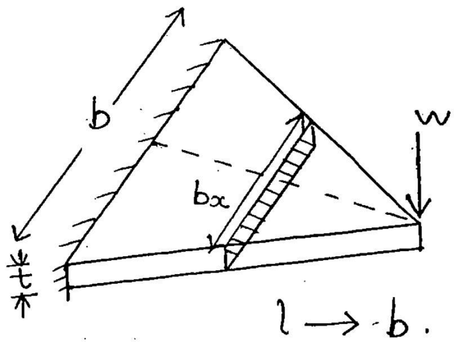
To obtain slope & deflection under point load, put $x = 3$.

$$\Rightarrow \theta_B = \left(-10\times3 - \frac{5}{6}\times3^3 + 187.5\right)\frac{1}{EI} = \frac{135}{EI}$$

$$y_B = \left(-5\times3^2 - \frac{5\times3^4}{24} + 187.5\times3 + -658.96\right)\frac{1}{EI}, = \underline{\underline{\frac{-158}{EI}}}$$

Q.





$l \rightarrow b.$

$x \rightarrow b_x = \dfrac{bx}{l}.$

$$I_x = \frac{b_x t^3}{12} = \frac{bx\,t^3}{12\,l} = \frac{bt^3}{12l}(x)$$

$$E\left(\frac{d^2y}{dx^2}\right) = \frac{M_x}{I_x} = \frac{-Wx}{\frac{bt^3}{12l}x} = \frac{-12wl}{bt^3}$$

$$E \frac{dy}{dx} = \frac{-12 wl x}{bt^3} + C_1$$

$$E\, y = \frac{-12\, wl x^2}{bt^3 \times 2} + C_1 x + C_2.$$

At $x = l$, $\frac{dy}{dx} = 0$ & $y = 0$. $\{$ fixed end $\}$

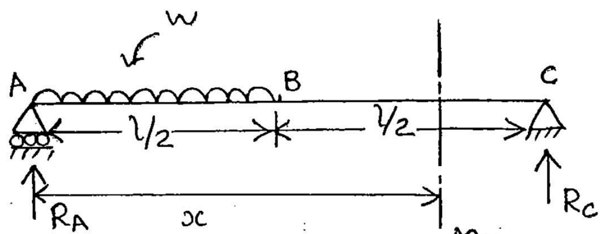
$$0 = \frac{-12\, wl^2}{bt^3} + C_1 \Rightarrow C_1 = \frac{12\, wl^2}{bt^3}.$$

$$0 = -\frac{6\, wl^3}{bt^3} + \frac{12\, wl^3}{bt^3} + C_2. \Rightarrow C_2 = -\frac{6\, wl^3}{bt^3}.$$

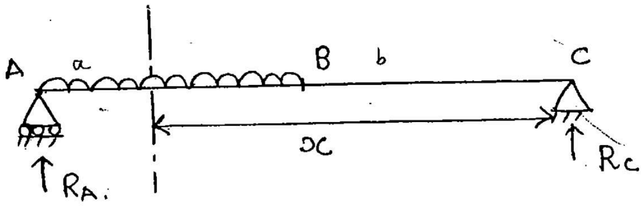
$$\theta_{max} = \frac{C_1}{E} = \frac{12\, wl^2}{bt^3\, E} \quad (\text{at } x = 0)$$

$$y_{max} = \frac{C_2}{E} = \frac{-6\, wl^3}{bt^3\, E} \quad (\text{at } x = 0).$$

Q.



Mx $\rightarrow$ against rule. So choose a different section.
(udl not cut).



$\frac{wb}{l}$

$$R_A = \frac{\frac{wl}{2} \times \frac{3l}{4}}{l} = \frac{3wl}{8}$$

$$R_C = \frac{wl}{8}$$

$$M_x = R_C(x) - \frac{w}{2}(x - l/2)^2.$$

$$EI \frac{d^2y}{dx^2} = \frac{wl}{8} x - \frac{w}{2}\left(x - l/2\right)^2.$$

$$EI \frac{dy}{dx} = \frac{wl x^2}{16} - \frac{w}{6}\left(x - \frac{l}{2}\right)^3 + C_1$$

$$EI\, y = \frac{wl\, x^3}{48} - \frac{w}{24}\left(x - \frac{l}{2}\right)^4 + C_1 x + C_2$$

Boundary conditions :

At C, $x = 0$ & $y = 0$.

At A, $x = l$ & $y = 0$.

$$0 = C_2$$

$$0 = \frac{wl^4}{48} - \frac{wl^4}{384} + C_1 l + 0.$$

$$-C_1 = \frac{-7wl^3}{384}$$

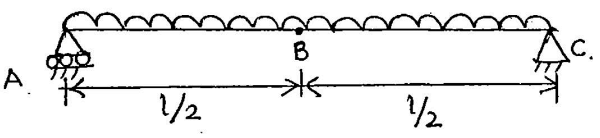
To obtain deflection and slope at B, put $x = \frac{l}{2}$.

$$\theta_B = \left(\frac{wl^3}{64} - \frac{wl^3}{48} - \frac{7wl^3}{384} \right) \frac{1}{EI} = \frac{-3wl^3}{128}$$

$$y_B = \frac{wl^4}{384} - \frac{wl^4}{384} \times 0 + - \frac{7wl^4}{768} + 0$$

$$y_B = \frac{-5wl^4}{768\,EI} \quad (\text{not } y_{max})$$

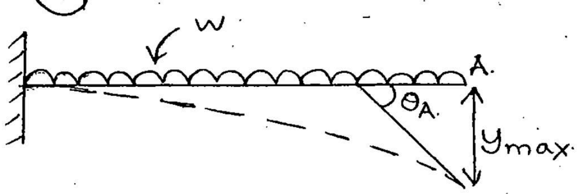
Q.



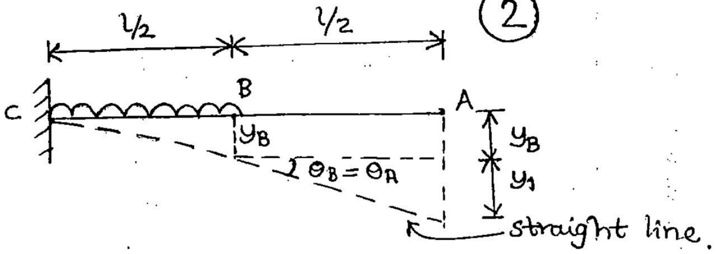
$$y_B = y_{max} = \frac{5wl^4}{384\,EI}.$$

(1)

Q.



(2)



$$y_{max} = \frac{wl^4}{8EI}.$$

$$\theta_{max} = \frac{wl^3}{6EI}.$$

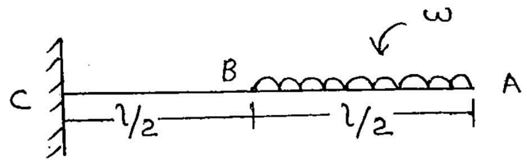
$$\theta_B = \theta_A = \frac{w(l/2)^3}{6EI}.$$

$$y_B = \frac{w(l/2)^4}{8EI}$$

$$y_A = y_B + y_1$$

$$= y_B + \theta_B \times \frac{l}{2} = \frac{wl^4}{128\,EI} + \frac{wl^4}{96EI} = \underline{\underline{\frac{7\,wl^4}{384\,EI}}}$$

Q.



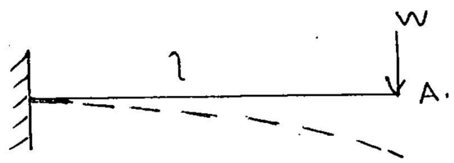
$$y_A = (y_A)_1 - (y_A)_2$$

$$= \frac{wl^4}{8EI} - \frac{7\,wl^4}{384\,EI} = \underline{\underline{\frac{41\,wl^4}{384\,EI}}}$$

$$\theta_{max} = \theta_B = (\theta_B)_1 - (\theta_B)_2$$

$$= \frac{wl^3}{6EI} - \frac{wl^3}{48EI} = \underline{\underline{\frac{7}{48\,EI}}}$$

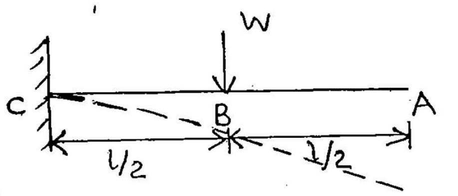
Q.



$$y_{max} = y_A = \frac{Wl^3}{3EI}$$

$$\theta_{max} = \theta_A = \frac{Wl^2}{2EI.}$$

Q



$$y_B = \frac{W(l/2)^3}{3EI} = \frac{Wl^3}{24\,EI}$$

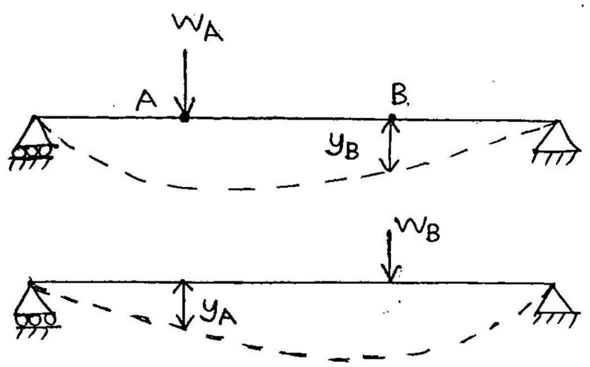
$$\theta_B = \frac{W(l/2)^2}{2EI} = \underline{\underline{\frac{Wl^2}{8EI.}}} = \theta_A$$

$$\tan\theta_B = \theta_B = \left(\frac{y_1}{l/2}\right)$$

$$y_A = y_B + y_1 = y_B + \theta_B \times \frac{l}{2},$$

$$= \frac{Wl^3}{24EI} + \frac{Wl^3}{16EI} = \underline{\underline{\frac{5\,Wl^3}{48\,EI}}}$$

→ Maxwell's Reciprocal Theorem

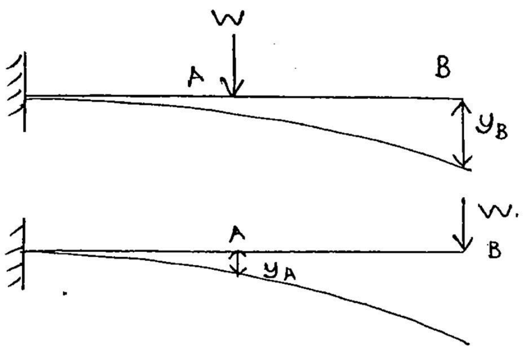


$$\boxed{W_A \, y_A = W_B \, y_B}$$

- It's applicable to conc. loads & moments.

If $w_A = w_B = w$, then $y_A = y_B$.

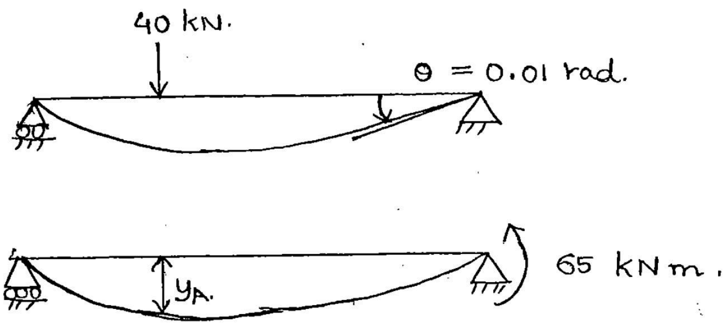
Q.



$$y_B = \frac{5wl^3}{48EI}$$

$$\underline{\underline{y_A = y_B}}$$

Q.



$$40 \times y_A = 65 \times 0.01$$

$$\Rightarrow y_A = \underline{\underline{16.25}} \text{ mm}$$

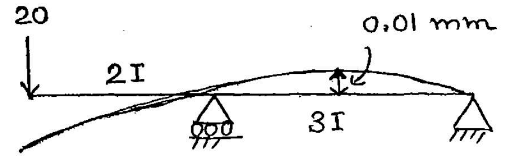
Limitations:

(i) Load must be upto proportionality limit. (Hooke's Law is valid). Load $\propto$ deflection for linear elastic members

(ii) Slopes and deflections should be negligibly small.

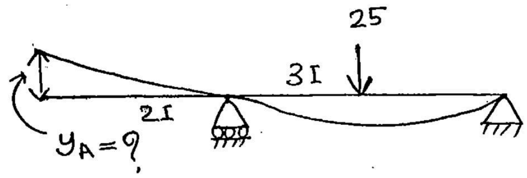
(iii) Applicable for prismatic & non prismatic beams. (in both cases, same beam with same material & c/s should be used

Q.



$$20 \times y_A = 25 \times 0.01$$

$$\underline{\underline{y_A = 0.0125 \text{ mm}}}$$



Relations :

$y \rightarrow$ deflections.

$\dfrac{dy}{dx} \rightarrow$ slope

$\dfrac{d^2y}{dx^2} \rightarrow$ curvature or $\dfrac{1}{R} = \dfrac{M}{EI}$

$= M$ (if $EI = 1$)

$\dfrac{d^3y}{dx^3} = \dfrac{dM}{dx} = F$ (if $EI = 1$).

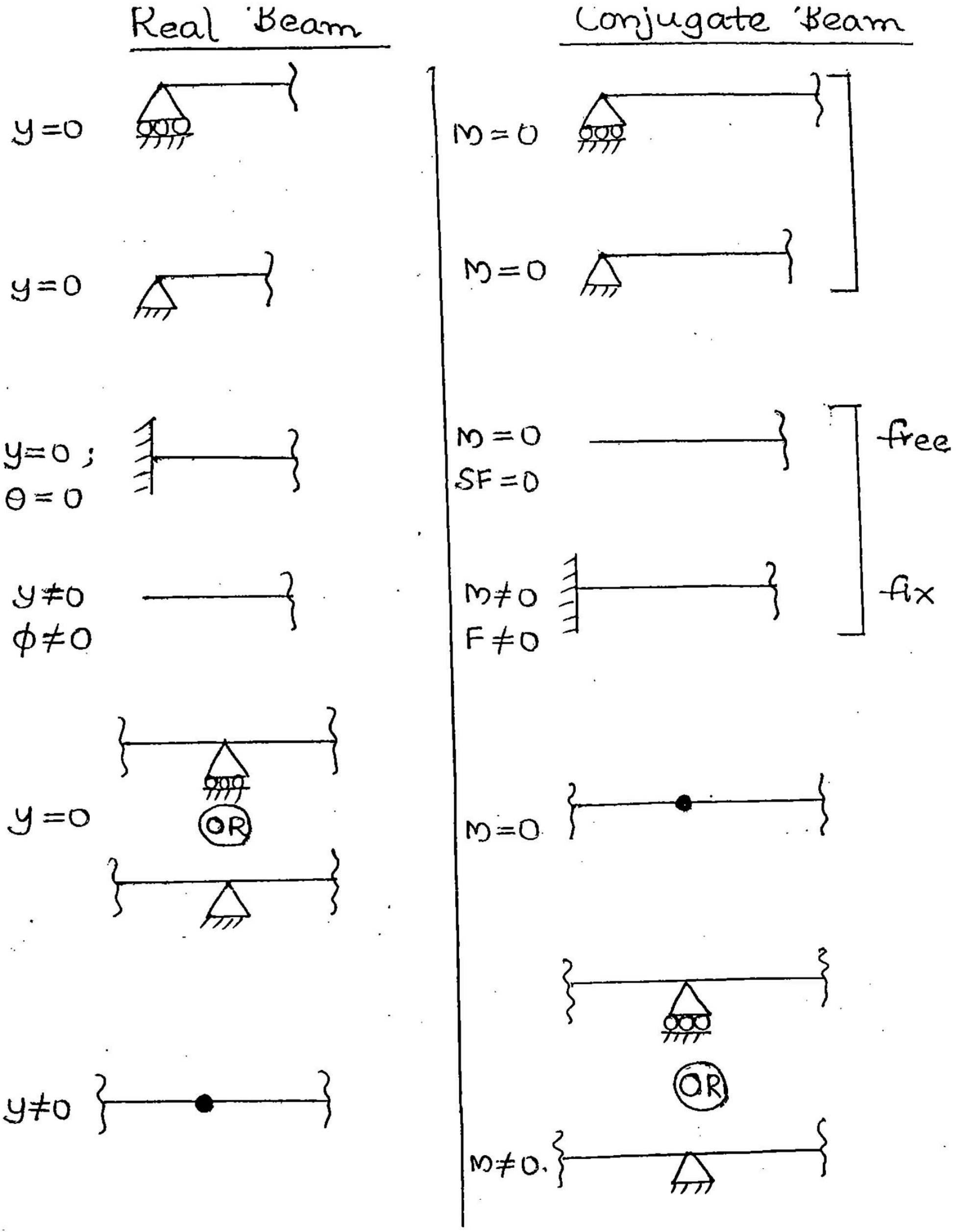
$\dfrac{d^4y}{dx^3} = \dfrac{dF}{dx} = w$ (if $EI = 1$)

- For y to be <u>maximum</u>, $\dfrac{dy}{dx} = 0$; slope $= 0$

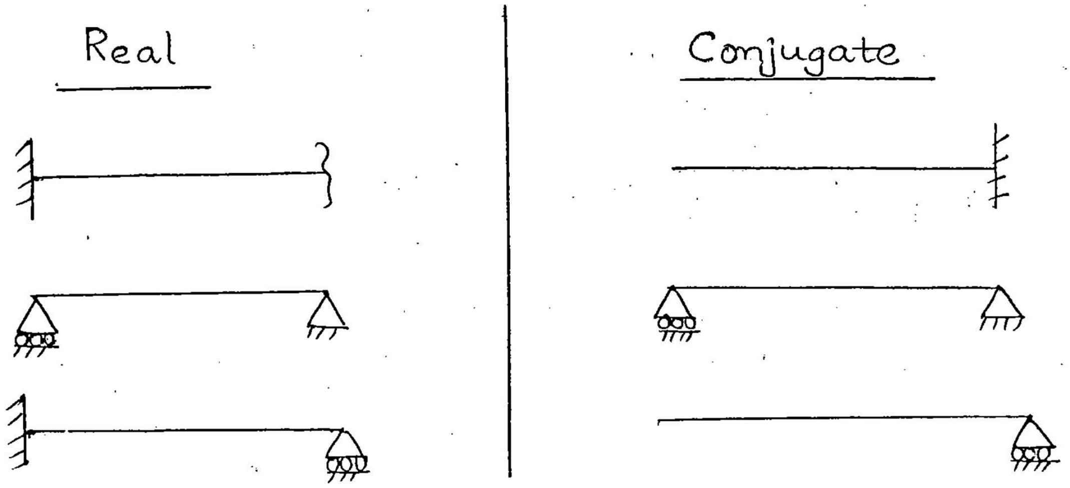
At the point of max. magnitude of deflection, slope must be zero. At the point of maximum slope, deflection need not be zero. The above condition is not valid for concentrated moments acting over the beam. (valid only for lateral or transverse loading)

2. Conjugate Beam Method.

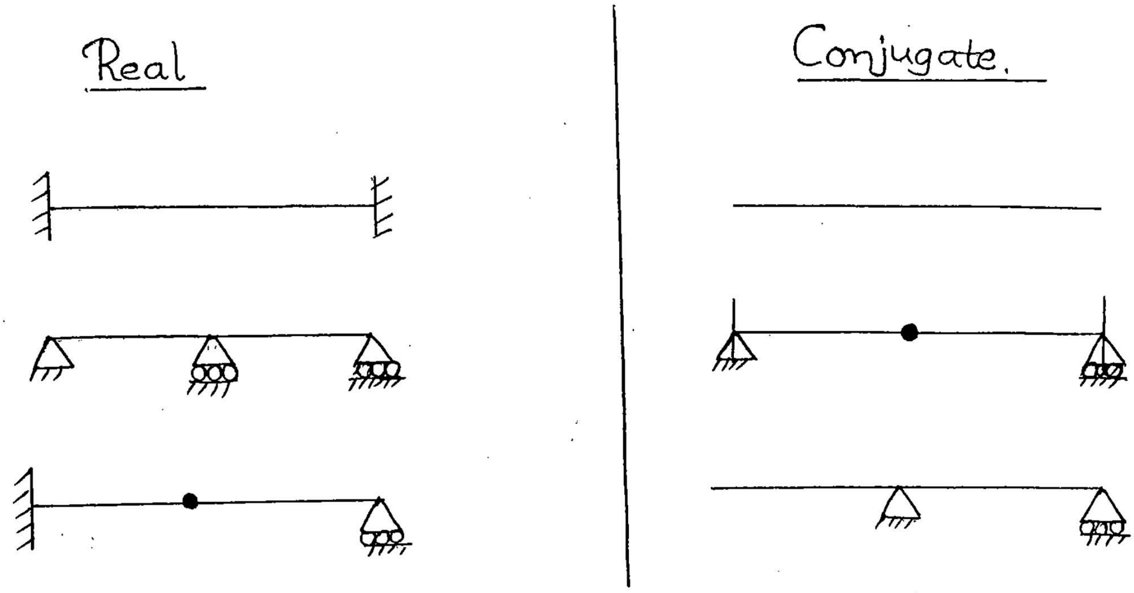
- imaginary beam.
- conjugate beam can be made by changing supports.



Real Beam	Conjugate Beam.
Slope	Shear force
Deflection.	Bending moment

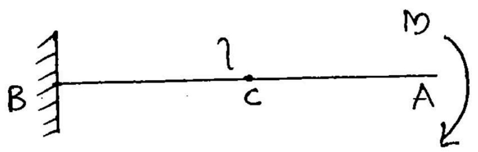


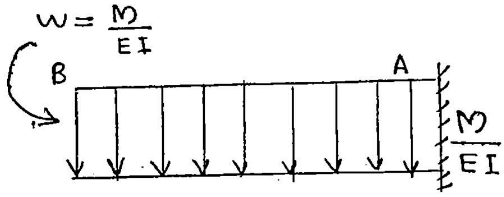
⊙ For conjugate beams, stability is not required.



* Load on Conjugate beam = $\frac{M}{EI}$ diagram.

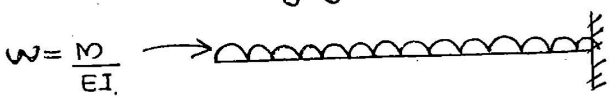
Q.





EI = constant.

▪ conjugate beam



$\theta_{max} = \theta_A \Rightarrow (SF)_A$ on conjugate beam.

$$= \frac{Ml}{EI}.$$

$y_{max} = y_A \Rightarrow (M)_A$ on conjugate beam.

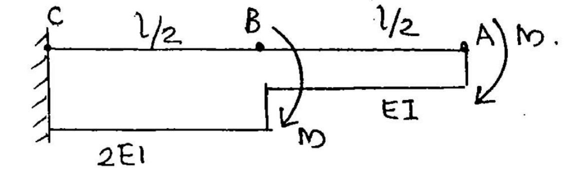
$$= \frac{Ml^2}{2EI}$$

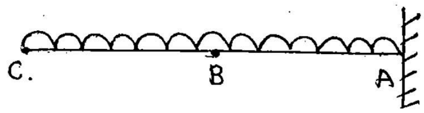
At midpoint of beam:

$\theta_c = \frac{Ml}{2EI}$. ($SF_c$ on CB)

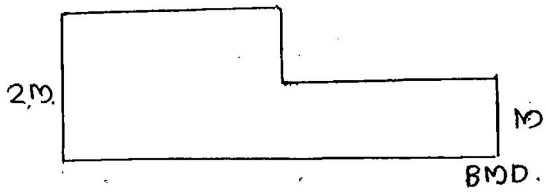
$y_c = \frac{Ml^2}{8EI}$. (M_c on CB).

Q.





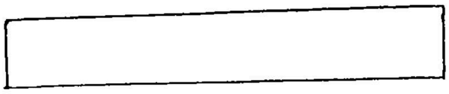
$\frac{M}{EI}$ dia.



BMD.

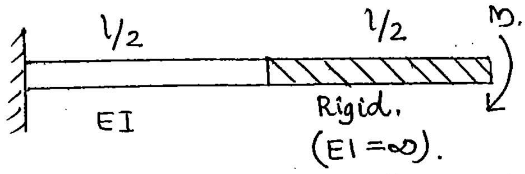
$$y_{max} = y_A = \frac{ML^2}{2EI}$$

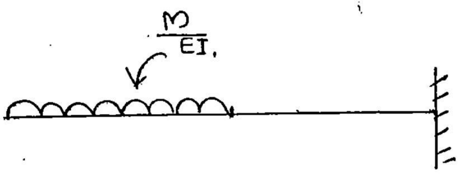
$$\theta_A = \theta_{max} = \frac{ML}{EI}$$



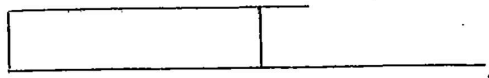
$\frac{M}{EI}$ dia.

Q.





BMD



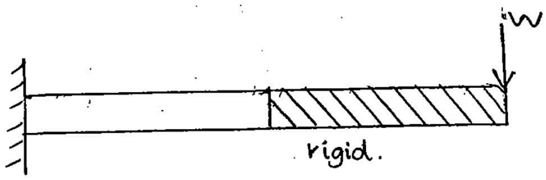
$\frac{M}{EI}$

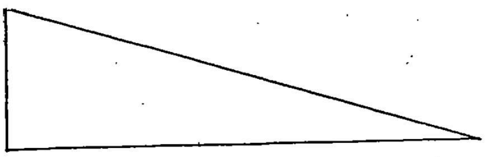
$$\theta_{max} = \frac{ML}{2EI}$$

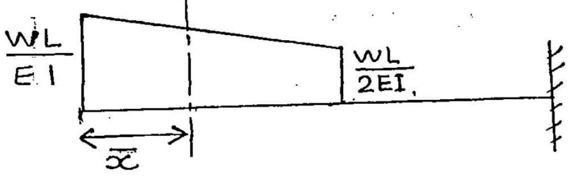
$$y_{max} = \frac{3ML^2}{8EI}$$

$\frac{ML}{2} \times \frac{3L}{4}$

Q.







$$\theta_{max} = (SF)_A \text{ on } CB.$$

$$= \frac{1}{2} \times \frac{L}{2}\left(\frac{WL}{EI} + \frac{WL}{2E}\right).$$

$$= \frac{3WL^2}{8EI}.$$

$$y_{max} = (M_A) \text{ on } CB.$$

$$\cancel{= \frac{3WL^2}{8EI} \times \left(\frac{4L}{18} + \frac{L}{2}\right)}$$

$\frac{2a+b}{a+b}$

$\frac{1}{6} \times \frac{\frac{1WL}{EI} + \frac{WL}{EI}}{\frac{3WL}{2EI}}$

$$\cancel{= \frac{13WL^3}{48EI}} = \frac{WL^3}{12EI}$$

$\frac{1}{6}\left(\frac{2\times 2}{3}\right) = \frac{4L}{18}$

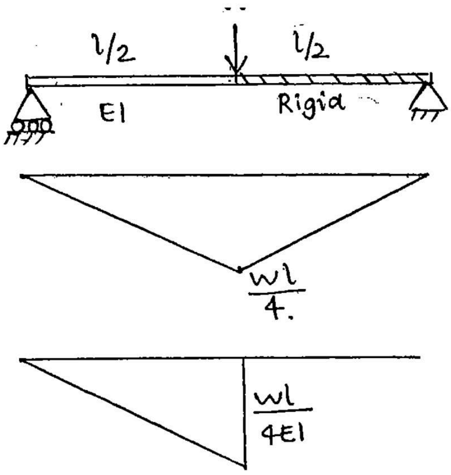
$$\bar{x} = \frac{h}{3}\left(\frac{2a+b}{a+b}\right)$$

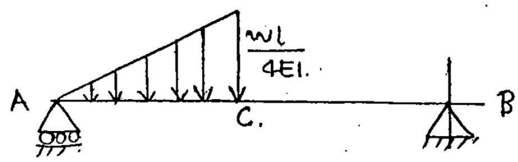
$$= \frac{l/2}{3}\left(\frac{2\times\frac{WL}{2EI} + \frac{WL}{EI}}{\frac{WL}{2EI} + \frac{WL}{EI}}\right) = 0.222L$$

$$= 0.277 L$$

$$y_{max} = \frac{3WL^2}{8EI}\left(0.277L + 0.5L\right) = \frac{7WL^2}{24EI}$$

Q.





$$\frac{1}{2}\times\frac{wl}{4EI}\times\frac{l}{2}\times\frac{2}{3}\left(\frac{l}{2}\right) = R_B\times l.$$

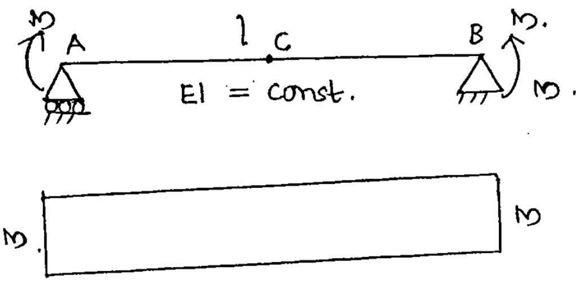
$$R_B = \frac{wl^2}{48EI.}$$

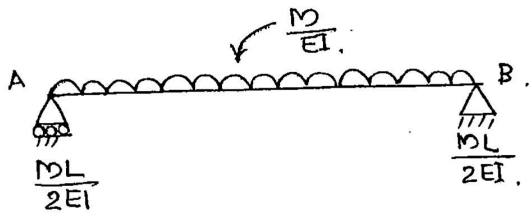
$\theta_c = (SF)_c$ on conjugate beam.

$$= \frac{wl^2}{48EI.}$$

$$y_c = \frac{wl^2}{48EI}\times\frac{l}{2} = \frac{wl^3}{96EI}$$

Q





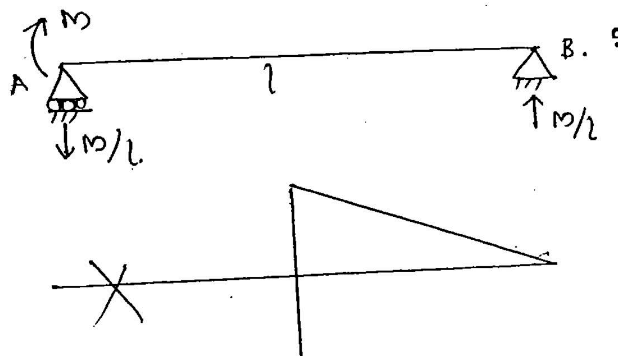
$$\theta_{max} = \theta_A = (SF)_A \text{ on. CB.}$$

$$= \frac{ML}{2EI.} = \theta_B.$$

$$y_{max} = y_c = \frac{ML}{2EI}\times\frac{L}{2} - \frac{M}{EI}\times\frac{L^2}{8}$$

$$= +\frac{ML^2}{8EI}$$

Q



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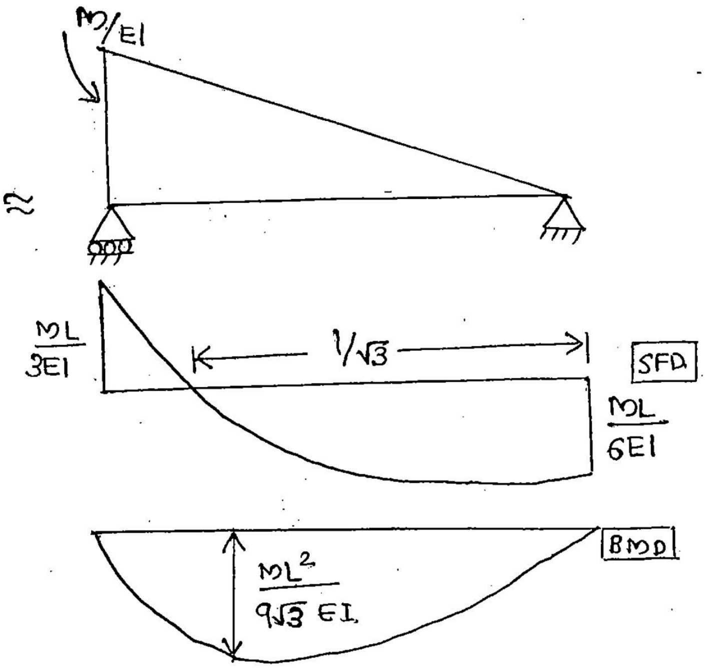
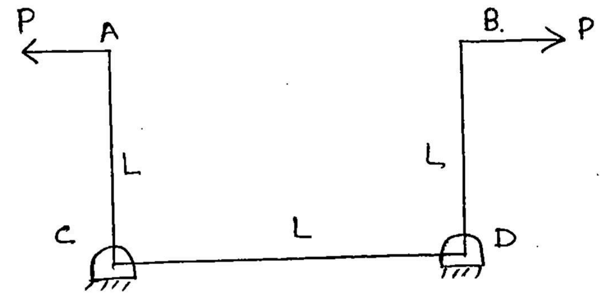


$$\theta_A = R_A = \frac{ML}{3EI}$$

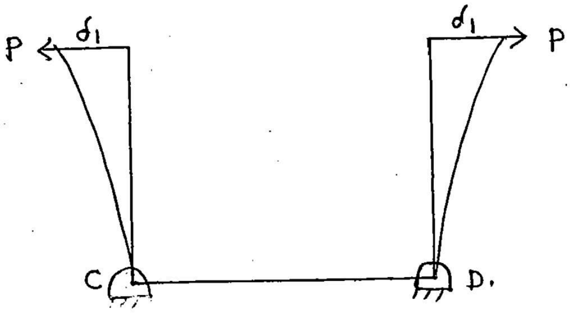
$$\theta_B = R_B = \frac{ML}{6EI.}$$

$$y_{max} = M_{max} = \frac{ML^2}{9\sqrt{3}\,EI}$$

Q. 

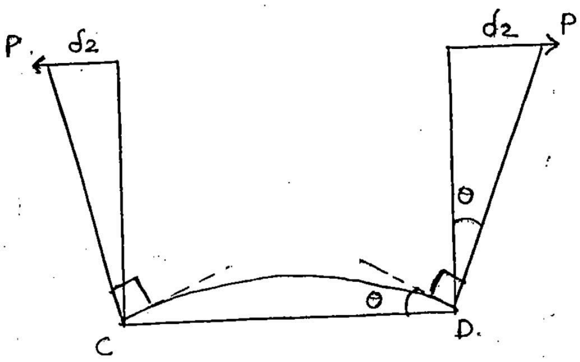
Determine relative displacement between A & B.

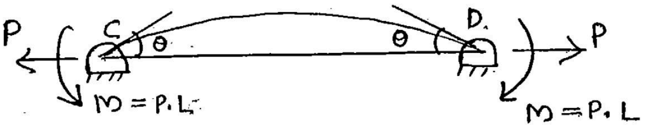
Initially assume CD is rigid, AC & BD are deflecting like cantilevers with fixed ends at C & D.



$$\delta_1 = \frac{PL^3}{3EI}$$

Now assume AC & BD are rigid, only CD is deflecting due to the loads.





$$\Rightarrow \theta = \frac{PL^2}{2EI}$$

For SSB with moments at both ends, $\theta = \frac{ML}{2EI}$

$$\tan\theta \simeq \theta = \frac{\delta_2}{L}$$

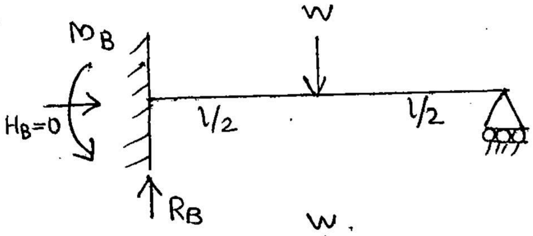
$$\frac{ML}{2EI} = \frac{PL^2}{2EI} = \frac{\delta_2}{L} \Rightarrow \delta_2 = \frac{PL^3}{2EI};$$

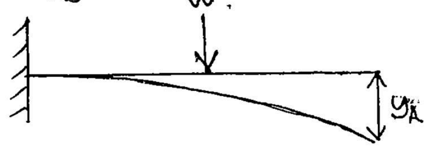
PROPPED CANTILEVER.

Boundary Conditions :

$y_A = 0$; $y_B = 0$; $\theta_B = 0$.

Q.

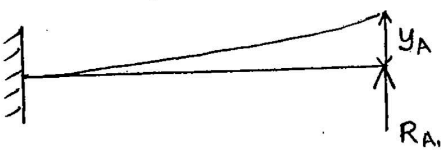




At A, $y_A = 0$.

$y_w - y_{RA} = 0$.

$$\frac{5wl^3}{48EI} - \frac{R_A l^3}{3EI} = 0$$



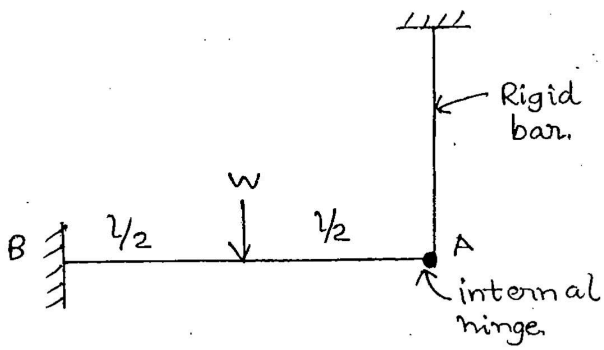
$$R_A = \frac{5w}{16}$$

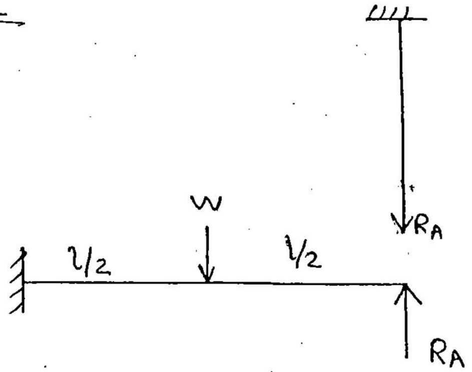
$R_A + R_B = w$

$\Rightarrow R_B = \frac{11w}{16}$.

$M_B = R_A l - w\frac{l}{2} = -\frac{3wl}{16}$ (hogging).

Q



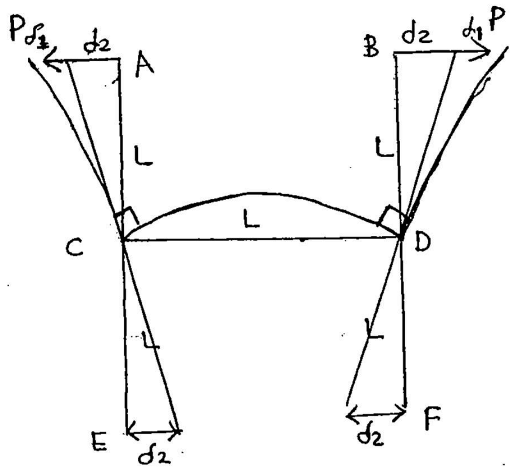


$\Rightarrow R_A = \frac{5w}{16}$

Relative displacement b/w A & B $= 2\delta_1 + 2\delta_2$

$$= \frac{5PL^3}{3EI}$$

Q.

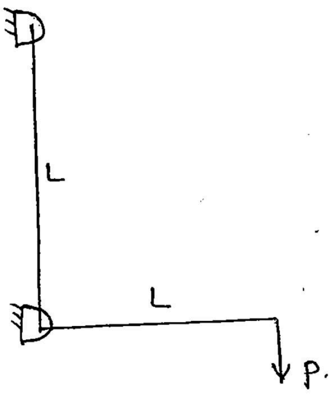


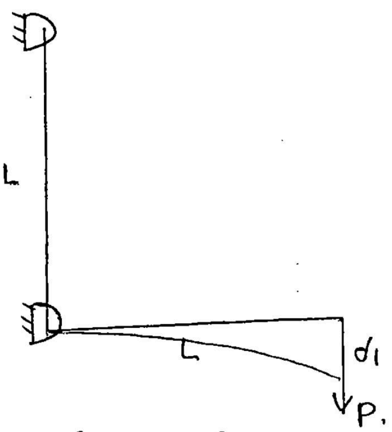
Find relative displacement b/w E & F ?

Relative displacement b/w E & F

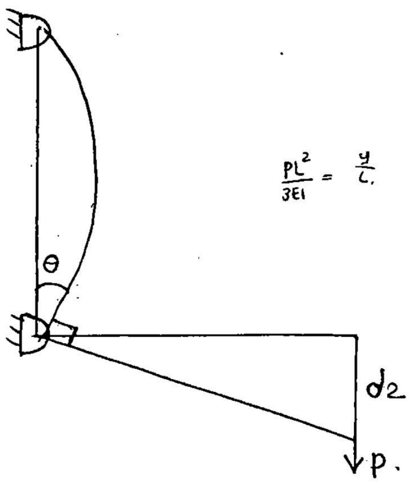
$$= 2\delta_2 = \frac{PL^3}{3EI}$$

Q.





$$\delta_1 = \frac{PL^3}{3EI}$$



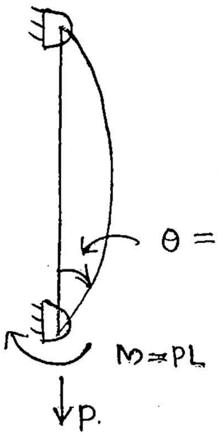
$\frac{PL^2}{3EI} = \frac{y}{L}$

$$\delta_2 = L\theta$$

$$= \frac{PL^2}{3EI} \times L = \frac{PL^3}{3EI}.$$

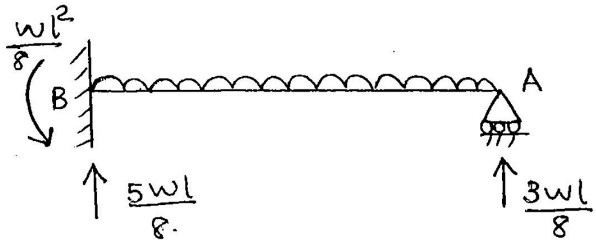
$$\delta = \delta_1 + \delta_2$$

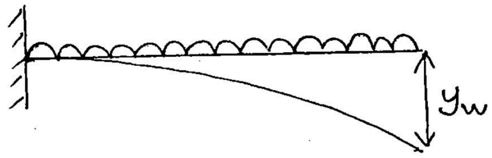
$$= \frac{PL^3}{3EI} + \frac{PL^3}{3EI} = \frac{2PL^3}{3EI}$$

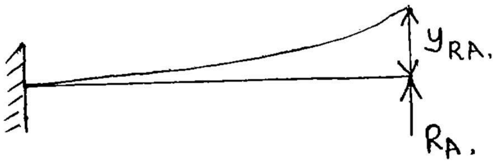


$\theta = \frac{PL^2}{3EI}.$

Q.







$$y_w - y_{RA} = 0$$

$$\frac{wl^4}{8EI} - \frac{R_A \cdot l^3}{3EI} = 0.$$

$$R_A = \frac{3wl}{8}$$

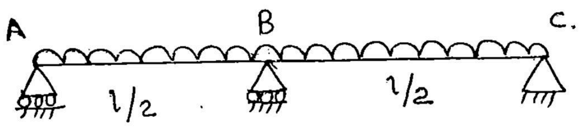
$$R_B = wl - \frac{3wl}{8}$$

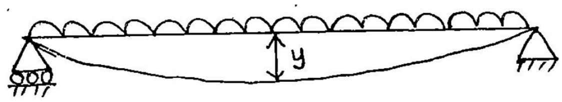
$$= \frac{5wl}{8}$$

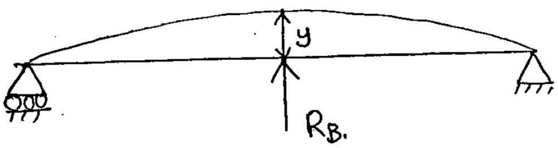
$$M_B = \frac{3wl}{8} \times l - \frac{wl^2}{2}$$

$$= -\frac{wl^2}{8} \text{ (hogging)}$$

Q.







$$\frac{5wl^4}{384EI} - \frac{R_B \times l^3}{48EI} = 0.$$

$$R_B = \frac{5wl}{8EI}.$$

$$M_B = R_C \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{l}{4}.$$

$$= \frac{3wl}{16} \times \frac{l}{2} - \frac{wl^2}{8}$$

$$= -\frac{wl^2}{32} \text{ (hogging).}$$

From original beam,

$\Sigma F_y = 0.$

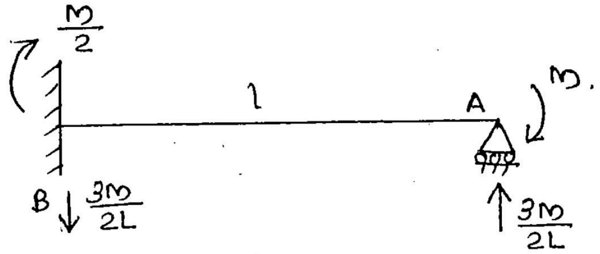
$$R_A + R_B + R_C = wl.$$

$$2R_A + \frac{5wl}{8EI} = wl. \text{ (due to symmetry, } R_A = R_C\text{).}$$

$$R_A = R_C = \frac{3wl}{16EI}$$

- Only hogging moments are developed at fixed supports due to gravity loads.

Q.



$y_A = 0$

$$\Rightarrow \quad \frac{ML^2}{2EI} - \frac{R_A L^3}{3EI} = 0.$$

$$R_A = \frac{3M}{2L}$$

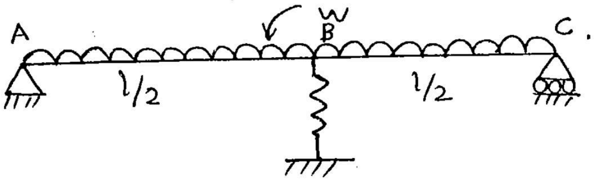
$\Sigma F_y = 0$

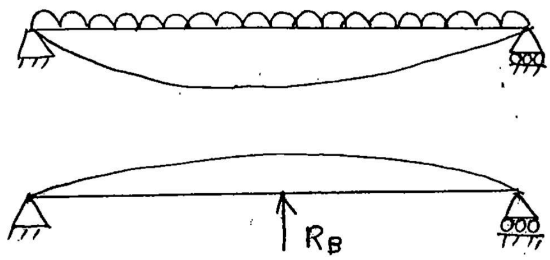
$R_A + R_B = 0$

$$\Rightarrow R_B = -\frac{3M}{2L}$$

$$M_B = \frac{3M}{2L} \times l - M. = 0.5M.$$

Q.





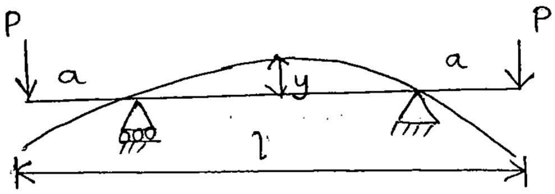
$$y_{udl} - y_{RB} = \frac{R_B}{k.}$$

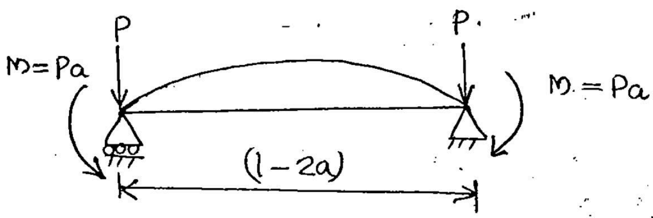
$$\frac{5wl^4}{384EI} - \frac{R_B \times l^3}{48EI} = \frac{R_B}{K}$$

$$\Rightarrow \frac{R_B}{K} + \frac{R_B l^3}{48EI} = \frac{5wl^4}{384EI.}$$

P-65

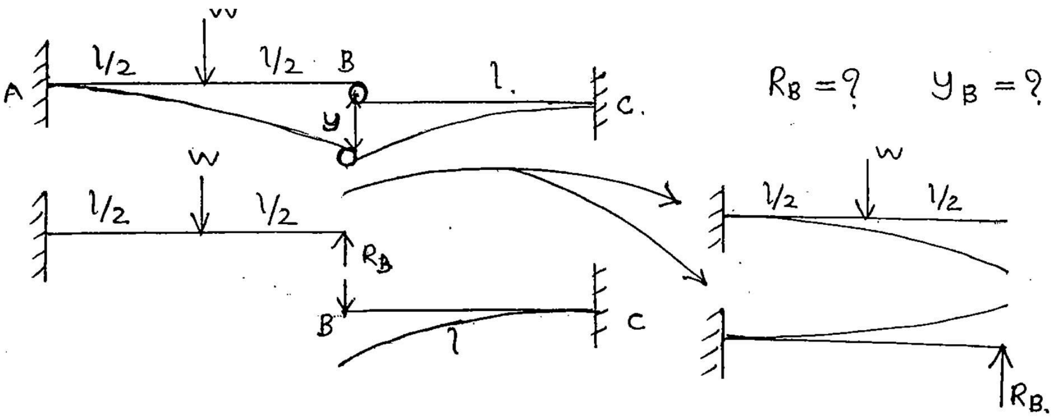
Q.6.





$$y_{max} = y_{centre} = \frac{Pa(l-2a)^2}{8EI}$$

$$\left\{ \frac{ML^2}{8EI} \right\}$$

Q. 

$R_B = ?$ $y_B = ?$

Compatibility condition at B:

$$(\downarrow y_{AB}) \text{ at } B = (\downarrow y_{BC}) \text{ at } B.$$

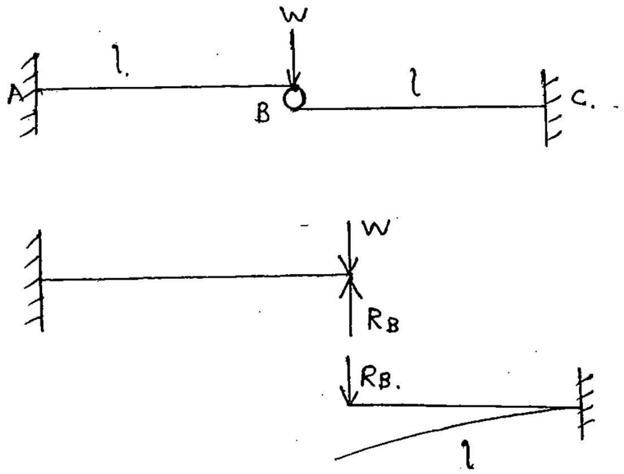
$$(\downarrow y_w - \uparrow y_{RB})_B = (y_{BC})_B.$$

$$\frac{5Wl^3}{48EI} - \frac{R_B l^3}{3EI} = \frac{R_B l^3}{3EI}.$$

$$\Rightarrow R_B = \underline{\underline{\frac{5W}{32}}}$$

y_B = substitute R_B in LHS/RHS of compatibility condition.

$$= \frac{5W}{32} \times \left(\frac{l^3}{3EI}\right) = \underline{\underline{\frac{5Wl^3}{96EI}}}$$

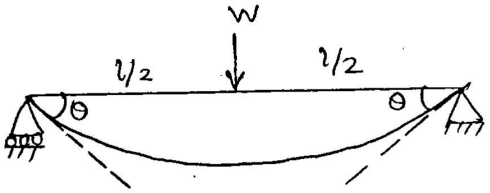
Q. 

$$\frac{Wl^3}{3EI} - \frac{R_B l^3}{3EI} = \frac{R_B l^3}{3EI} \Rightarrow R_B = \underline{\underline{\frac{W}{2}}}$$

$$y_B = \frac{R_B l^3}{3EI} = \frac{Wl^3}{6EI}$$

P-66

Q.12.



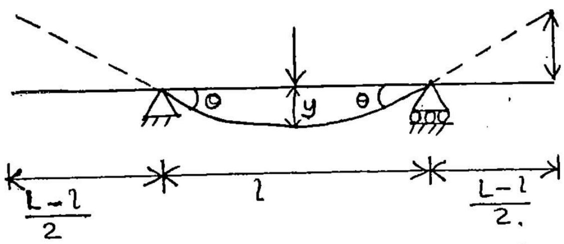
$$\theta = \frac{\pi}{180} \text{ rad} = \frac{Wl^2}{16EI}.$$

$$y_{max} = \frac{Wl^3}{48EI} = \frac{\pi}{180} \times \frac{l}{3} = \frac{\pi}{180} \times \frac{4000}{3}$$

$$= 23.27 \text{ mm}$$

P-67

Q.5.



$$\tan\theta = \theta = \frac{\uparrow y}{\left(\frac{L-l}{2}\right)}.$$

$(\uparrow y)$ at free ends $= (\downarrow y)$ mid span.

$$\theta \times \left(\frac{L-l}{2}\right) = \frac{Wl^3}{48EI}.$$

$$\frac{Wl^2}{16EI}\left(\frac{L-l}{2}\right) = \frac{Wl^3}{48EI}.$$

$$\frac{Wl^3}{32EI}\left(\frac{L}{l} - 1\right) = \frac{Wl^3}{48EI}.$$

$$\frac{L}{l} = 1 + \frac{32}{48} = \frac{5}{3}$$

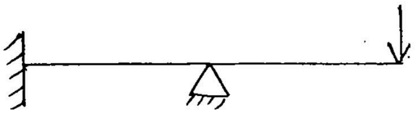
$1 - \frac{4}{6}.$

Q.9. $$\frac{WL^3}{3EI} = \frac{WL^3}{48EI}.$$

$$\frac{233^3}{3 \times 20 \times 40^3} = \frac{L^3}{48 \times 15 \times 30^3} \Rightarrow L = 400 \text{ mm}$$

P-9

Q.3.



Indeterminacy, $R = 1$.

No: of boundary conditions required to calculate deflection $= R+2$

$= 1+2 = \underline{\underline{3}}$

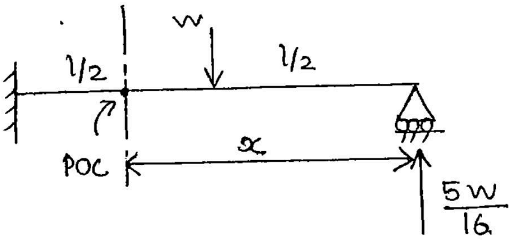
	No. of compatibility condition.
To analyse	R
Slope.	$R+1$
Deflection.	$R+2$.

8-94.

03. $\frac{M}{2}$ (hogging).

04 $\frac{3M}{2L}$ (downward).

05.



@ POC,

$$M_x = R_A x - W(x - l/2) = 0.$$

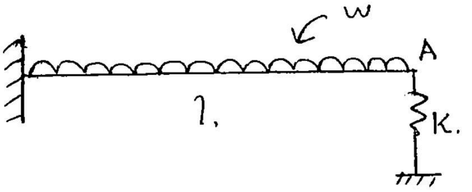
$$\Rightarrow \frac{5W}{16}x - W\left(x - \frac{l}{2}\right) = 0.$$

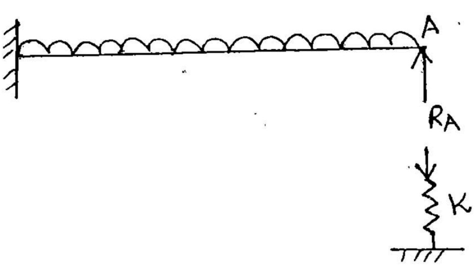
$$\Rightarrow x = \frac{8L}{11} \text{ (from hinge).}$$

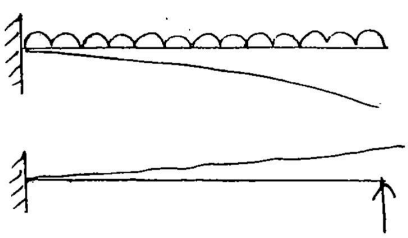
$$= \frac{3L}{11} \text{ (from fixed support).}$$

$\frac{11}{16} - \frac{1}{2}$

06.







Net deflection at A = Compression of spring.

$$\downarrow y_{udl} - y_{RA} = \frac{R_A}{K}.$$

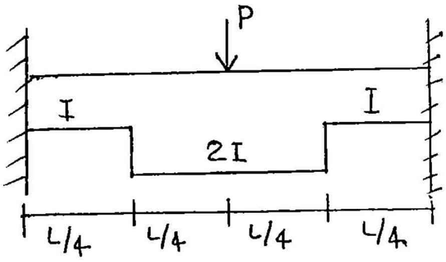
$$\frac{wl^4}{8EI} - \frac{R_A l^3}{3EI} = \frac{R_A}{K}.$$

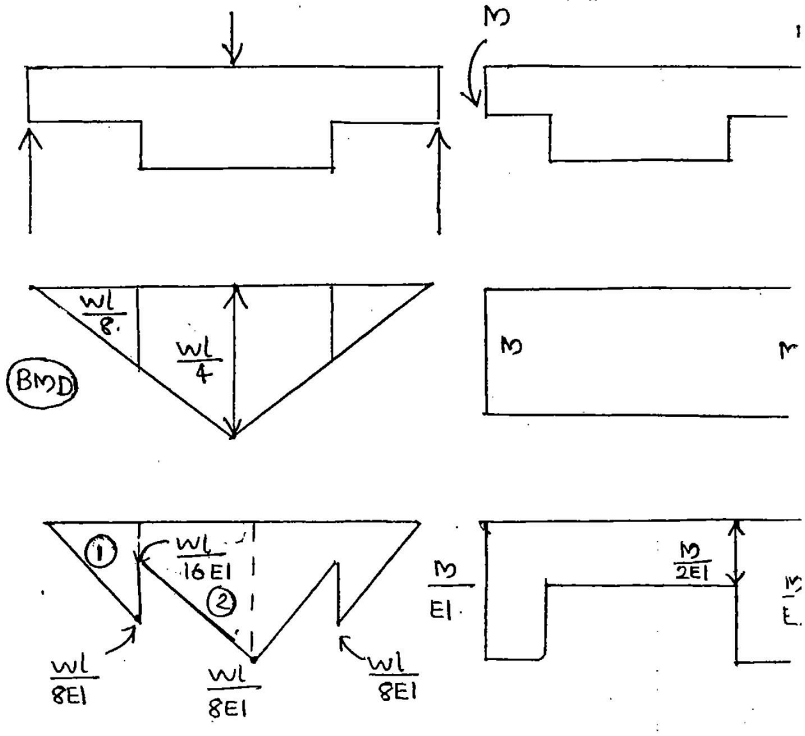
$$R_A\left(\frac{l^3}{3EI} + \frac{1}{K}\right) = \frac{wl^4}{8EI}.$$

$$R_A \; \frac{Kl^3 + 3EI}{K \times 3EI} = \frac{wl^4}{8EI}.$$

$$R_A = \frac{3wl^4 \times K}{8(Kl^3 + 3EI)} = \underline{\underline{\frac{3wl/8}{1 + 3EI/Kl^3}}}$$

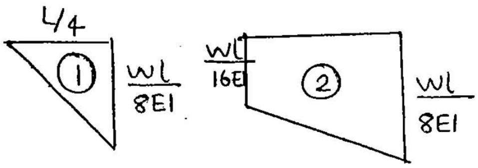
07.





M/EI

$$(A_{ss})_{\frac{M}{EI}} = (A_{fix})_{\frac{M}{EI}}$$



$$\frac{1}{2} \times \frac{L}{4} \times \frac{wl}{8EI} + \frac{1}{2} \times \left(\frac{wl}{16EI} + \frac{wl}{8EI}\right) \times \frac{L}{4} = \frac{M}{EI} \times \frac{L}{4} + \frac{M}{2EI} \times \frac{L}{4}$$

$$M = \underline{\underline{\frac{5wl}{48}}}$$

$3M = 2.5 \frac{wl}{8 \times}$ $\quad M = \frac{5}{48}$

9th Oct,
SATURDAY

04 CENTRE OF GRAVITY & MOMENT OF INERTIA

Centroid:

The point through which entire area is concentrated. This is applicable for plane surface areas.

Centre of Gravity:

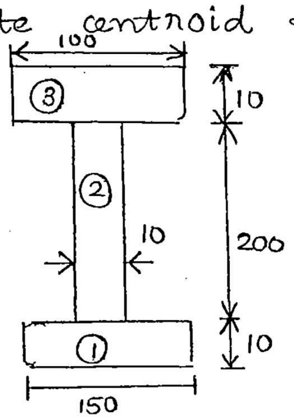
The point through which entire mass or weight is concentrated. Applicable for solids.

Centroids of Compound Areas:

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

Q. Locate centroid from base.

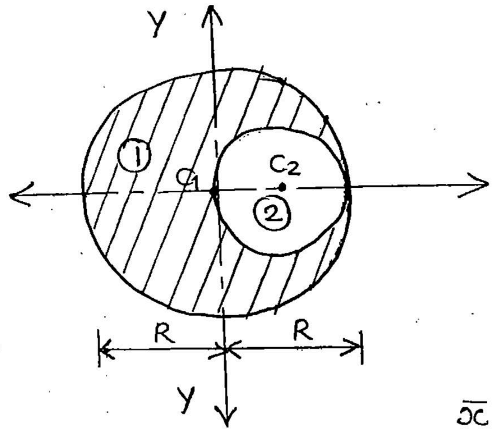


$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}.$$

$$= \frac{150 \times 10 \times 5 + 200 \times 10 \times 110 + 100 \times 10 \times 215}{150 \times 10 + 200 \times 10 + 100 \times 10}$$

$$= 98.33 \text{ mm}$$

Q.



Locate centroid from y-axis.

$x_1 = 0 \;;\; x_2 = R/2$

$A_1 = \pi R^2 \;;\; A_2 = \pi \left(\frac{R}{2}\right)^2.$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{A_1 \times 0 - \pi\left(\frac{R}{2}\right)^2 \times \frac{R}{2}}{\pi R^2 - \pi\left(\frac{R}{2}\right)^2}$$

$\frac{R/2}{4R^2 - 1}$

$$= \underline{\underline{-\frac{R}{6}}} \text{ (towards left)}$$

→ Moment of Inertia

1. Area MI (I) – for plane areas.
2. Mass MI (I_m) – for solids.

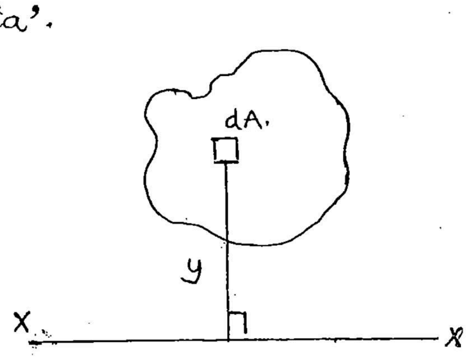
* Area MI (I) :

Resistance of a member against externally applied moment is moment of inertia. The possible moments in a member are: Bending Moments & Twisting moments,

OR

Second moment of a given area about a reference axis is also 'Area Moment of Inertia'.

$$\boxed{I_x = \int dA \cdot y^2}$$

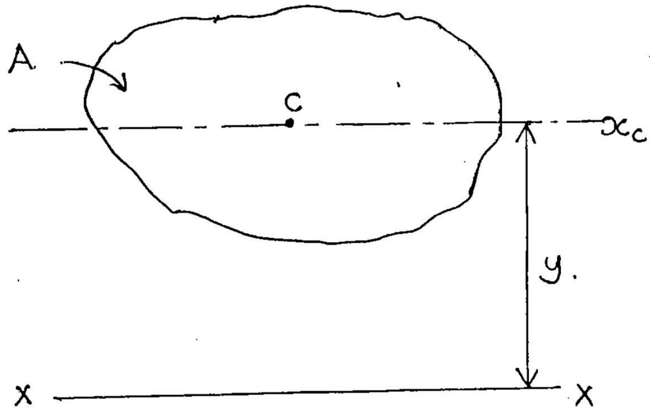


Unit : m^4

- Moment of inertia indicates the distribution of a given area about a reference axis.

As <u>I increases</u>, <u>stability increases</u> against external moments.. Similarly, strength against external moment also increases.

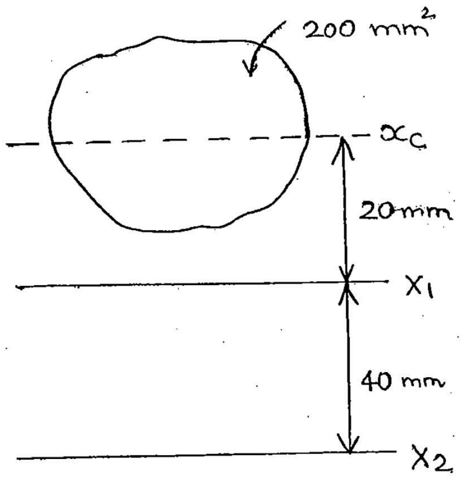
* Parallel Axis Theorem: (transfer formula)



$$\boxed{I_x = I_{xc} + Ay^2}$$

⊙ The least moment of inertia of a given area will be with respect to centroidal axis.

Q.



$I_{x_1} = 2 \times 10^6\ mm^4$; $I_{x2} = ?$

$$I_{x1} = I_{xc} + Ay^2$$

$$2 \times 10^6 = I_{xc} + 200 \times 20^2.$$

$$I_{xc} = \underline{\underline{1.92 \times 10^6\ mm^4}}$$

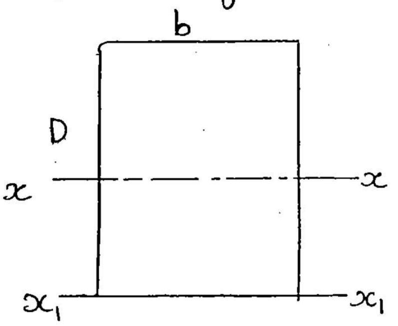
$$I_{x2} = I_{xc} + 200 \times 60^2.$$

$$= 1.92 \times 10^6 + 200 \times 60^2$$

$$= \underline{\underline{2.64 \times 10^6}}\ mm^4$$

⊙ Transfer formula is applicable to transfer centroidal moment of inertia to any other parallel axis.

Q.

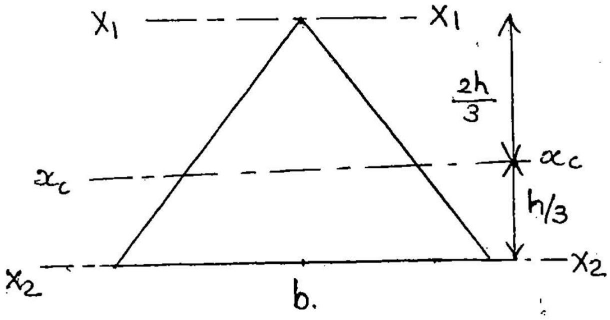


$$I_{xc} = \frac{bd^3}{12}$$

$$I_{x1} = I_{xc} + Ay^2$$

$$= \frac{bd^3}{12} + bd.\frac{d^2}{4} = \frac{bd^3}{3}.$$

Q.



$$I_{xc} = \frac{bh^3}{36}$$

$$I_{x1} = \frac{bh^3}{36} + A\left(\frac{2h}{3}\right)^2$$

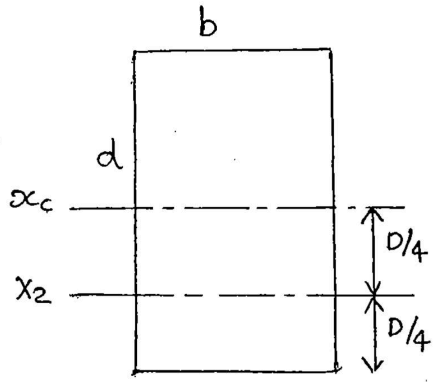
$$= \frac{bh^3}{36} + \frac{1}{2}bh\left(\frac{2h}{3}\right)^2$$

$$= \frac{bh^3}{4}$$

$$I_{x2} = \frac{bh^3}{36} + \frac{1}{2}bh\left(\frac{h}{3}\right)^2$$

$$= \underline{\underline{\frac{bh^3}{12}}}$$

Q.

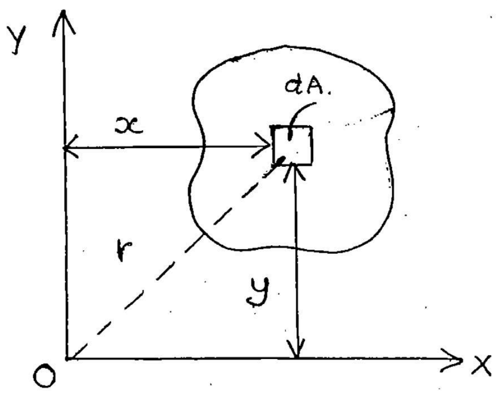


$$I_{x2} = I_{xc} + Ay^2$$

$$= \frac{bd^3}{12} + bd\left(\frac{d}{4}\right)^2$$

$$= \underline{\underline{\frac{7bd^3}{48}}}$$

* Perpendicular Axis Theorem.



$$I_x = \int dA \cdot y^2$$

$$I_y = \int dA \cdot x^2$$

$$I_z = \int dA \cdot r^2$$

$$\Rightarrow \boxed{I_z = \int dA\,(x^2 + y^2)}$$

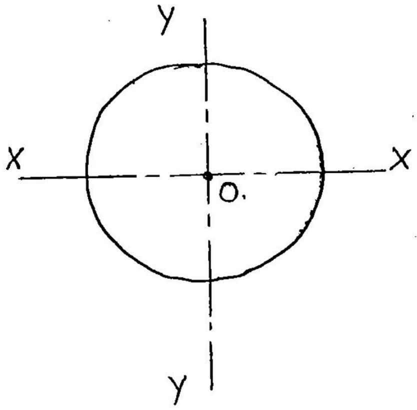
Polar moment of inertia $= I_z = I_p = J = I_x + I_y$

The moment of inertia about a perpendicular axis to the plane of area is 'Polar moment of Inertia'.

- I_x & I_y are used in bending problems.
- I_z used in torsion problems

- I_x, I_y & I_z are always non-zero positive values

Q.

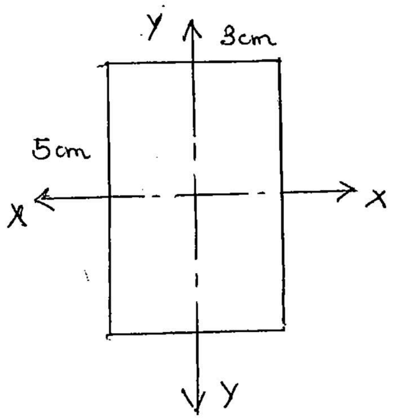


$$I_x = I_y = \frac{\pi}{64} D^4.$$

$$I_z = J = I_x + I_y$$

$$= \underline{\underline{\frac{\pi}{32} D^4}}$$

Q.

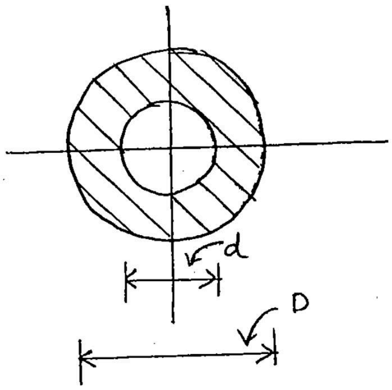


$$I_x = \frac{3 \times 5^3}{12}$$

$$I_y = \frac{5 \times 3^3}{12}$$

$$I_z = \frac{3 \times 5^3}{12} + \frac{5 \times 3^3}{12} = \underline{\underline{42.5 \text{ cm}^4}}$$

Q.



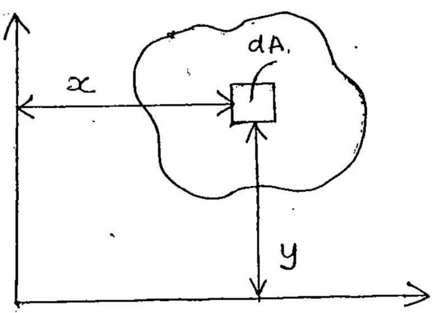
$$I_x = I_y = \frac{\pi}{64}(D^4 - d^4).$$

$$I_z = J = I_x + I_y.$$

$$= \frac{\pi}{64} \times 2 \, (D^4 - d^4).$$

$$\therefore \; \underline{\underline{I_z = \frac{\pi}{32}(D^4 - d^4)}}.$$

→ Product of Inertia (I_{xy})



$$I_x = \int dA \cdot y^2$$

$$I_y = \int dA \cdot x^2$$

$$\boxed{I_{xy} = \int dA. \; xy}$$

Unit : m^4

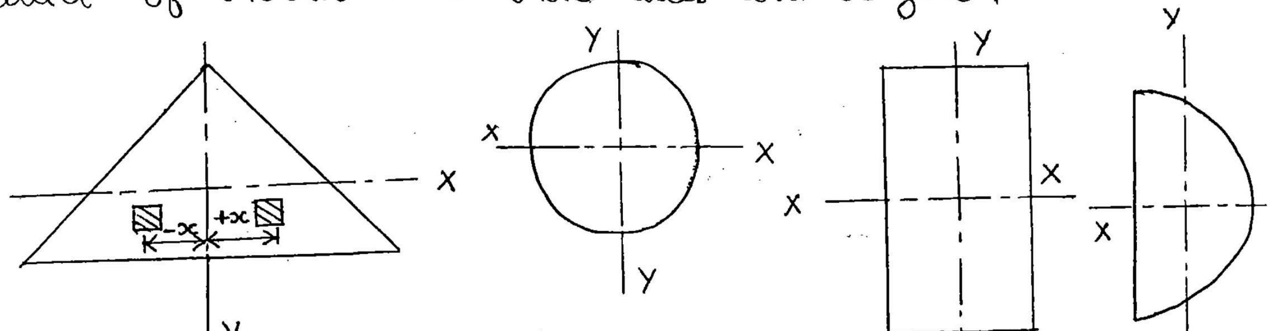
- Product of inertia may be +ve or −ve or zero also depending upon the position of a given area w.r.t. axis.

Uses:

1. Unsymmetrical / Skew / Bi-axial bending
   Eg: Purlins.
2. Principal MI.
3. Inertia tensor.

- For product of inertia any two mutually perpendicular axes in the <u>plane of area</u> are required.

- Among the two axes, if anyone is symmetrical the product of inertia w.r.t those axes will be zero.



$I_{xy} = 0$.
(symmetric about Y).

$I_{xy} = \int (dA(-x)y + dA(x)y)$

$= 0$

$I_{xy} = 0$
(symmetric)

$I_{xy} = 0$
(symmetric about X & Y)

$I_{xy} = 0$

→ Principal MI :

Max or min MI for a given c/s area.

Stresses	σ_x	σ_y	τ_{xy}
Inertia	I_x	I_y	I_{xy}

$$\boxed{\left.\begin{array}{l} I_1 = I_{max} \\ I_2 = I_{min} \end{array}\right\} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}}$$

* Principal. Axes :

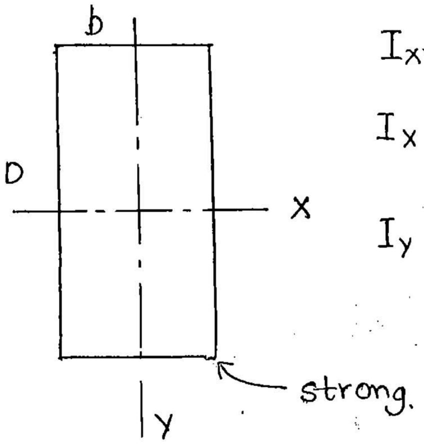
The axes about which principal moment of inertias will be acting.

About these axes, product of inertia (I_{xy}) is zero.

∴ they are symmetrical axes

⊙ Principal axes are mutually perpendicular.

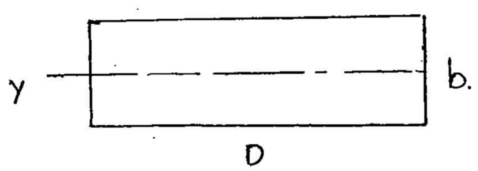
Q.



$I_{XY} = 0$

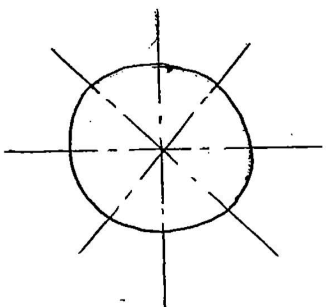
$I_X = \frac{bD^3}{12} = I_1 = I_{max}$

$I_Y = \frac{Db^3}{12} = I_2 = I_{min}$.



$I_Y = \frac{Db^3}{12}$

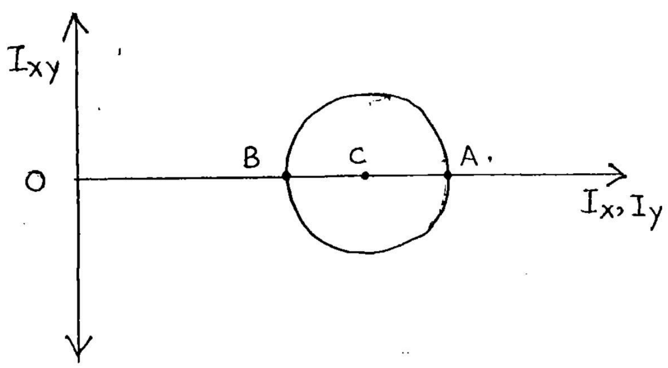
Q.



$I_{max} = I_{min} = \frac{\pi D^4}{64}$.

All the diametric axes are principal axes (symmetric axes)

-* Mohr Circle of Inertia.



⊙ $OA = I_{max}$

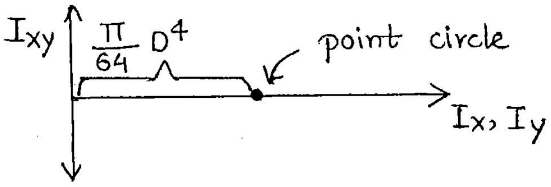
$OB = I_{min}$.

⊙ Radius $= (I_{xy})_{max}$

$= \frac{I_{max} - I_{min}}{2}$

⊙ $OC = I_{avg} = \frac{I_{max} + I_{min}}{2}$

Mohr's circle of inertia for a circular c/s is a point circle.



→ Inertia Tensor

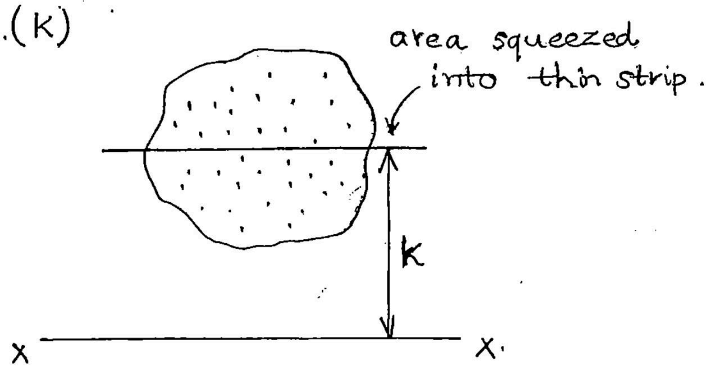
$$\begin{bmatrix} I_x & I_{xy} \\ I_{yx} & I_y \end{bmatrix}_{2\times2}$$

For symmetry, $I_{xy} = I_{yx}$.

→ Radius of Gyration (K)

$$K = \sqrt{\frac{I}{A}}$$

unit: m

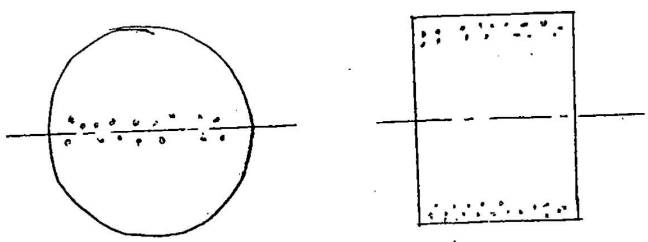


The fixed distance from a reference axis where all the particles of a given area are squeezed to be concentrated

As K increases, distance of particles from axis increases. This increases stability and hence the strength.

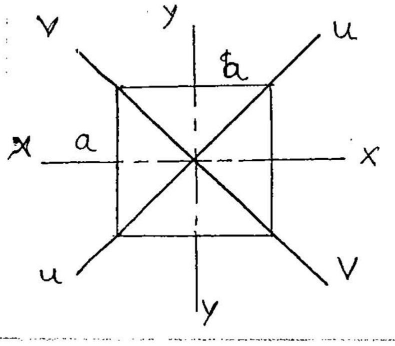
P-38

3.



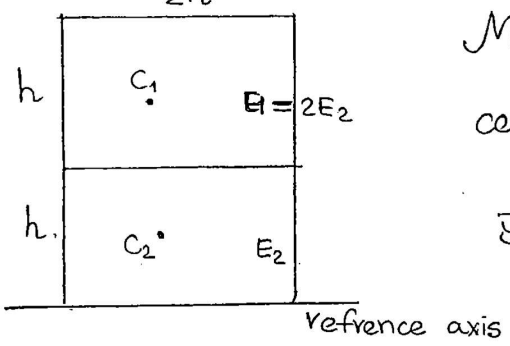
Weaker Stronger

8.



$$I_x = I_y = I_u = I_v = \frac{a \cdot a^3}{12}$$

01.

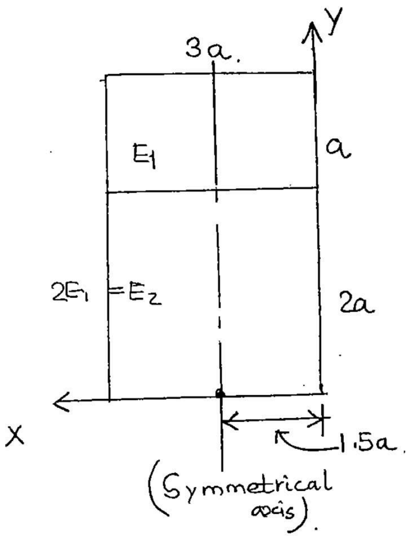


Max shear @ NA ; ie passes through 54 centroid.

$$\bar{y} = \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2} = \frac{2E_2\left(\frac{h}{2} + h\right) + E_2\left(\frac{h}{2}\right)}{2E_2 + E_2}$$

$$\bar{y} = \frac{3.5h}{3} = \underline{\underline{1.167h}}$$ (from bottom

02. Always, centroid lies on symmetrical axis.



$$\bar{x} = \frac{A_1E_1x_1 + A_2E_2x_2}{A_1E_1 + A_2E_2}$$

$\frac{a\times 3a \times \frac{3a}{2} + 2a\times 3a\times \frac{3a}{2}\times 2}{3a^2\times E_1 + 2\times 6a^2}$ $\frac{1.5a + 6a}{3}$

$$= \frac{(a\times 3a)\times \frac{3a}{2}\times E_1 + (2a\times 3a)\times \frac{3a}{2}\times 2E_1}{a\times 3a\times E_1 + 2\times 6a^2E_1}$$

$$= \underline{\underline{1.5\,a}}$$

$$\bar{y} = \frac{3a^2\times E_1\times 2.5a + 6a^2\times 2E_1\times a}{3a^2E_1 + 6a^2\times 2E_1} = \underline{\underline{1.3\,a}}$$

03.

$\frac{7\times 2b\times 27a^3}{4825}$

22nd Oct,
WEDNESDAY

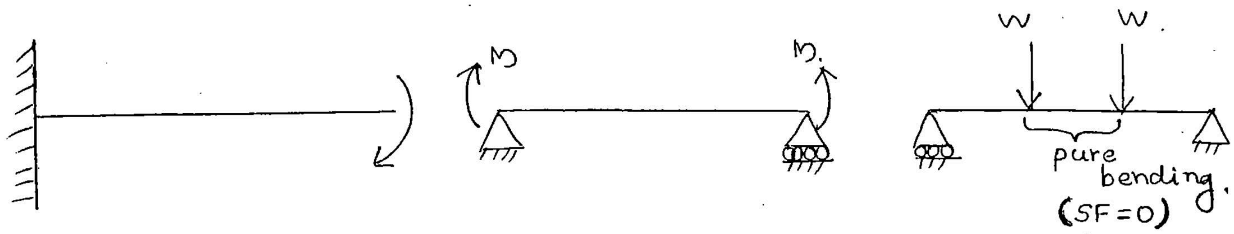
05. THEORY OF SIMPLE BENDING

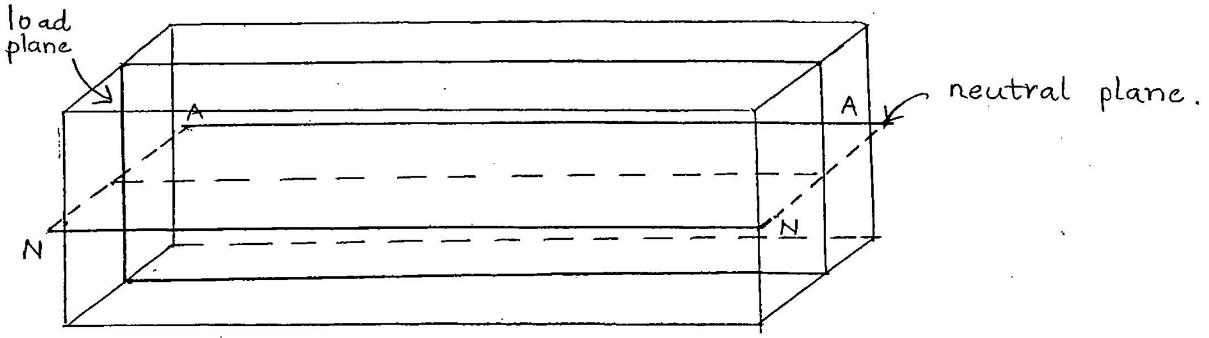
For pure bending,

SF = 0

BM = non zero constant. & MAX

Elastic curve = arch of a circle.





A line joining centroids of all cross sections along the length of a beam is centroidal axis (or) longitudinal axis (or) axis

- If load is applied, the centroidal axis deflects in the form of elastic curve or deflected shape.

- The axis in the c/s perpendicular to axis of the beam is the neutral axis

- The plane containing neutral axis and the axis of beam is neutral plane. Any point on neutral plane, ha no bending stress and no bending strain. (Shear stress and shear strain may be there).

In circular members subj. to torsion, Bernoulli assumption is valid.

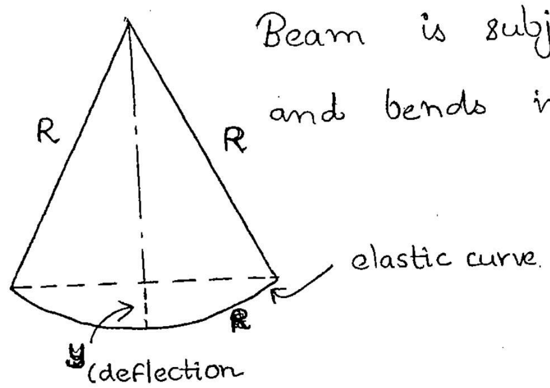
2. It is assumed that beam comprising of layers and they are free to slide one over the other without friction ∴ SF can be eliminated.

3. The material properties are remaining the same in tension and compression. ($E_{tension} = E_{compression}$)

4. Radius of curvature is more compared to dimensions of c/s of beam. ($R >>> b$ & D).

slopes ↓ } superposition
deflections ↓ } is applicable.

5. Beam is subjected to pure bending and bends in an arc of a circle.



→ Flexural Equation (or) Bending Equation.

$$\boxed{\frac{E}{R} = \frac{M}{I} = \frac{f}{y}}$$

R → radius of curvature.

$\frac{1}{R} = \rho$ → curvature.

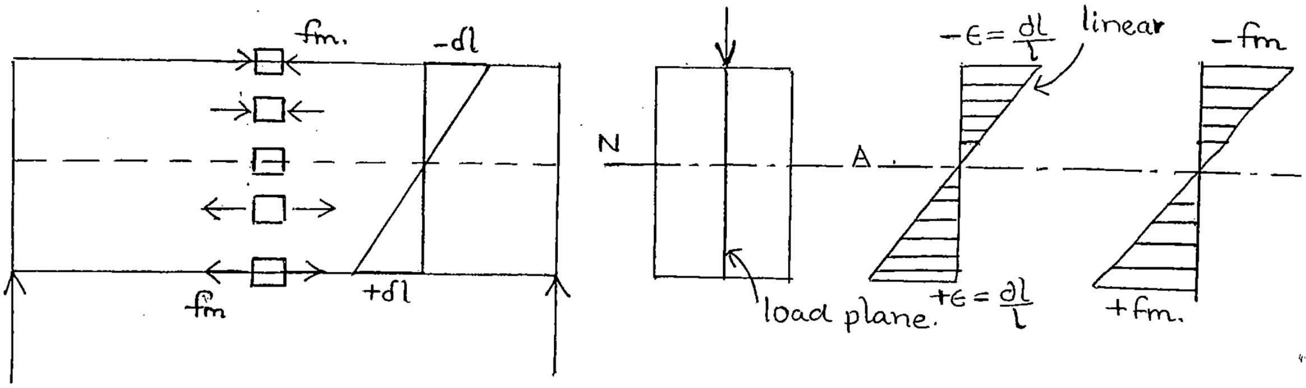
I → MI of entire c/s area about NA

f → bending stress (indirect normal stress). {tensile or comp}

y → linear distance from NA, where f is required.

Due to loading, c/s of beam rotates wrt neutral axis.

But NA always remains straight.



- Vertical plane through which load is applied to avoid torsion in the c/s is called 'Load plane'.

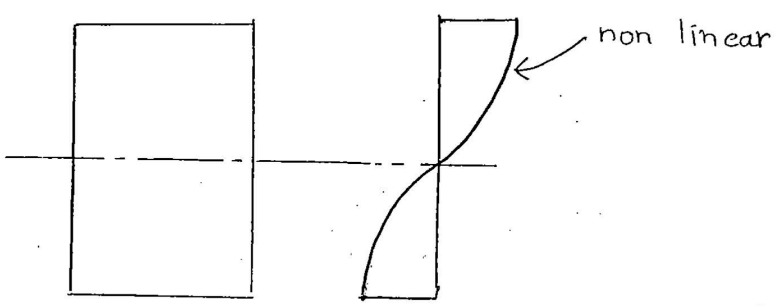
* Assumptions:

1. Euler- Bernoullie:

As per Bernoullie, there is no distortion in the shape of c/s due to bending. As per the assumption, strain distribution is linear along the depth with zero strain at the axis and max. at extreme fibres. As per Bernoulli, the linear distribution of strain is valid in all bending theories upto failure. (WSM of RCC, LSM of RCC, Ultimate Load Method of RCC, Plastic theory in steel)

2. Bernoulli's assumption is valid for composites beams like RCC also. But proper bond is required b/w different materials.

Not valid For:-



① Deep Beam (D > 750 mm).

Strain distrubution

② Non circular c/s subjected to torsion.

$$f = \text{const.} * y. \Rightarrow f \propto y.$$

NOTE :

In a beam, stresses developed are only in longitudinal direction. Even though an element is taken just below the load, no normal stress in the load direction on the element.

* Limitations :

1. Valid only upto PL.
2. Not valid for composites (like RCC).
3. Only gradual load. (no impact loads).
4. Only prismatic beams.

→ Section Modulus (Z)

First moment of area about neutral axis.

$$Z = \frac{I}{y_{max}} \quad (\text{Unit : } m^3)$$

As Z ↑, strength in bending ↑.

→ Flexural Rigidity (EI) (Unit: N-mm²)

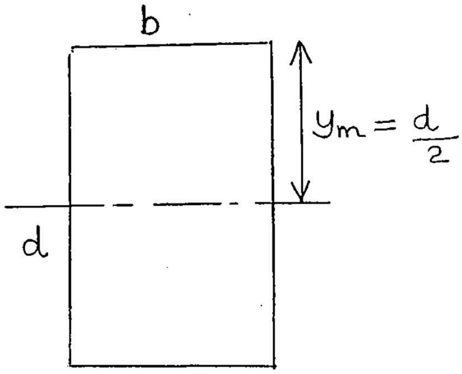
As EI ↑, rigidity in bending ↑
Stiffness. ↑
slopes & deflections ↓

- In a beam, strength parameter is Z.
  stiffness parameter is EI

→ Axial Rigidity (AE)

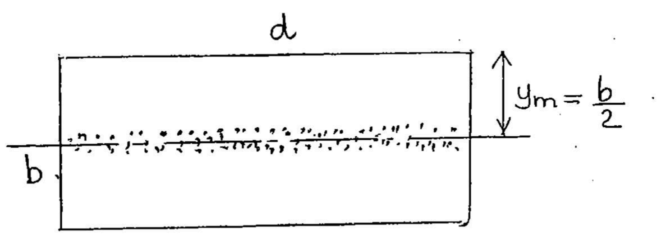
Unit : N

As AE ↑, axial deformation ↓

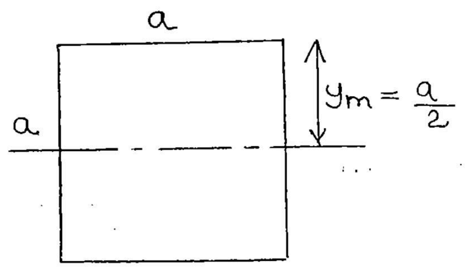


$$Z = \frac{I_{NA}}{y_{max}}$$

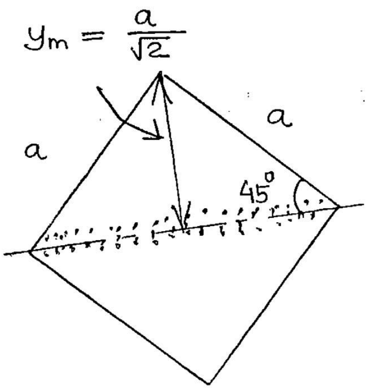
$$= \frac{\left(\frac{bd^3}{12}\right)}{\left(\frac{d}{2}\right)} = \underline{\underline{\frac{bd^2}{6}}}$$



$$Z = \frac{\left(\frac{db^3}{12}\right)}{\left(\frac{b}{2}\right)} = \underline{\underline{\frac{db^2}{6}}}$$



$$Z = \underline{\underline{\frac{a^3}{6}}}$$

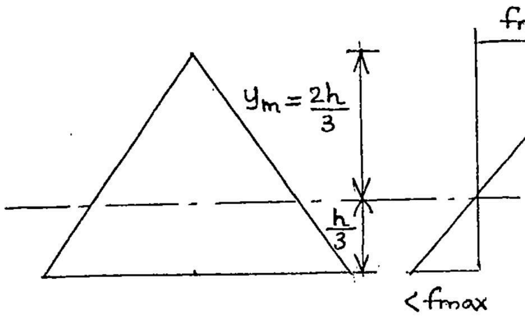


$$Z = \frac{I}{y_{max}} = \frac{\frac{a \cdot a^3}{12}}{\frac{a}{\sqrt{2}}}$$

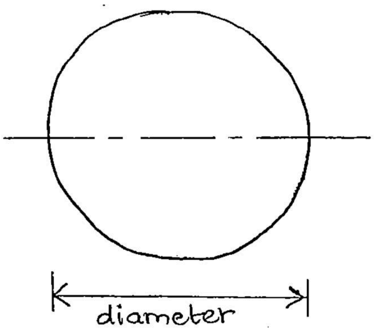
$$Z = \underline{\underline{\frac{a^3}{6\sqrt{2}}}}$$

$$\odot \quad \frac{(\text{Strength})_{sq}}{(\text{Strength})_{Di}} = \frac{(Z)_{sq}}{(Z)_{Di}} = \sqrt{2} = 1.414$$

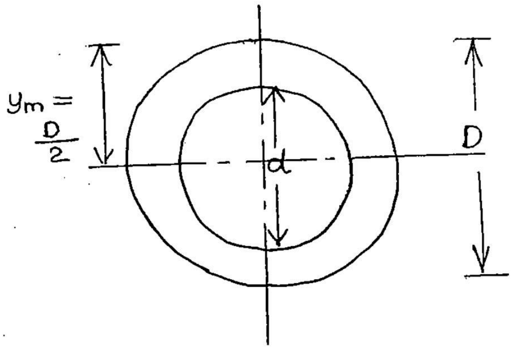
$$(\text{Strength})_{sq} = 41.4\,\% \uparrow (\text{strength})_{dia}$$



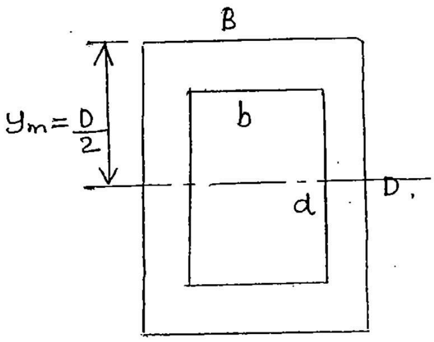
$$Z = \frac{\frac{bh^3}{36}}{\frac{2h}{3}} = \underline{\underline{\frac{bh^2}{24}}}$$



$$Z = \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \underline{\underline{\frac{\pi d^3}{32}}}$$

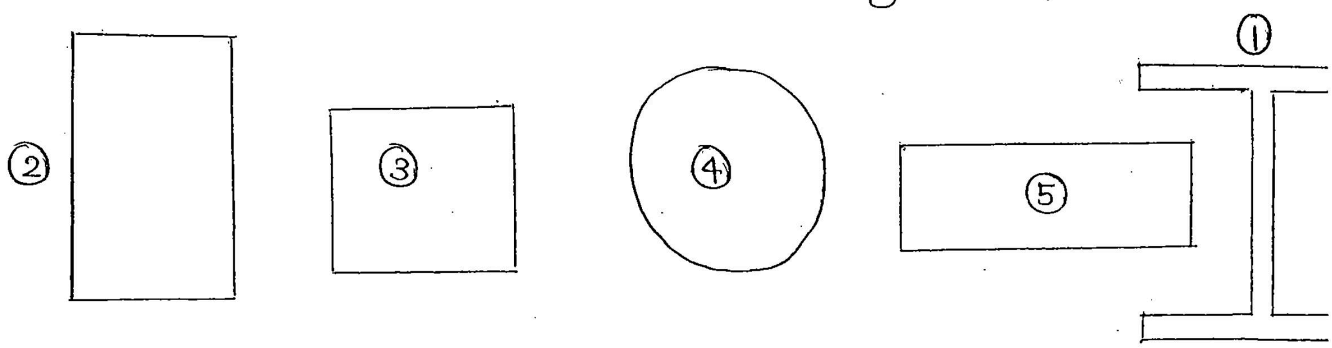


$$Z = \frac{\frac{\pi}{64}(D^4 - d^4)}{\frac{D}{2}} = \frac{\pi (D^4 - d^4)}{32 D}.$$



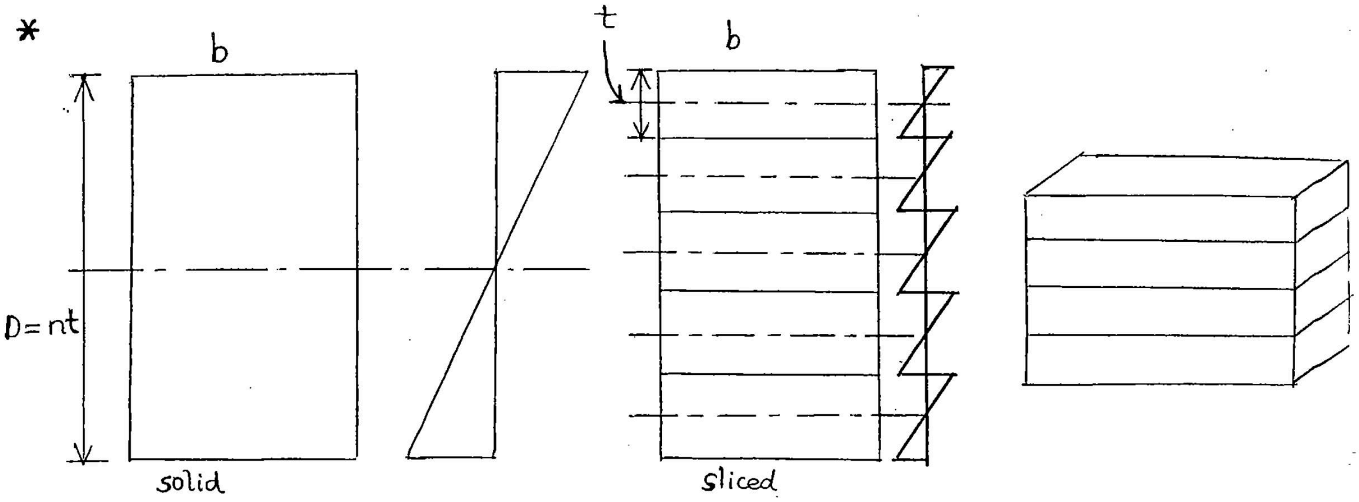
$$Z = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}} = \frac{BD^3 - bd^3}{6D}$$

* Same c/s area (Rankings in bending strength).



→ Sliced Beams.

*



$$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{sliced}} = \frac{(Z)_{solid}}{(Z)_{sliced}} = \frac{\frac{b(nt)^2}{6}}{n\left(\frac{bt^2}{6}\right)} = n.$$

$$\rho = \frac{1}{R} = \frac{M}{EI}$$

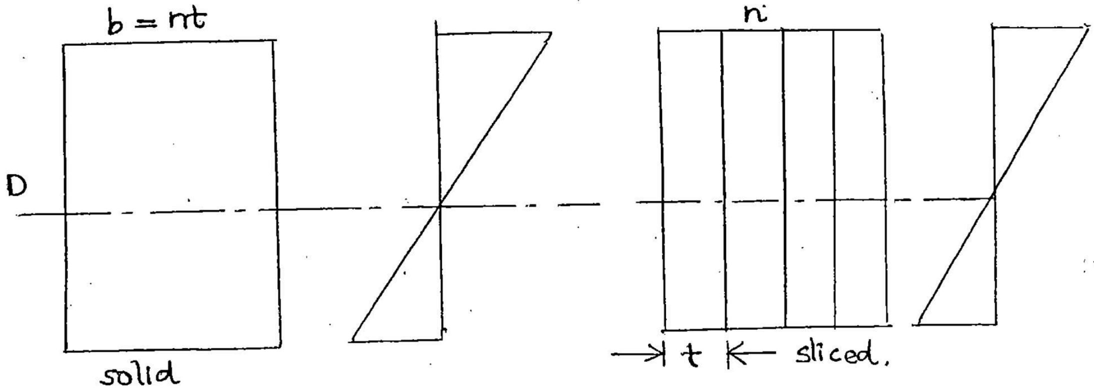
$$\Rightarrow \quad \rho \propto \frac{1}{I}$$

$$\frac{\rho_{solid}}{\rho_{sliced}} = \frac{I_{sliced}}{I_{solid}} = \frac{n\left(\frac{bt^3}{12}\right)}{\frac{b\,(nt)^3}{12}} = \frac{1}{n^2}$$

$\rho_{sliced} = \rho_{solid} * n^2$ (Take the example of a book!)

$(\text{Stiffness})_{solid} = (\text{Stiffness})_{sliced} * n^2$

*



$$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{sliced}} = \frac{(Z)_{solid}}{(Z)_{sliced}} = \frac{\frac{(nt)D^2}{6}}{n\left(\frac{tD^2}{6}\right)} = \underline{\underline{1}}$$

$$\frac{P_{solid}}{P_{sliced}} = \frac{I_{sliced}}{I_{solid}} = \frac{n\left(\frac{tD^3}{12}\right)}{(nt)\frac{D^3}{12}} = 1$$

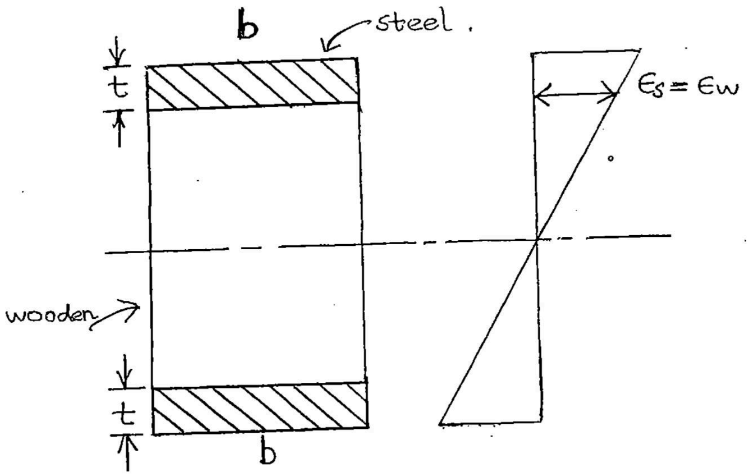
$\therefore$ $P_{solid} = P_{sliced}$.

$I_{sliced} = I_{solid}$.

$(\text{Stiffness})_{solid} = (\text{Stiffness})_{sliced}$.

→ Flitched Beams (composite beams)

Example : RCC



$\epsilon_s = \epsilon_w$

$$\frac{f_s}{E_s} = \frac{f_w}{E_w}$$

$$\boxed{f_s = \left(\frac{E_s}{E_w}\right) f_w}$$

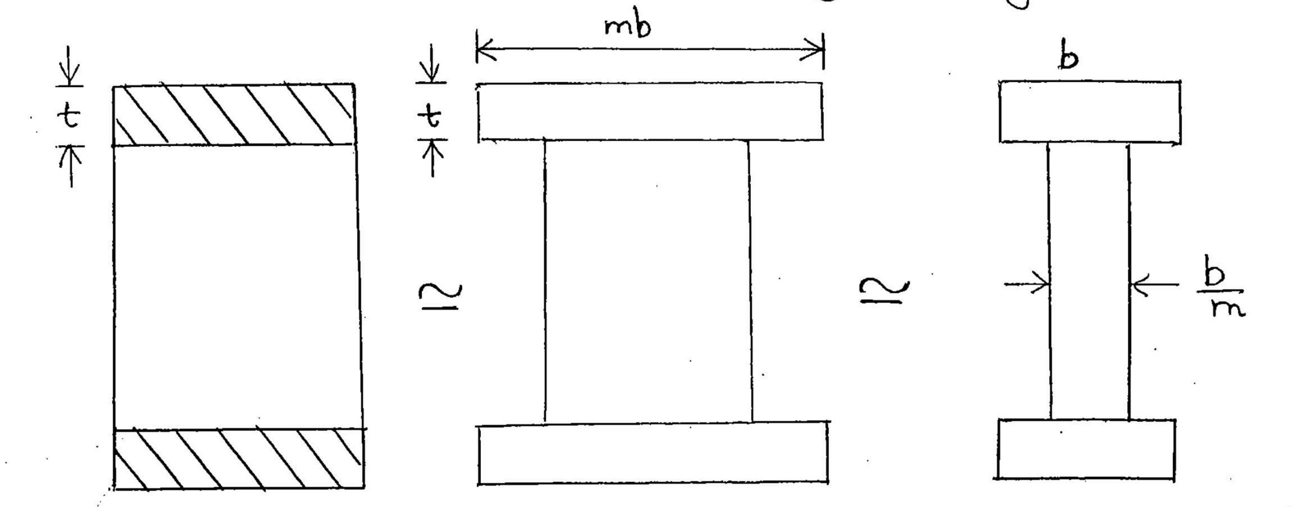
In a composite beam, different material should be bonded together so that the load can be shared.

- Bernoulli's assumption is valid for composite beams.

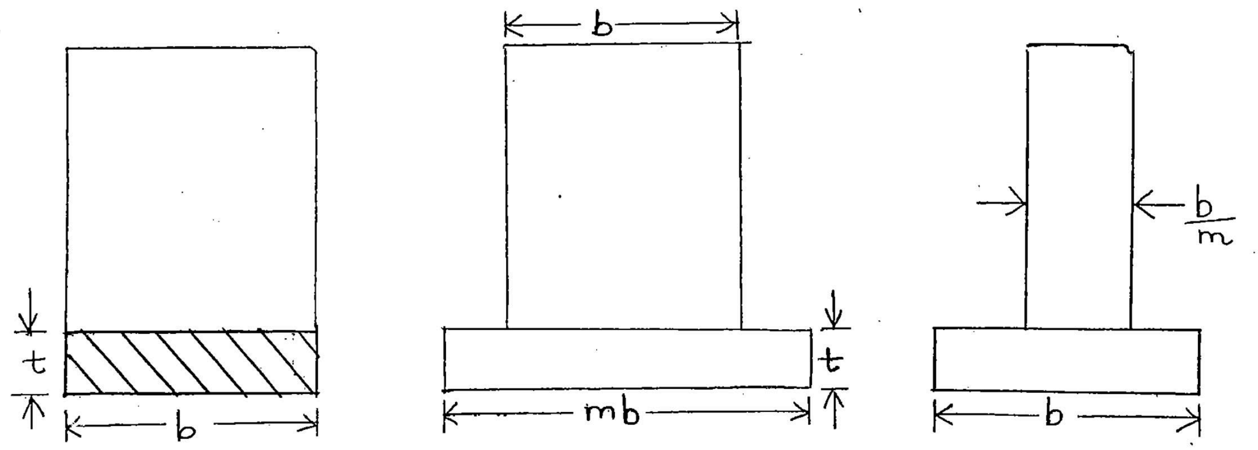
$$\boxed{\text{Modular ratio, } m = \frac{E_{strong}}{E_{weak}}}$$

$$\boxed{f_s = m f_w}$$

For the analysis of composite beams, equivalent area method is used. Total c/s is divided into equivalent ~~material~~ area of single material and analysed using bending equation.



▪ Equivalent in Wood ▪ Equivalent in steel.



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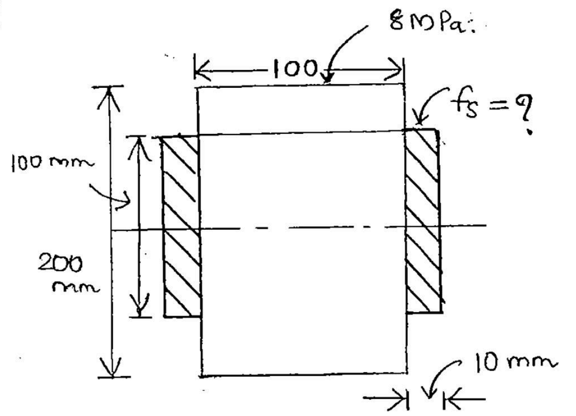
17



$m = 20$

From linear variation of stress,

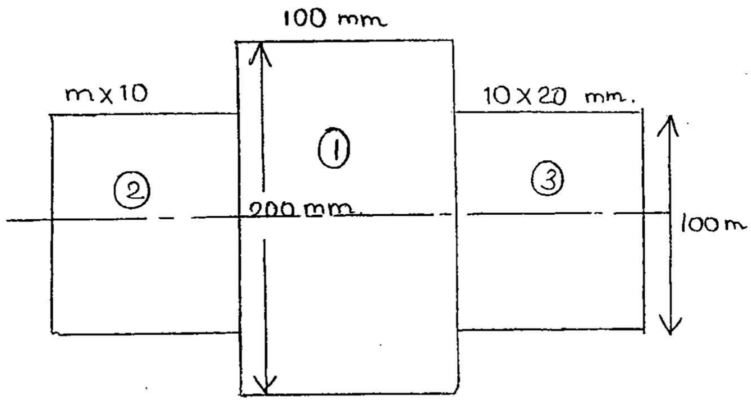
100 mm $\longrightarrow$ 8 MPa.

50 mm $\longrightarrow$? (From NA)

$= 8 \times \frac{50}{100} = \underline{\underline{4 \text{ MPa}}}$

$f_s = m . f_w$

$= 20 \times 4 = \underline{\underline{80 \text{ MPa}}}$



MI of equivalent wooden beam about NA

$$I = I_1 + 2 I_2$$

$$= 100 \times \frac{200^3}{12} + 2 \times \frac{200 \times 100^3}{12}$$

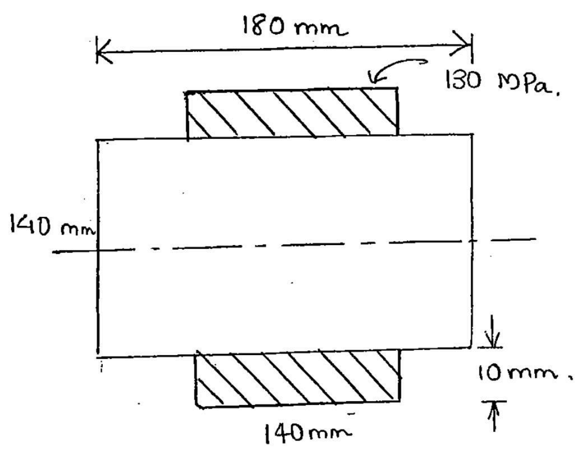
$$= \underline{\underline{10^8 \ mm^4}}$$

$$y_{max} = \frac{200}{2} = 100 \ mm$$

$$\Rightarrow \quad \frac{M}{I} = \frac{f}{y}.$$

$$M = \frac{fI}{y} = \frac{8 \times 1 \times 10^8}{100} = \underline{\underline{8 \ kN \ m}}$$

09.



$$\left.\begin{array}{l} f_w = 8 \ MPa \\ f_s = 130 \ MPa. \end{array}\right\} \text{max. values}$$

80 – 130
70 –

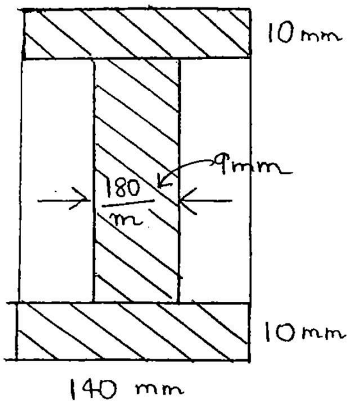
80 mm $\longrightarrow$ 130 MPa.

70 mm $\longrightarrow$?

$$f_s = \frac{70 \times 130}{80} = 113.75 \ MPa.$$

Stress in wood, $f_w = \frac{f_s}{m} = \frac{113.75}{20} = 5.6875 \ MPa \ \underline{\underline{< 8 \ MPa}}$

If $f_w = 8$ MPa, stress in steel (f_s) goes beyond 130 MPa, which is practically not possible as steel fails if its stress = 130 MPa. ∴ in the design stress in the steel is the deciding criteria.



MI of equivalent steel beam about NA,

$$I = \frac{140 \times 160^3}{12} - \frac{(140-9)\,140^3}{12}$$

$$= \underline{\underline{17.82 \times 10^6 \text{ mm}^4}}$$

From bending equation, (using eq. steel section).

$$\frac{M}{I} = \frac{f}{y} \Rightarrow M = 130 \times \frac{17.82 \times 10^6}{80} = 28.95 \times 10^6 \text{ Nmm}.$$

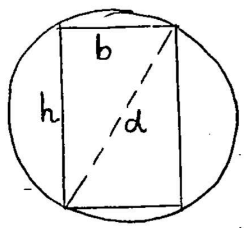
$$= \underline{\underline{28.95 \text{ kNm}}}$$

→ Beam of Uniform Strength.

Along the length of a beam, if the bending stress developed is const, it is the beam of uniform strength.

3rd Oct,
HURSDAY

-43. ⊙ In order to obtain a rectangle of maximum strength in pure bending from a circular log of wood,



$$d^2 = b^2 + h^2$$

$$h^2 = d^2 - b^2 \longrightarrow (1)$$

$$z = \frac{bh^2}{6} = \frac{b(d^2 - b^2)}{6}.$$

For strongest rectangular section, z should be maximum.

$$\frac{dz}{db} = 0$$

$$= \frac{d^2 - 3b^2}{6} = 0.$$

$$\Rightarrow \quad b = \frac{d}{\sqrt{3}} \longrightarrow (2)$$

$$h^2 = d^2 - b^2$$

$$= d^2 - \left(\frac{d}{\sqrt{3}}\right)^2$$

$$h = \sqrt{\frac{2}{3}}\, d \longrightarrow (3)$$

$$\Rightarrow \boxed{\frac{h}{b} = \sqrt{2}}$$

Area of strongest rectangle $= bh$

$$= \left(\frac{1}{\sqrt{3}} d\right) \times \left(\sqrt{\frac{2}{3}}\, d\right)$$

$$= \frac{\sqrt{2}}{3} d^2$$

P-44

9. $\frac{M}{I} = \frac{f}{y}$.

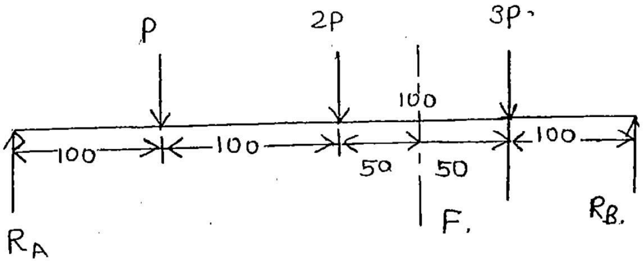
$$M = f \cdot \frac{I}{y} = fz = f \cdot \frac{bd^2}{6}.$$

Given $f =$ const. & $d =$ const.

$$\therefore M \propto b$$

45.

03. $R_B \times 400 = P \times 100 + 2P \times 200 + 3P \times 300$



$$R_B = \frac{14}{4} P.$$

$$R_A = \frac{5}{2} P.$$

$$M_F = R_B \times 150 - 3P \times 50.$$

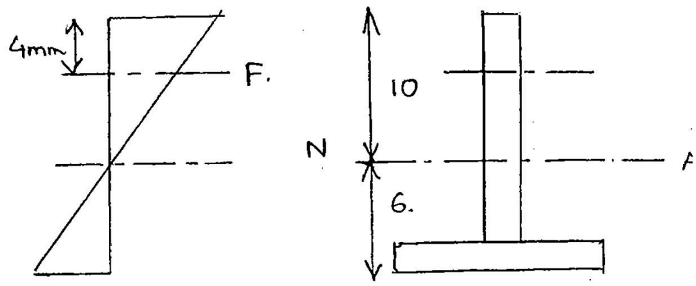
$$= \frac{14}{4} P \times 150 - 3P \times 50 = 375\, P$$

$$\epsilon_F = 1.5 \times 10^{-6}.$$

$$f_F = \epsilon_F \times E$$

$$= (1.5 \times 10^{-6})(200 \times 10^{+3})$$

$$= 0.3 \text{ N/mm}^2.$$



Using bending equation (@ F),

$$\frac{M}{I} = \frac{f_F}{y_F}$$

$$\frac{375P}{2176} = \frac{0.3}{6}$$

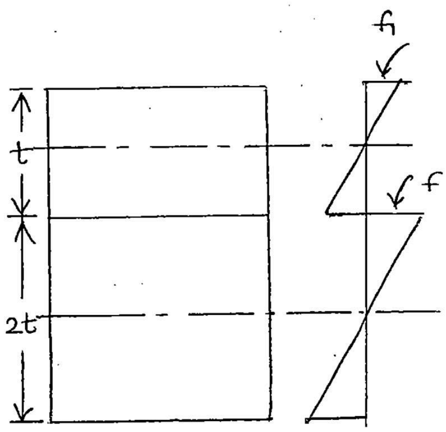
$$\underline{\underline{P = 0.290 \text{ N}}}$$

9.

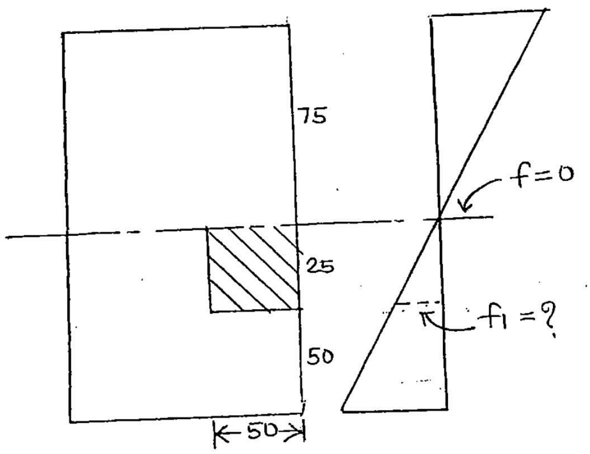
$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y} = \text{const.}$$

$$f = ky.$$

$$\frac{f_1}{f_2} = \frac{(y_{max})_1}{(y_{max})_2} = \frac{t/2}{2t/2} = \underline{\underline{\frac{1}{2}}}$$



14.



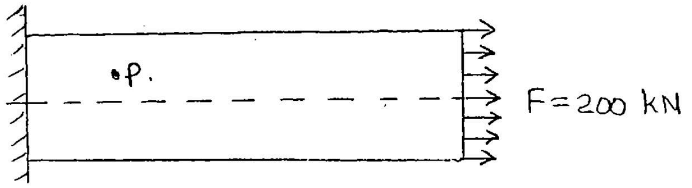
$$\frac{f_1}{y_1} = \frac{M}{I.}$$

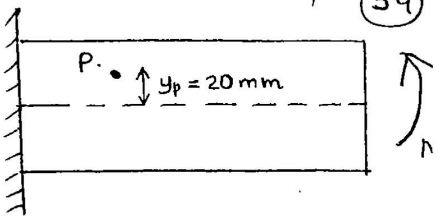
$$\frac{f_1}{25} = \frac{16 \times 10^6}{\left(\frac{100 \times 150^3}{12}\right)}.$$

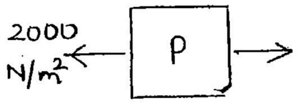
$$f_1 = \underline{\underline{14.2}} \text{ MPa}.$$

Force on hatched area = avg stress * hatched area

$$= \frac{1}{2}(0 + f_1) \times 25 \times 50 = \underline{\underline{8.9 \text{ kN}}}$$

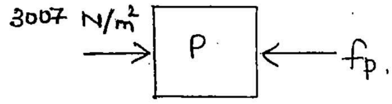






$\sigma = \frac{F}{A}$ (tensile).

$= \frac{200}{0.1} = 2000$ N/m²

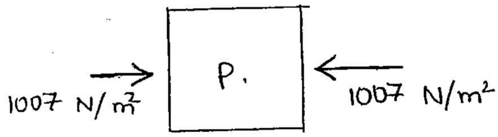


$f_p = \frac{M}{I} y_P.$

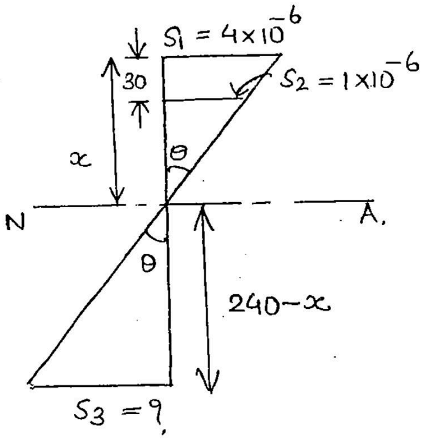
$= \frac{200}{1.33\times10^{-3}} \left(\frac{20}{1000}\right).$

$= 3007$ N/m².

Resultant stress @ P :



02.



$\tan\theta = \frac{4\times10^{-6}}{x} = \frac{1\times10^{-6}}{x-30} = \frac{S_3}{240-x}.$

$x = 40$ mm.

$S_3 = 20\times10^{-6}$

5. $\frac{E}{R} = \frac{f}{y}.$

$\frac{2\times10^5}{500/2} = \frac{f}{0.5/2} \Rightarrow f = 200$ N/mm²

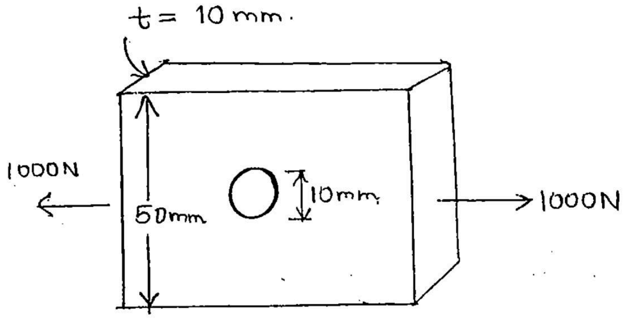
P-14.

4. $(dl)_{sw} = \frac{Wl}{2AE}$ (elongation)

$(dl)_{ext} = \frac{Wl}{AE}$ (contraction).

$(dl)_{net} = dl_{sw} - dl_{ext} = \frac{Wl}{2AE} - \frac{Wl}{AE} = \ominus\frac{Wl}{2AE}$ (contraction).

8.

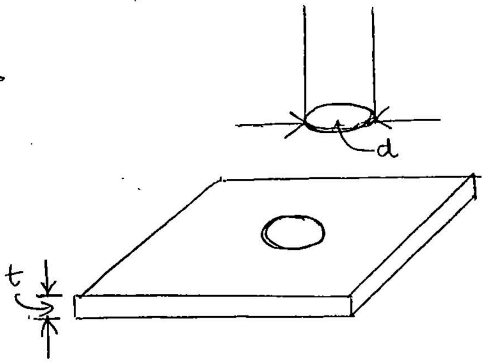


$$\sigma_{max} = \frac{P}{A_{min.}}$$

$$= \frac{1000}{(500-10)\ 10} = \underline{\underline{2.5\ MPa}}$$

Level 2

5.



Punching ~~head~~ force = Shear resistance.

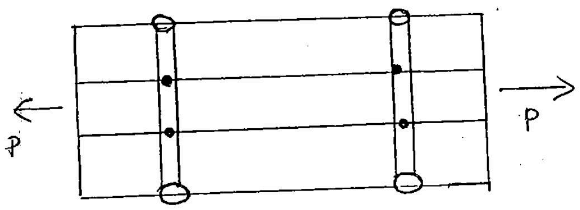
σ (c/s area of head) = τ (shearing area.

$$\sigma\left(\frac{\pi}{4}d^2\right) = \tau\,(\pi d t).$$

$$4\tau\left(\frac{\pi}{4}d^2\right) = \tau\,(\pi d t).$$

$$\Rightarrow \cancel{t} = d = \underline{\underline{10\ mm}}$$

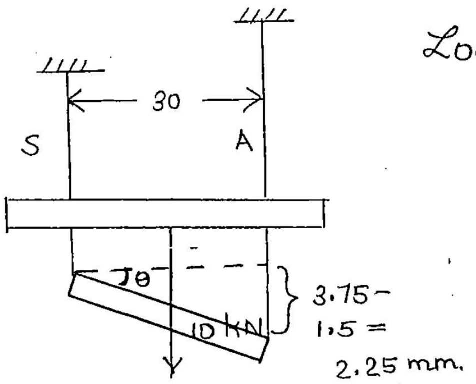
7



Rivet in double shear.

Force for each cut $= \underline{\underline{\frac{P}{2}}}$

16.



Load is acting at centre,

$$P_S = P_a = \frac{P}{2} = \frac{10}{2} = 5\ kN.$$

$$\sigma_S = \frac{P_S}{A_S} = \frac{5\times10^3}{0.1\times10^2} = 500\ kN/mm^2$$

$$\sigma_A = \frac{P_A}{A_A} = \frac{5\times10^3}{0.2\times10^2} = 250\ kN/mm^2$$

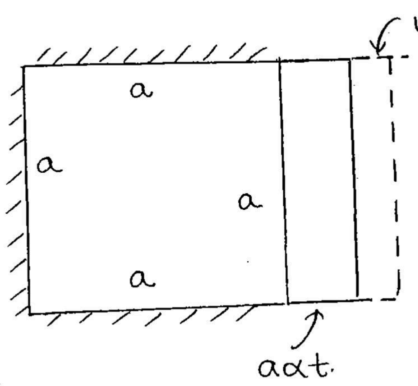
$$\delta l_A = \left(\frac{Pl}{AE}\right)_A = \frac{5\times10^3\times1000}{(0.2\times10^2)(66667)} = 3.75\ mm$$

$$\delta l_S = \left(\frac{Pl}{AE}\right)_S = \frac{5\times10^3\times600}{0.1\times10^2\times2\times10^5} = 1.5\ mm.$$

$\sin\theta = \frac{2.25}{300} \Rightarrow \theta = 0.43 \ (cw)$ (60) 62

Q- 17

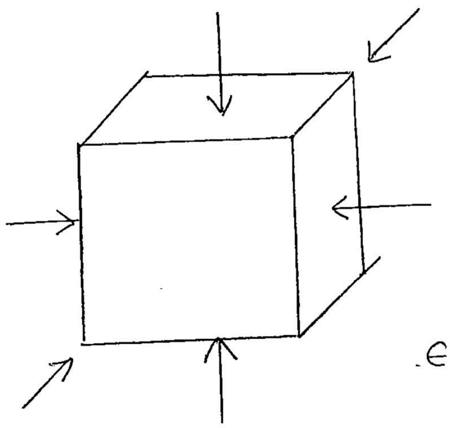
3.



Total expansion $= a\alpha t + \mu a\alpha t$.

$= \alpha a t (1+\mu)$.

Q0.



Due to temperature change,

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha \Delta T \rightarrow (1)$$

Due to expansion prevented,

$$\epsilon_x = \epsilon_y = \epsilon_z = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}.$$

$$\epsilon_x = \frac{-\sigma}{E} - \mu\left(\frac{-\sigma}{E}\right) - \mu\left(\frac{-\sigma}{E}\right) \rightarrow (2).$$

Equating (1) & (2),

$$-\frac{\sigma}{E} + \mu\frac{\sigma}{E} + \mu\frac{\sigma}{E} = \alpha\Delta T.$$

$$\sigma = \frac{\ominus E\alpha\Delta T}{(1-2\mu)}.$$

If cube is free to expand in all directions, what is the temperature stress developed ?

Zero

23 Oct,
THURSDAY

06 SHEAR STRESS IN BEAMS

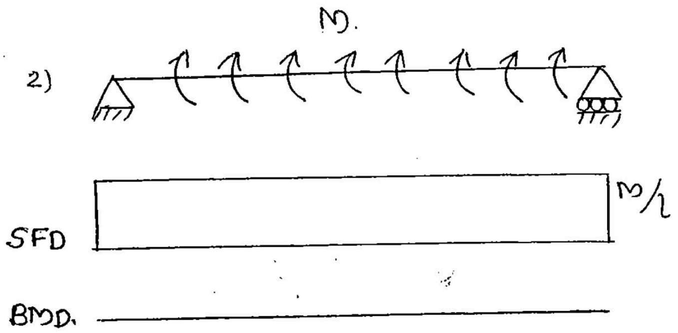
- Flexural shear stress (or) Indirect shear stress due to bending action in a beam.
- Pure shear occurs when;

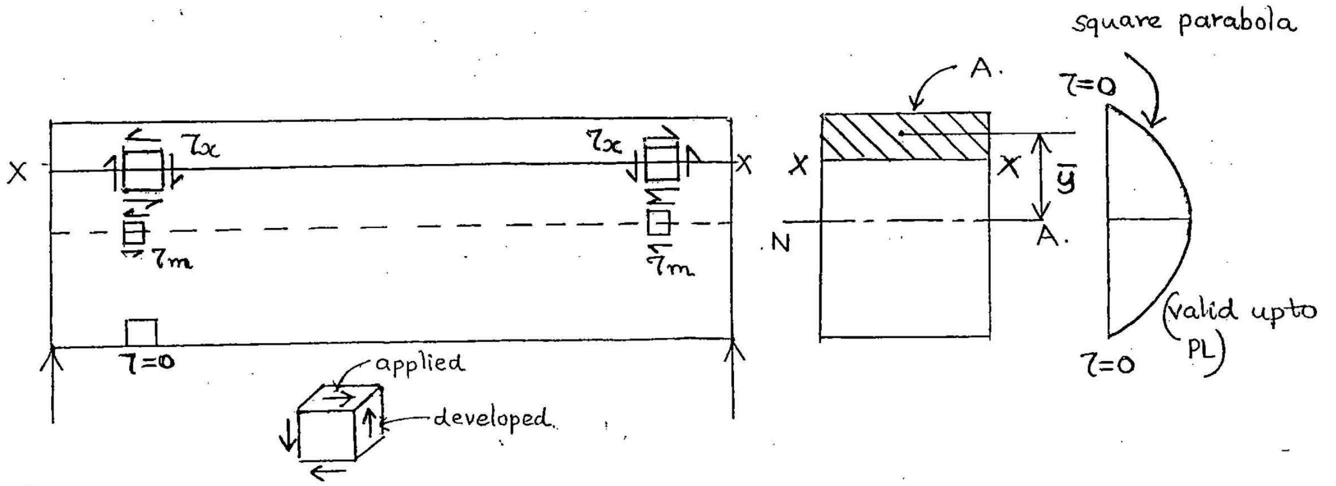
  SF = non zero const. and maximum.

  BM = 0

Eg: 1) Deep beam (D>750 mm) {Bending moment is almost ignored}

2)

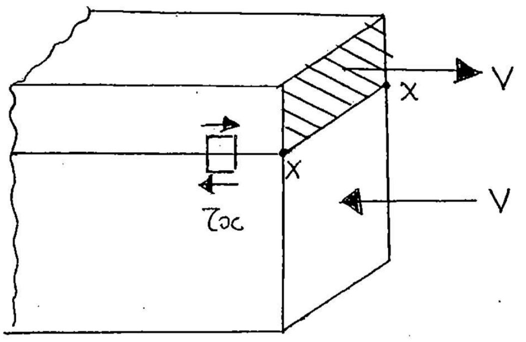




In a beam, the loading will be in transverse direction which causes layers of the beam move one over the othe in the axial or longitudinal direction.

∴ the critical shear stress in a beam is in axial directi directi of beam only.

To balance this shear, a complementary shear stress of equal magnitude and opposite in direction develops on vertical planes as shown in fig.



$$\boxed{\tau_x = \frac{V A \bar{y}}{I b}}$$

where V → SF at a c/s due to vertical or transverse loading.

A → the area either above or below the section X–X in the c/s. $\left\{\begin{array}{l} A \text{ above NA} - (+ve) \\ A \text{ below NA} - (-ve) \end{array}\right\}$ net area is considered

$\bar{y}$ → Distance to centroid of area from NA.

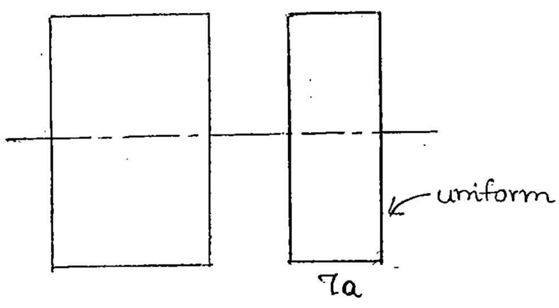
I → MI of entire c/s area (not the hatched area) about NA

b → width of c/s parallel to NA where shear stress is required

$$\tau_x = \underbrace{\frac{V}{I}}_{\text{const.}} \; \underbrace{\frac{A\bar{y}}{b}}_{\text{variables @ a c/s}} \qquad \left\{ \text{unit: } \frac{m^2 \cdot m}{m} = m^2 \right\}$$

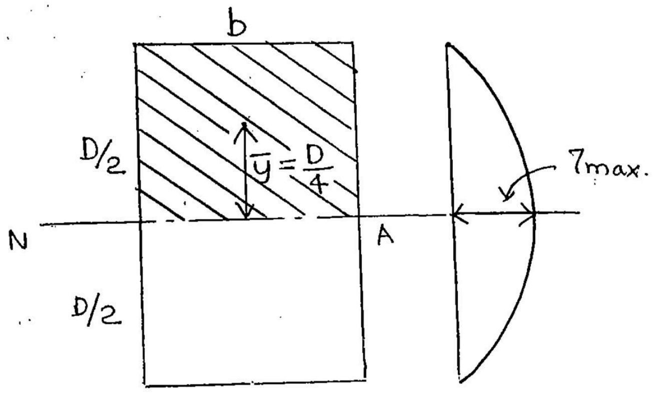
* Average Shear Stress :

$$\boxed{\tau_a = \frac{V}{\text{c/s area}}} \quad ; \text{uniform in c/s}$$



→ Relation b/w τ_m & τ_{avg}.

1. Rectangular / Square.



$$\tau_m = \frac{VA\bar{y}}{Ib}$$

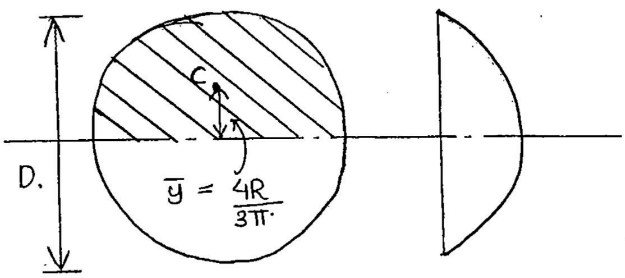
$$= \frac{V.\left(b.\frac{D}{2}\right)\left(\frac{D}{4}\right)}{\frac{bD^3}{12}.b}$$

$$\tau_a = \frac{V}{bD}$$

$$\Rightarrow \boxed{\frac{\tau_m}{\tau_a} = \frac{3}{2}}$$

$\tau_m = 1.5\ \tau_a$. (50% more than τ_a).

2. Solid Circular.



$$\tau_m = \frac{V.\ \frac{\pi d^2}{8} \times \frac{2d}{3\pi}}{\frac{\pi d^4}{64}.\ d}$$

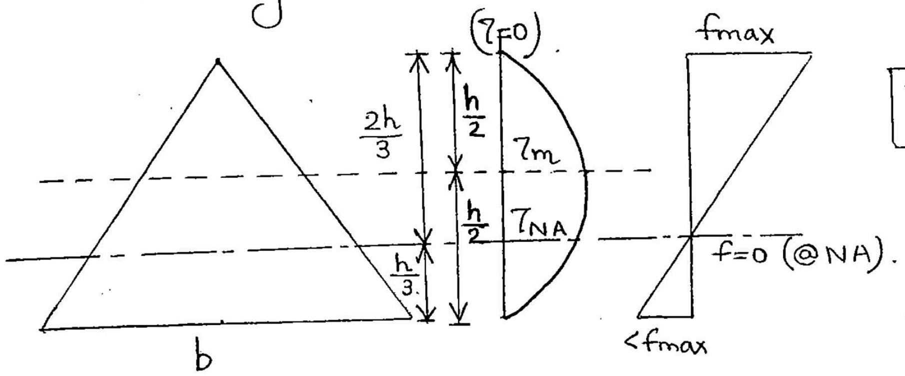
$$\tau_a = \frac{V}{\frac{\pi d^2}{4}}$$

$$\boxed{\frac{\tau_m}{\tau_a} = \frac{4}{3}}$$

$\tau_m = 1.33\ \tau_a$ (33% more than τ_a)

⊙ In a beam, shear stress is secondary criteria, and main design criteria is bending. So τ_a is considered instead of τ_m.

3. Triangular.



$$\boxed{\tau_{NA} < \tau_{max}}$$

$$\boxed{\frac{\tau_m}{\tau_a} = \frac{3}{2} = \text{Same as square / rect.}}$$

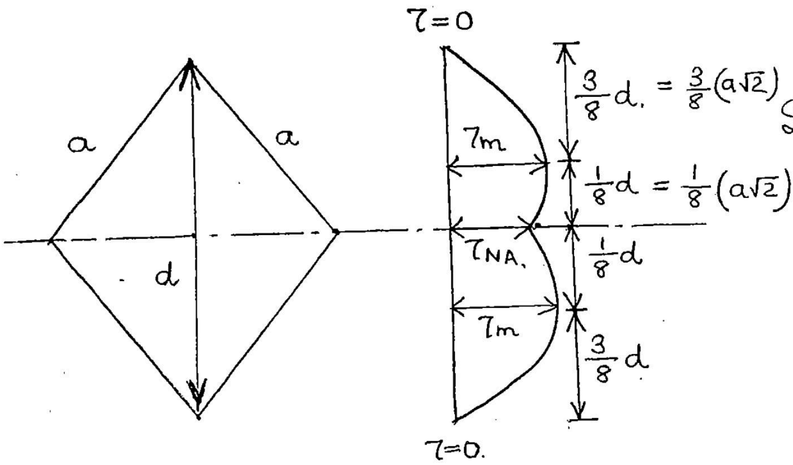
$$\boxed{\frac{\tau_m}{\tau_{NA}} = \frac{9}{8}}$$

$$\boxed{\frac{\tau_{NA}}{\tau_a} = \frac{4}{3} = \text{same as solid circular section}}$$

At the point of max bending stress, (fmax), shear stress must be zero (τ=0).

At the point of max shear stress (τ_m), bending stress need not be zero.

4. Diamonds.

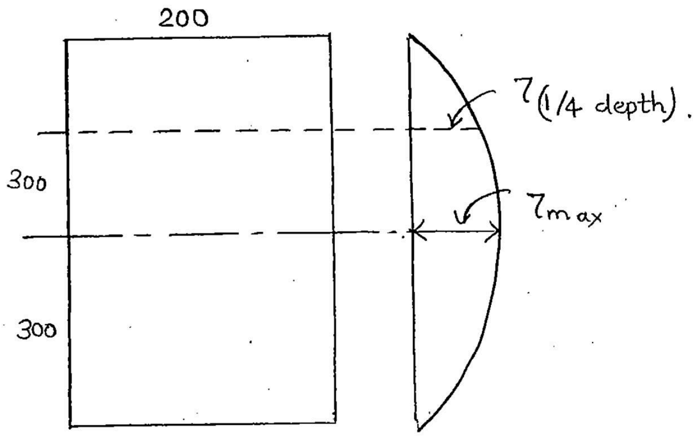


Diagonal, $d = \sqrt{2}\,a$.

$$\boxed{\frac{\tau_m}{\tau_a} = \frac{9}{8}} \qquad \boxed{\frac{\tau_{NA}}{\tau_{avg}} = 1} \qquad \boxed{\frac{\tau_m}{\tau_{NA}} = \frac{9}{8}}$$

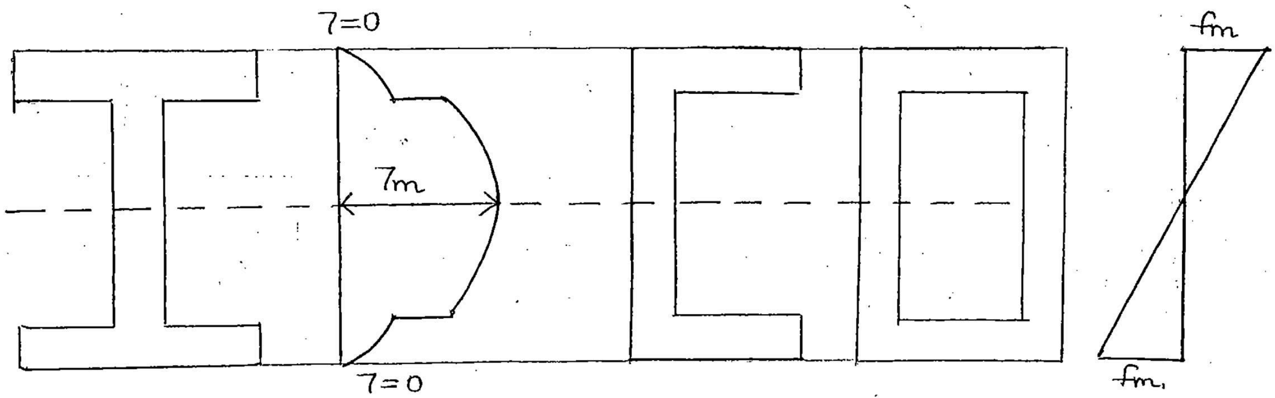
$$\tau_m = \frac{9}{8}\tau_a = 1.125\,\tau_a \left(12.5\% \text{ more than } \tau_a\right)$$

Section	τ_m/τ_a	τ_N/τ_a
Rectangular / Square	3/2	3/2
Circular	4/3	4/3.
Triangle.	3/2	4/3.
Diamond.	9/8	1

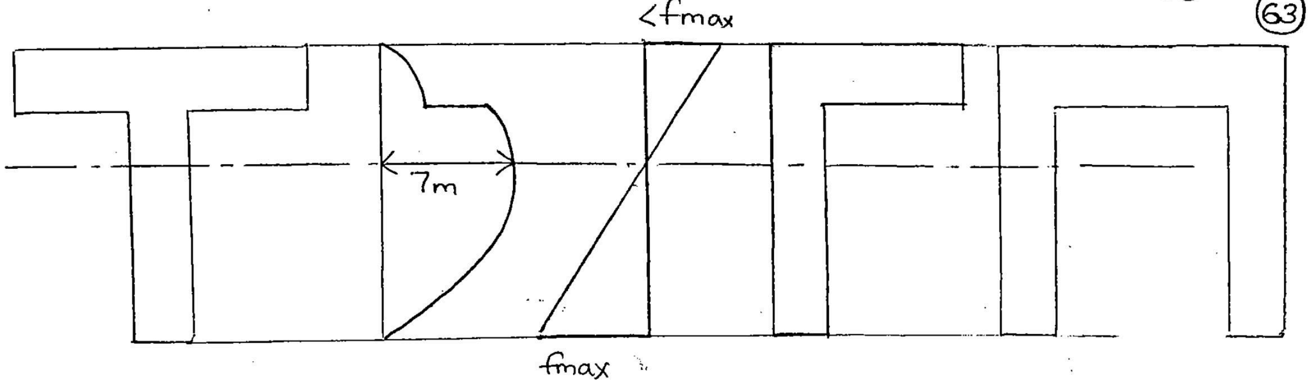


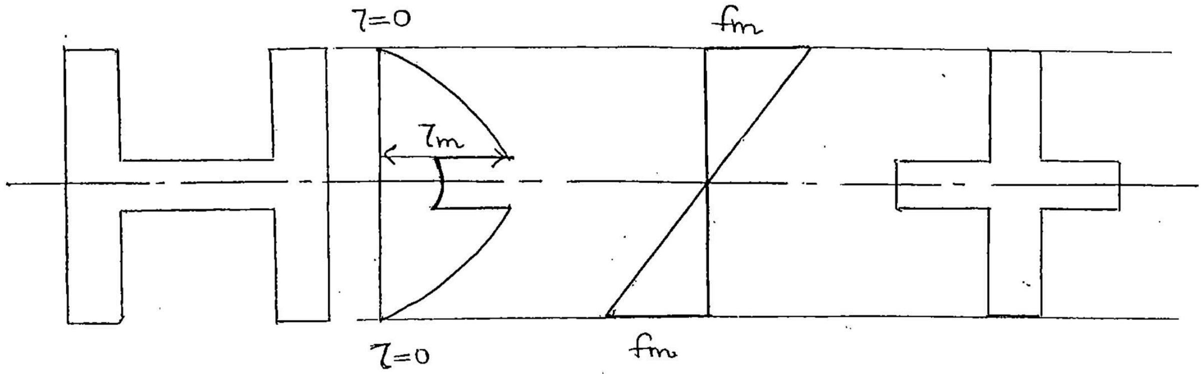
$$\frac{\tau_{(1/4\ \text{depth})}}{\tau_m} = \frac{\dfrac{V\cdot(200\times150)(75+150)}{Ib}}{\dfrac{V(200\times300)(150)}{Ib}} = \frac{3}{4}$$

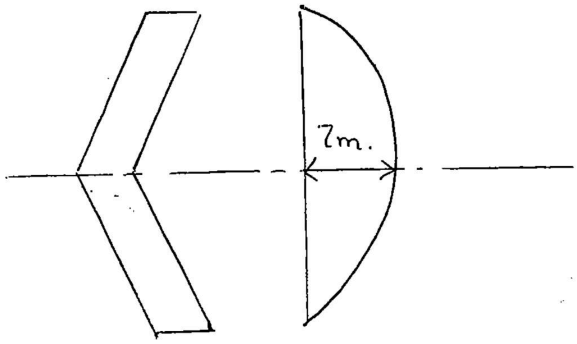
→ Flanged Beams



In flanged beams, max. shear stress is taken by web, max bending stress taken by flange.



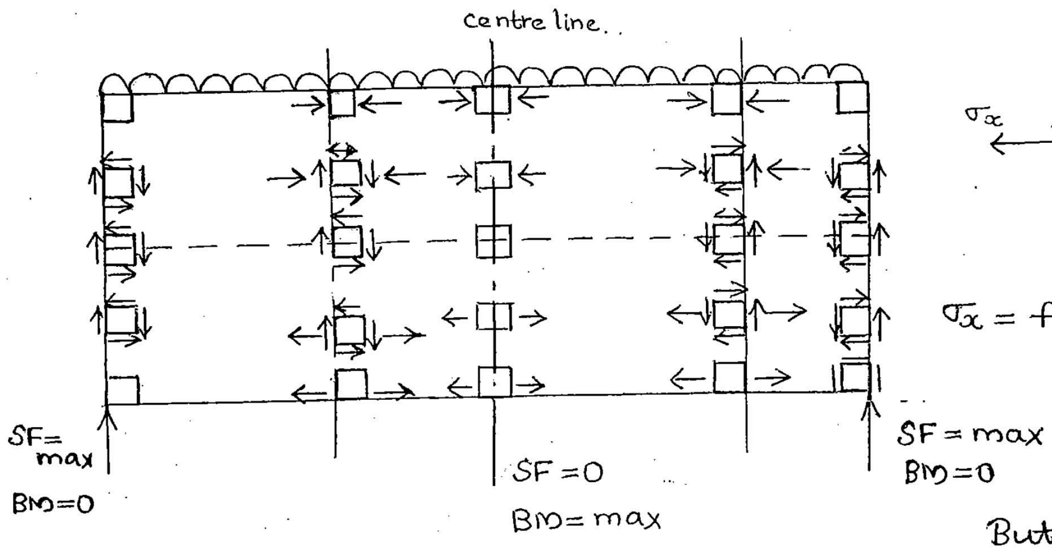


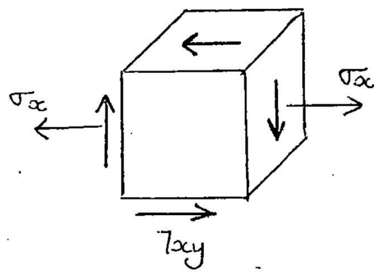


$$\tau = \frac{V A \bar{y}}{I b} \Rightarrow \tau \propto \frac{1}{b}$$

ie

$b \uparrow$	$\tau \downarrow$
$b \downarrow$	$\tau \uparrow$





$\sigma_x = f$; $\tau_{xy} = \tau_{yx} = \tau$

$\sigma_y = 0$; $\tau_{xz} = 0 = \tau_{zx}$

$\sigma_z = 0$; $\tau_{yz} = 0 = \tau_{zy}$

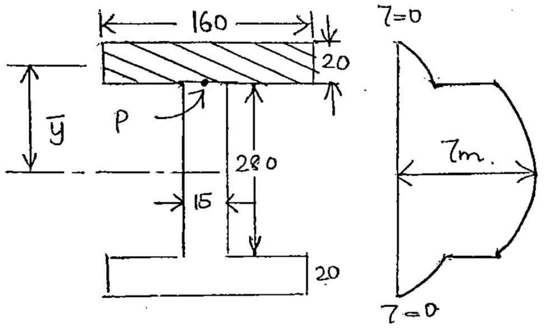
But $\epsilon_x \neq 0$

$\epsilon_y \neq 0$

$\epsilon_z \neq 0$

Q52

8.



NOTE :

- In a beam, stresses in the width direction (z direction) will be zero. ∴ beam can be taken as a plane stress system. However, the strain in the width of (or z direction) directio is not zero.

8. $$I_{NA} = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12} = \underline{\underline{171.6 \times 10^6}} \text{ mm}^4$$

$$\tau_p = \frac{VA\bar{y}}{I b_p} = \frac{200 \times 10^3 \times (160 \times 20).(140+10)}{171.6 \times 10^6 \times \underline{(15)}}$$

(in web).

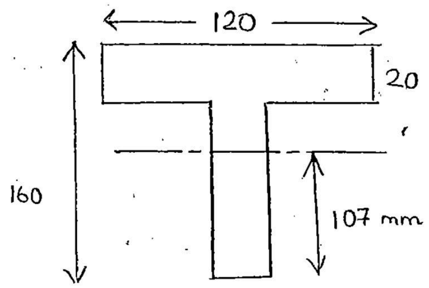
$$= \underline{\underline{37.296}} \text{ MPa}$$

10. $$\tau_p = \frac{200 \times 10^3 \times 160 \times 20\,(150)}{171.6 \times 10^6 \times \underline{160}} = \underline{\underline{3.496}} \text{ MPa}$$

(in flange).

9. $$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{160 \times 20 \times 150 + 140 \times 15 \times 70}{160 \times 20 + 140 \times 15} = 118.30 \text{ mm}$$

$$\tau_m = \frac{200 \times 10^3 \times (160 \times 20 + 140 \times 15).118.30}{171.6 \times 10^6 \times 15} = \underline{\underline{48.71}} \text{ MPa}$$

11. 

$$\tau_{max} = \frac{VA\bar{y}}{I b} = \frac{140 \times 10^3 \times 107 \times 20 \times \frac{107}{2}}{13 \times 10^6 \times 20}$$

$$= \underline{\underline{61.65}} \text{ MPa}$$

Q3. $f = \frac{M}{Z} = \frac{wl/4}{\frac{bd^2}{6}}$

$= \frac{3wl}{2bd^2} = 12.$

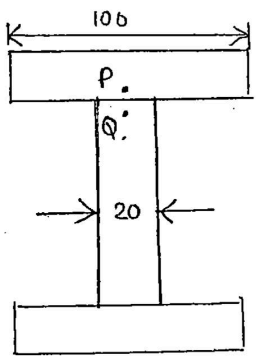
$q = \frac{VA\bar{y}}{Ib} = \frac{\frac{w}{2} \times b\,d/2 \times d/2}{\frac{bd^3}{12} \times b.} = 1.2.$

$= \frac{3w}{bd} = 1.2.$

$\frac{f}{q} = \frac{12}{1.2} = \frac{3wl/bd^2}{2 \times 3w/bd.}$ $\qquad \frac{bdl}{bd^2}$

$\frac{10}{2} = \frac{l}{d} \Rightarrow \frac{l}{d} = 5$

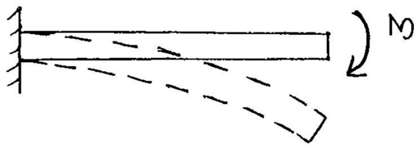
Q4.



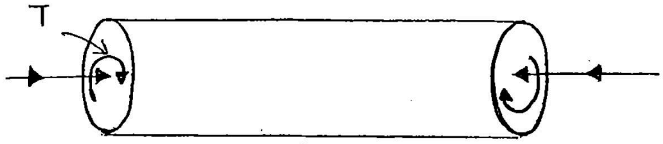
$\tau_Q = \tau_P \times \frac{100}{20} = 60 \text{ MPa}$

24th Oct,
FRIDAY

07. TORSION

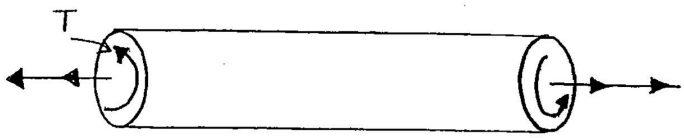


BM : along axis



Torsion : about axis
Clockwise : +ve

Torsion also called as
Twisting moment (or).
Axial couple (or).
Torque.



Anticlockwise : −ve

* Pure Torsion (impossible)

T = non zero const. & max

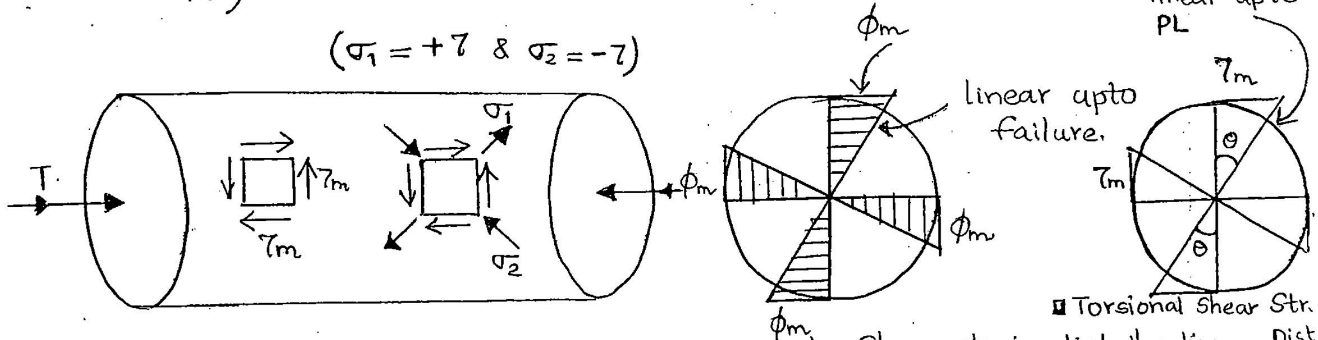
$SF = 0$; $BM = 0$; $AF = 0$

→ Assumptions :

1. Euler – Bernoullie

As per Bernoullie, there is no distortion in the shape of c/s after the torsion (no warping and no bending in the c/s)

$(\sigma_1 = +\tau$ & $\sigma_2 = -\tau)$



▫ Torsional Shear Str. Dist

▫ Torsional Shear strain distribution

As per Bernoulli, shear strain is linear in the c/s with zero at centre of shaft and max. at all extreme points on the surface of shaft.

* Limitations :

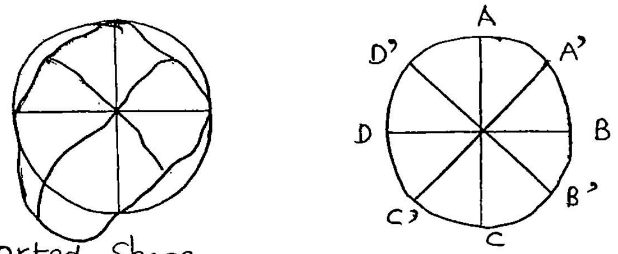
(i) Applicable for gradually applied torsion. (invalid for torsion with impact).

(ii) Applicable only for circular (solid or hollow). shafts. only

2. Torsion is constant along length of shaft.

3. Material is isotropic, homogenous and follows Hooke's Law

4. Radii remain straight after torsion (no distortion in c/s)



Distorted Shape
(Bernoulli's Assumption not valid).

5. Torsion applied must be within proportionality limit.

→ Torsion Equation.

$$\boxed{\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}}$$

$J \rightarrow$ Polar MI $= I_z = I_p = I_x + I_y$.

$\theta \rightarrow$ angle of twist (in rad)

$\tau \rightarrow$ Torsional shear stress (indirect shear stress)

$r \rightarrow$ radial distance from centre of shaft.

⊙ Equation is valid only for circular shafts (both solid & hollow)

⊙ Not valid for composite shafts made of different materials

$$\boxed{\tau \propto r}$$

- Due to torsion, shear stress is developing b/w the layers. The max. torsional shear stress is b/w the outermost thin layer and the layer below it.

- Any element on the surface of shaft will be under pure shear (if normal stress on τ_{max} plane is zero, then it is called pure shear).

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2} = 0.$$

where σ_1 & σ_2 are principal stresses.

- Due to torsion, all the stresses are b/w the layers only, there is no stress developed in the plane of c/s.

$$\boxed{\frac{T}{J} = \frac{G}{(l/\theta)} = \frac{\tau}{r}}$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

→ Polar Section Modulus

$$Z_p = \frac{J}{r_{max}} \qquad \left(Z = \frac{I}{y_{max}}\right).$$

Unit : m^3, mm^3

$\uparrow Z_p \Rightarrow \uparrow$ strength in torsion

→ Torsional Rigidity (GJ)

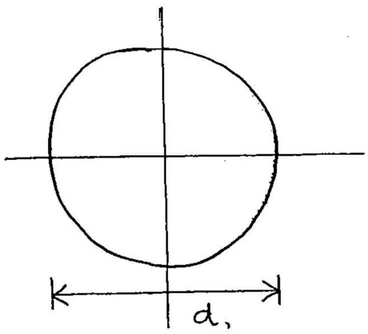
Unit : Nm^2.

$\uparrow GJ \Rightarrow \uparrow$ rigid shaft.

$\uparrow$ stiffness

$\downarrow \theta$

→ Solid Shaft.



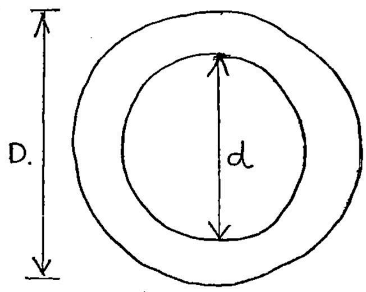
$$I_x = I_y = \frac{\pi}{64} d^4$$

$$I_z = I_x + I_y = \frac{\pi}{32} d^4$$

$$Z_p = \frac{J}{d/2} = \frac{\frac{\pi}{32} d^4}{d/2}$$

$$\Rightarrow \boxed{Z_p = \frac{\pi d^3}{16}} \qquad \left\{ Z = \frac{\pi d^3}{32} \right\}$$

→ Hollow Shaft.



$$\boxed{Z_p = \frac{\pi (D^4 - d^4)}{16 D}}$$

→ Power Transmission.

$$P = \omega T$$

$$\boxed{P = 2\pi N T}$$

T → average torque (after losses). (Nm or J)

N → rps (or) Hz (or) cycles/sec.

P → average power = Nm/s
= J/s = W

⊙ 1 watt (W) = 1 Nm/s = 1 J/s

kW = kNm/s.

⊙ HP = 746 W = 746 Nm/s.
= 0.746 kW = 0.746 kNm/s

◉ If N is given in rpm,

$$P = \frac{2\pi NT}{60}$$

(Theoretical)

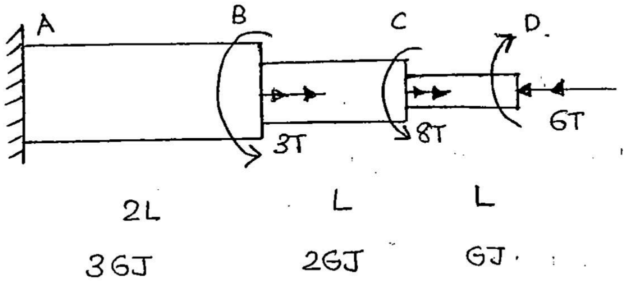
Max. Torque (without losses) $\rightarrow$ $$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}.$$

◉ If losses are not given in a problem, consider

$$T_{max} = T_{avg}$$

→ Arrangement of Shafts.

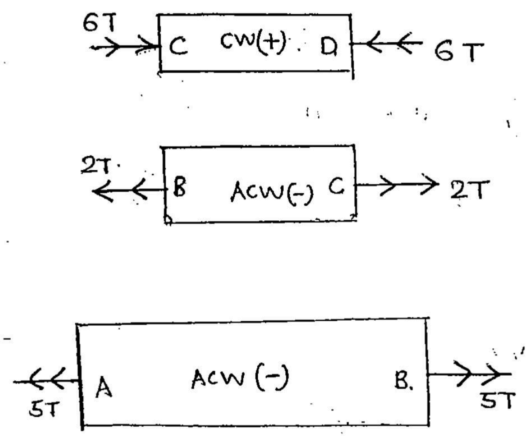
1. Series.



$\theta_A = 0$

θ @ free end = ?

$\theta_c = ?$



$$\theta = \frac{TL}{GJ}.$$

$$\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}.$$

$$\theta_D - \theta_A = \theta_{AB} + \theta_{BC} + \theta_{CD}.$$

$$\theta_D - 0 = \frac{-5T \times 2L}{3GJ} + \frac{-2T \times L}{2GJ} + \frac{6TL}{GJ}.$$

$$\theta_D = \theta_{max} \text{ @ free end} = \underline{\underline{\frac{5TL}{3GJ}}} \text{ (CW)}.$$

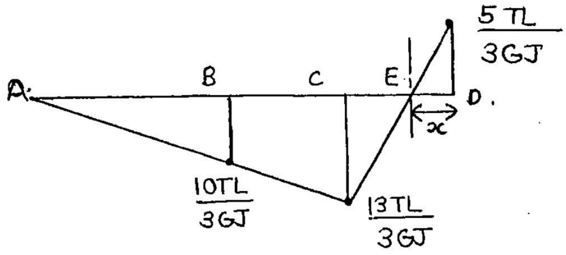
$$\theta_{AC} = \theta_{AB} + \theta_{BC},$$

$$\theta_C - \theta_A = \frac{-5T \times 2L}{3GJ} + \frac{-2T \times L}{2GJ}.$$

$$\therefore \theta_C = \underline{\underline{\frac{-13TL}{3GJ}}} \text{ (ACW)}$$

OR

$$\theta_{CD} = \frac{(6T)L}{GJ}$$



$$\theta_D - \theta_c = \frac{6TL}{GJ}$$

$$\frac{5TL}{3GJ} - \theta_c = \frac{6TL}{GJ} \Rightarrow \theta_c = \frac{-13TL}{3GJ}$$

$$\theta_{AB} = \theta_B - \theta_A.$$

$$\frac{-10TL}{3GJ} = \theta_B \quad (ACW).$$

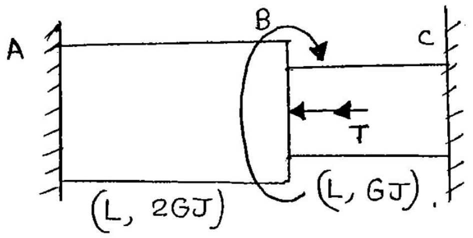
$$\frac{ED}{5/3} = \frac{CE}{13/3}.$$

$$\frac{x}{5} = \frac{l-x}{13}.$$

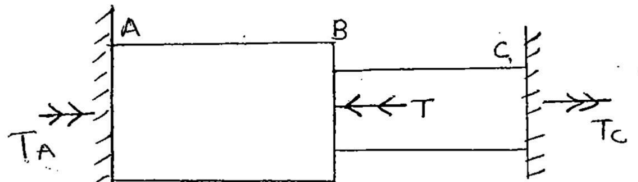
$$\Rightarrow x = \frac{5l}{18} \left\{\text{from free end } D\right\}$$

2. Parallel.

Q.



$T_A = ?$; $T_C = ?$; $\theta_B = ?$

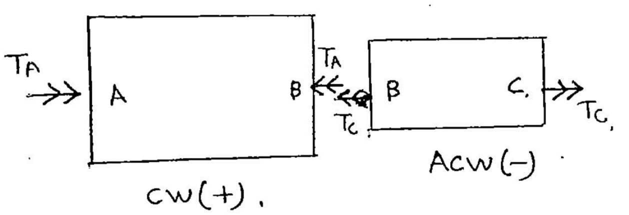


Equilibrium equation;

$$T_A + T_C = T.$$

Compatibility condition,

$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$



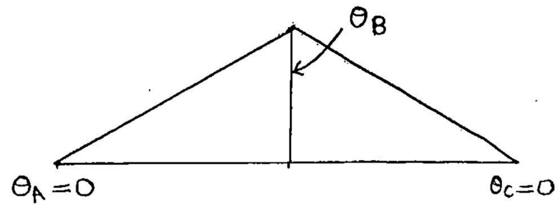
$$\cancel{\theta_C}^{0} - \cancel{\theta_A}^{0} = \theta_{AB} + \theta_{BC}.$$

$$\Rightarrow \theta_{AB} + \theta_{BC} = 0.$$

$$0 = \frac{T_A L}{2GJ} + \frac{-T_C L}{GJ}.$$

$$T_A = 2T_C,$$

$$\Rightarrow \quad T_C = \frac{T}{3} \quad \& \quad T_A = \frac{2T}{3}$$

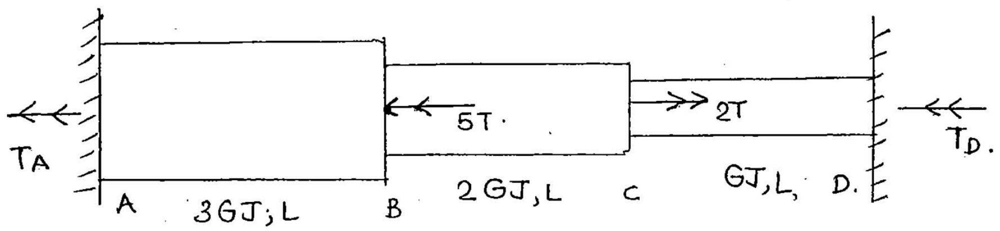


$$\theta_{AB} = \theta_B - \theta_A.$$

$$\frac{T_A L}{2GJ} = \theta_B - 0.$$

$$\Rightarrow \quad \theta_B = \frac{TL}{3GJ} \quad (CW).$$

Q.



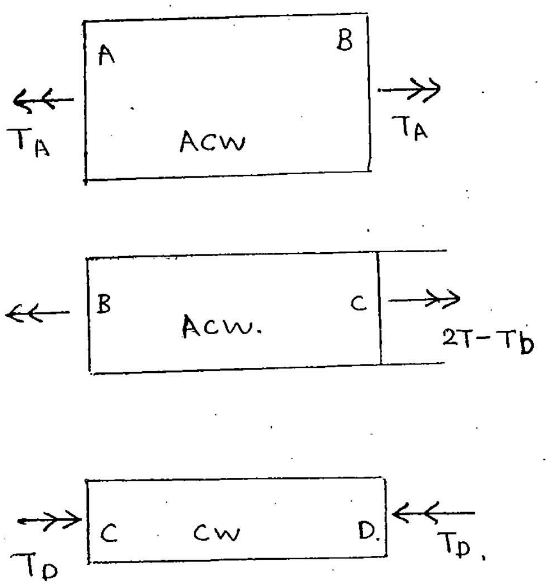
Compatibility condition :

$$\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}$$

$$0 = -\frac{T_A L}{3GJ} - \frac{(2T - T_D)}{2GJ} + \frac{T_D L}{GJ}.$$

$$-\frac{T_A}{3} - T + \frac{3T_D}{2} = 0.$$

$$-2T_A + 9T_D = 6T.$$



Equilibrium condition :

$$T_A + T_D + 5T = 2T,$$

$$T_A + T_D = -3T.$$

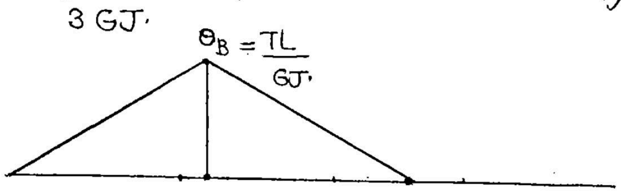
$$T_A = -3T \quad \& \quad T_D = 0,$$

$$\therefore \quad T_A = 3T \ (CW) \quad \& \quad T_D = 0$$

(68)

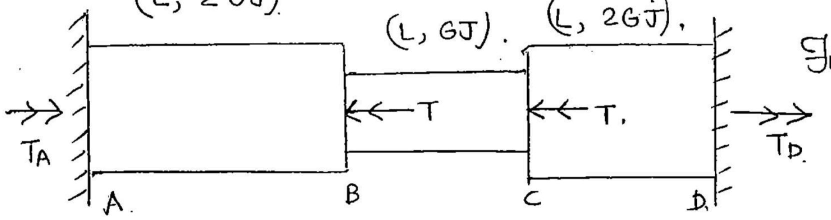
$$\theta_{AB} = \theta_B - \cancel{\theta_A}^{\,0} = \frac{T_A \cdot L}{3\,GJ} = \frac{3TL}{3\,GJ}$$

$$\theta_B = \frac{TL}{GJ}$$

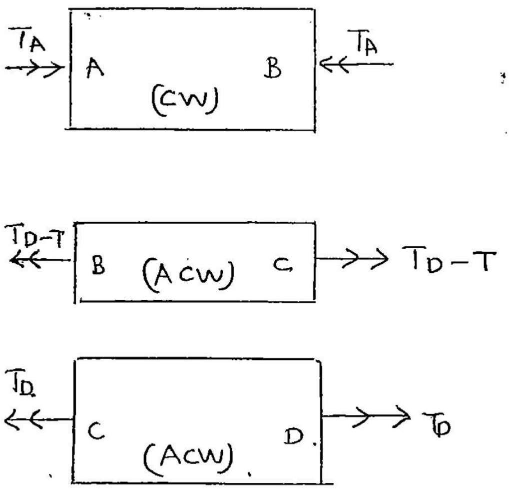


$$\theta_{CD} = \cancel{\theta_D}^{\,0} - \theta_C = \frac{-T_D L}{GJ} = \underline{\underline{0}}$$

$$\therefore \theta_C = 0$$

Q. 

Find torsion in BC ?



$$T_A + T_D = 2T.$$

$$0 = \frac{T_A \times L}{2GJ} - \frac{(T_D - T)L}{GJ} + \frac{-T_D \times L}{2GJ}$$

$$\frac{T_A}{2} - \frac{3T_D}{2} = -T.$$

$$\Rightarrow T_A = T$$

$$T_D = T$$

Tension in BC, $T_{BC} = T_D - T = T - T = \underline{\underline{0}}$

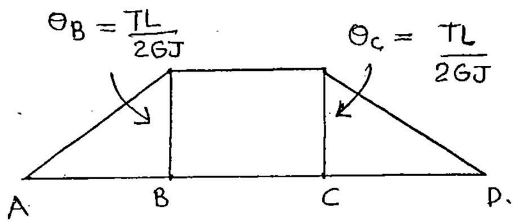
$$\theta_{AB} = \theta_B - \theta_A.$$

$$\theta_B = \frac{T_A L}{2GJ} = \frac{TL}{2GJ}$$

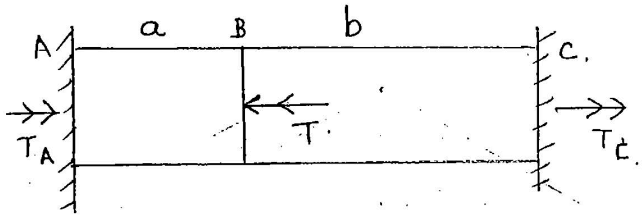
$$\theta_{CD} = \theta_D - \theta_C.$$

$$\frac{-T_D \times L}{2\,GJ} = -\theta_C.$$

$$\Rightarrow \theta_C = \frac{T \cdot L}{2GJ}$$



Q



$$T_A + T_C = T.$$

$$\theta_{AC} = \theta_{AB} + \theta_{BC},$$

$$0 = \frac{T_A\, a}{GJ} + \frac{T_B \cdot b}{GJ}.$$

$$a T_A = -b T_B.$$

$$T_A = \frac{Tb}{l},$$

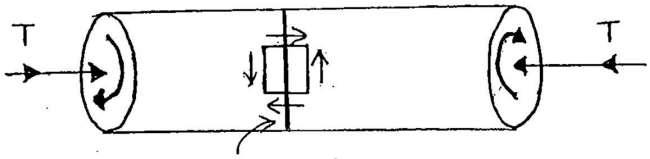
$$T_C = \frac{Ta}{l}$$

→ Failure Criteria.

1. Ductile Shaft.

Weak in shear.

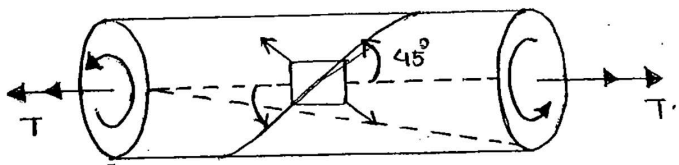
No failure in horizontal direction due to large area to resist the shear (length * diameter). So failure occurs as a vertical cut.



vertical cut (normal to axis). {for cw & ACW T}

2. Brittle Shaft. (CI, glass).

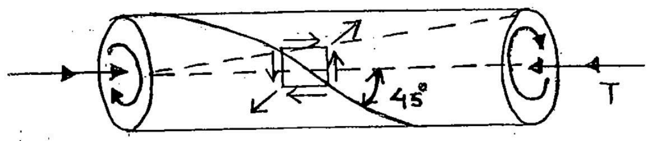
Weak in tension.



[FIX]

ACW torsion is applied

(45° ACW crack with axis)

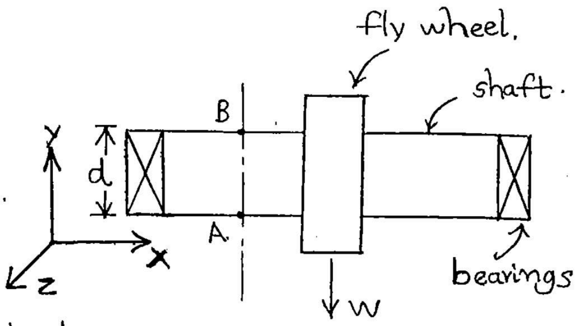


[FIX]

CW (+)

(45° cw cracks with axis)

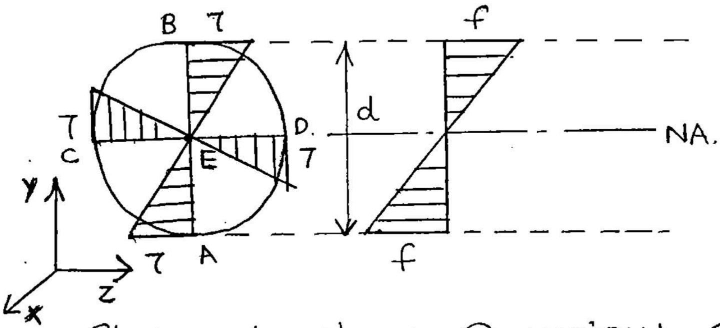
→ Combined Stresses.



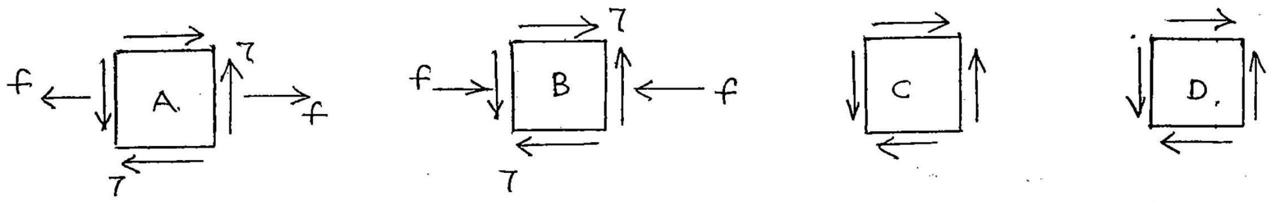
- Usually rotating shafts are subjected to torsion, BM & SF.

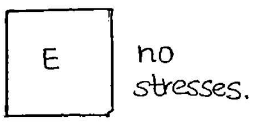
At the point of max. BM, SF is zero. ∴ the shaft must be designed for the combined effect of bending and torsion.

Assume diameter of shaft is **d**.



State of stress @ various points :-





The critical elements for the design of shaft are A and B

Now consider element A.

$$\sigma_x = f = \frac{M}{Z} \quad ; \quad \tau_{xy} = \tau = \frac{T}{Z_p}.$$

$$\sigma_y = 0$$

$$\sigma_x = \frac{M}{\frac{\pi d^3}{32}} = \frac{32M}{\pi d^3}.$$

$$\tau_{xy} = \frac{T}{\frac{\pi d^3}{16}} = \frac{16T}{\pi d^3}$$

⊙ Design is based on Principal Stresses:

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

$$= \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2+\tau^2}$$

$$= \frac{16M}{\pi d^3} \pm \sqrt{\left(\frac{16M}{\pi d^3}\right)^2+\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2+T^2}\right]$$

$$\boxed{\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}}$$

In any member subjected to bending action, major and minor principal stresses will be opposite in nature. Intermediate principal stress $= 0$ $(\sigma_2 = 0)$.

* Equivalent BM $= M_e = \dfrac{M+\sqrt{M^2+T^2}}{2}$

* Equivalent torsion, $T_e = \sqrt{M^2+T^2}$

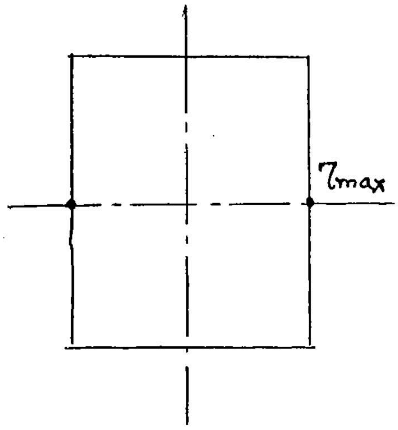
⊙ For a shaft, M & T act together to produce principal stress σ_1. But the equivalent moment, M_e, alone can produce the same value of σ_1 on the shaft.

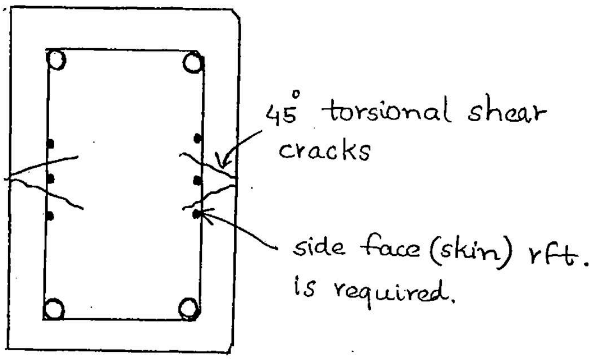
⊙ Similarly, M & T act together to produce max. shear stress, τ_{max}. But the equivalent torsion, T_e, alone can produce the same value of τ_{max} on the shaft.

25th Oct,
SATURDAY

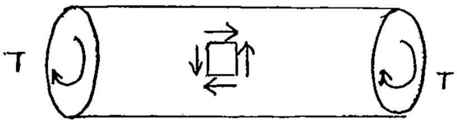
Q. 58.

9.





Deep beam (D > 750 mm).
[Torsion develops.]

11. 

For element, $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \underline{\underline{+\tau}}$$

For element on surface subjected to pure shear, $\sigma_1 = +\tau$
$\sigma_3 = -\tau$

14. $\sigma_1 = \tau = \frac{16T}{\pi d^3}$

13. In the c/s, no stresses.

8. $P = 2\pi N T$.

$452.8 \times 0.746 = 2\pi \times 2\ T$

$T = 26.89$ kNm.

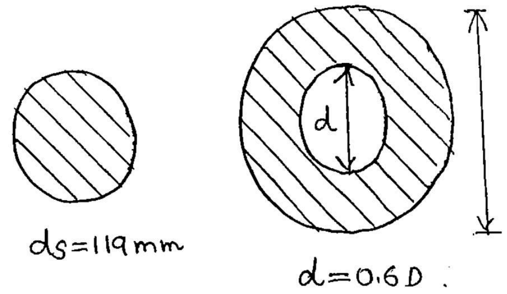
$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{Z_p} = \frac{T}{\frac{\pi}{16} d^3}$$

$$80 = \frac{16T}{\pi d^3} = \frac{16(26.89 \times 10^3)}{\pi d^3}$$

$d = \underline{\underline{119 \text{ mm}}}$

Replaced hollow shaft should transfer same torsion.



$$\tau_s = \tau_h.$$

$$\left(\frac{T}{Z_P}\right)_s = \left(\frac{T}{Z_P}\right)_h.$$

$$(Z_P)_h = (Z_P)_s.$$

$$\frac{\pi}{16D}(D^4 - d^4) = \frac{\pi}{16} d_s^3.$$

$$\frac{D^4 - (0.6D)^4}{D} = 119^3.$$

Outer diameter of hollow shaft, $D = \underline{\underline{124.635}}$ mm

Weight, $W = \gamma A l$.

For both the shafts, 'l' & 'γ' must be same.

$$\Rightarrow \quad W \propto A.$$

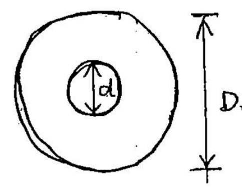
$$\frac{W_h}{W_s} = \frac{\frac{\pi}{4}(D^2 - d^2)}{\frac{\pi}{4} \times d_s^2} = \frac{D^2(1 - 0.6^2)}{119^2} = 0.702$$

$$W_h = 0.702\ W_s.$$

$\Rightarrow$ 30% savings in weight when solid shaft replaced by hollow shaft.

P-56 → Comparison of Hollow & Solid shaft:

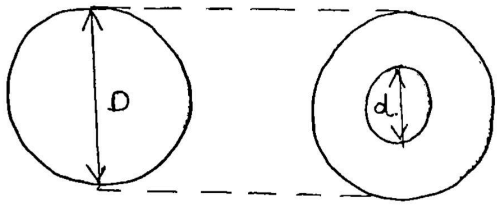
1. Areas are equal.



$$A_s = A_h \Rightarrow W_s = W_h.$$

$$\frac{T_h}{T_s} = \frac{P_h}{P_s} = \frac{(\text{Strength})_h}{(\text{Strength})_s} = \frac{(Z_P)_h}{(Z_P)_s} = \frac{1+K^2}{\sqrt{1-K^2}} \qquad K = \frac{d}{D}$$

2.



$$\frac{T_h}{T_s} = \frac{P_h}{P_s} = \frac{(\text{strength})_h}{(\text{strength})_s} = \frac{(Z_p)_h}{(Z_p)_s} = \underline{\underline{1 - K^4}}$$

3. Solid and hollow shaft of equal strength

$$T_h = T_s$$

$$P_h = P_s$$

$$(Str)_h = (Str)_s$$

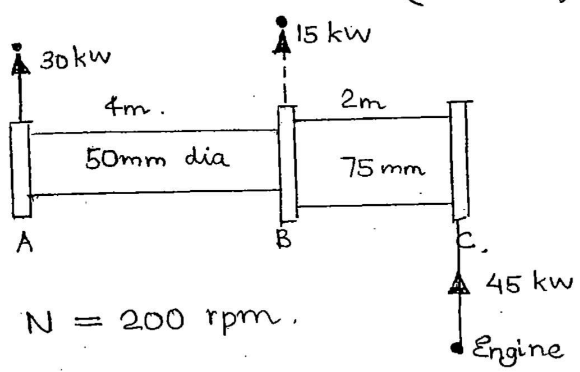
$$(Z_p)_h = (Z_p)_s$$

$$\Rightarrow \boxed{\frac{W_h}{W_s} = \frac{A_h}{A_s} = \frac{1-K^2}{(1-K^4)^{2/3}}}$$

8. $\frac{W_h}{W_s} = ?$ $\quad K = \frac{d}{D} = 0.6.$

$$\frac{W_h}{W_s} = \frac{1-0.6^2}{(1-(0.6)^4)^{2/3}} = \underline{\underline{0.702}}$$

09.



N = 200 rpm.

$P_{AB} = 30$ kw. $\quad P_{BC} = 45$ kw.

Shaft AB :

$$P = \frac{2\pi NT}{60} \Rightarrow 30 \times 1000 = \frac{2\pi \times 200\,(T)}{60}$$

$$T_{AB} = 1.43\ \text{kNm}.$$

$$\text{lly}\quad T_{BC} = 2.15\ \text{kNm}.$$

$$\tau_{AB} = \frac{16\,T_{AB}}{\pi d_{AB}^3} = \frac{16\times 1.43\times 10^6}{\pi\times 50^3} = \underline{\underline{58.3\ \text{MPa}}}$$

$$\tau_{BC} = \frac{16\,T_{BC}}{\pi d_{BC}^3} = \frac{16\times 2.15}{\pi\times 75^3} = \underline{\underline{25.9\ \text{MPa}}}$$

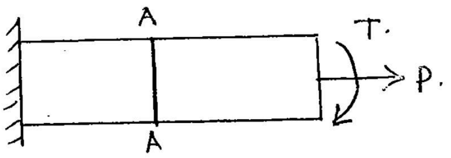
$$\tau_{max} = \tau_{AB} = \underline{\underline{58.2}}\ \text{MPa}.\ (\text{max}).$$

10. $$\theta_{AC} = \theta_{AB} + \theta_{AC}.$$

$$= \frac{1.43\times 10^6\times 4000}{8.5\times 10^4\times \frac{\pi}{32}(50^4)} + \frac{2.15\times 10^6\times 2000}{8.5\times 10^4\times \frac{\pi}{32}\times 75^4} = \underline{\underline{0.126}}\ \text{rad}$$

$$= \underline{\underline{7.14^\circ}}$$

11.



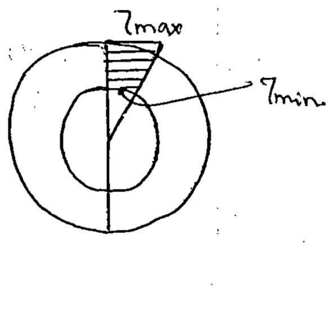
$$\sigma = \frac{P}{A} = \text{const.}$$

$$\tau = \frac{16T}{\pi d^3} = \text{const.}$$

∴ Both normal and shear stress are continuous at every section.

13. $$\tau_{max} = \frac{T}{J}\,r_{max} = \frac{100\times 10^3}{\frac{\pi}{32}(30^4 - 26^4)} \times \frac{30}{2}$$

$$= \underline{\underline{43.27}}\ \text{MPa}$$



$$\tau_{min} = \frac{T}{J}\,r_{min} = \frac{100\times 10^3}{\frac{\pi}{32}(30^4 - 26^4)} \times \frac{26}{2} = \underline{\underline{37.5\ \text{MPa}}}$$

4th Oct,
SATURDAY

09 THIN CYLINDERS

Pressure Vessels.

Thin.

$\boxed{t \leq \frac{D}{20}}$

Cylinders. Spheres.

Eg: storage tanks.
boilers.
LPG cylinder.
Balloons.

Thick.

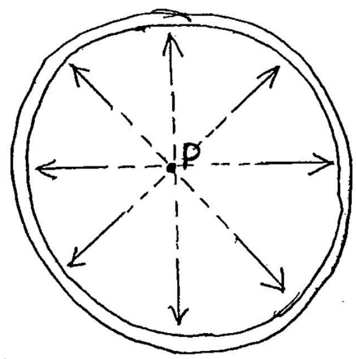
$\boxed{t > \frac{D}{20}}$

Cylinders Spheres.

Eg: Nozzles
Jets.

Warheads.

THIN CYLINDERS



- Applied fluid pressure is radial in any cylinder or sphere.

  P = internal applied pressure due to fluids inside.

→ Stresses Developed.

1. Hoop / Circumferential.

$$\sigma_h = \frac{PD}{2t} \text{ (tension)}$$

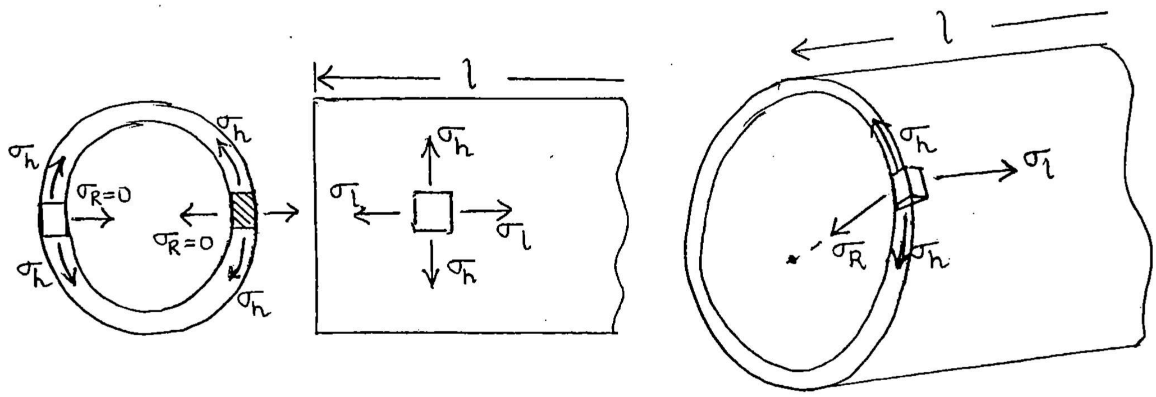
2. Longitudinal / Axial.

$$\sigma_l = \frac{\sigma_h}{2} = \frac{PD}{4t} \text{ (tension)}.$$

3. Radial Stress

$$\sigma_R = 0.$$

⊙ The thin cylinder cannot resist the stress in the radial or thickness direction, even though the applied pressure is in radial direction. ∴ such members can be called as Plane stress members.



→ Principal Stresses.

$$\sigma_1 = \sigma_h$$
$$\sigma_2 = \sigma_l$$
$$\sigma_3 = \sigma_R = 0$$

* Max. Shear stress., $\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2}$

$$= \frac{\sigma_h - \sigma_R}{2} = \frac{\sigma_h}{2} = \sigma_l.$$

$$\boxed{\tau_{max} = \frac{\sigma_h}{2} = \sigma_l = \frac{PD}{4t}}$$

⊙ Max shear stress acts on a cross sectional plane where σ_h and σ_R are acting.

→ Strains

1. Hoop Strain (ϵ_h)

$$\epsilon_h = \frac{\delta D}{D} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}.$$

$$\boxed{\epsilon_h = \frac{\sigma_h}{E} - \frac{\mu\sigma_l}{E}.}$$

2. Longitudinal Strain. (ϵ_l)

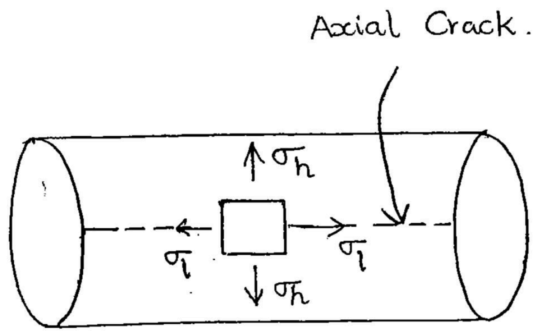
$$\epsilon_l = \frac{dl}{l} = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_3}{E}.$$

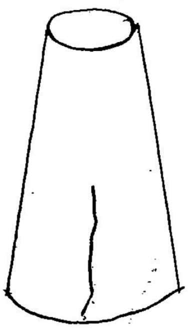
$$\boxed{\epsilon_l = \frac{\sigma_l}{E} - \mu\frac{\sigma_h}{E}}$$

$$\boxed{\epsilon_v = \frac{dV}{V} = \epsilon_l + 2\epsilon_h.}$$

→ Failure Criteria.

For both thin and thick cylinders, axial cracks will be formed.





P = const.
t = const.

$$\sigma_h = \frac{PD}{2t}. \Rightarrow \sigma_h \propto D.$$

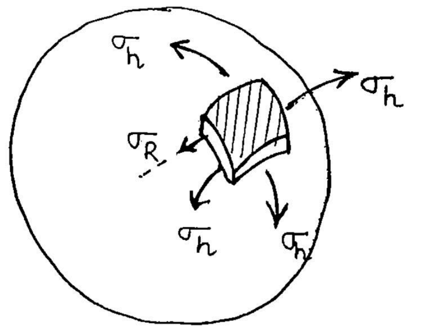
Max. hoop stress develops at bottom ∴ cracks are first formed at bottom and propogated towards top.

∴ chimneys are provided with, thicker plates at the bottom and thinner ones at the top.

THIN SPHERES

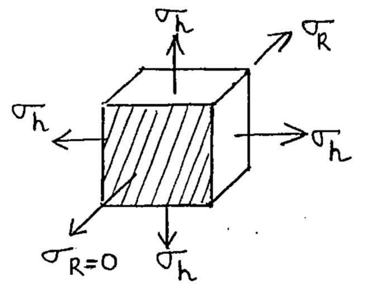
- Hoop Stress (σ_h)

$$\sigma_h = \frac{PD}{4t} \text{ (tension)}$$



- Radial Stress (σ_R)

$$\sigma_R = 0$$



▫ State of stress

→ Principal Stresses

$$\sigma_1 = \sigma_h$$

$$\sigma_2 = \sigma_h$$

$$\sigma_3 = \sigma_R = 0.$$

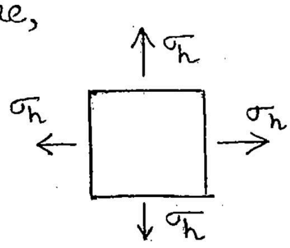
* Max. Shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_h - \sigma_R}{2}$

$$= \frac{\sigma_h}{2} = \frac{PD}{8t}.$$

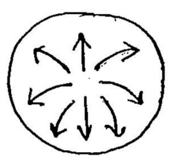
$$\boxed{\tau_{max} = \frac{\sigma_h}{2} = \frac{PD}{8t}.}$$; acts in the c/s.

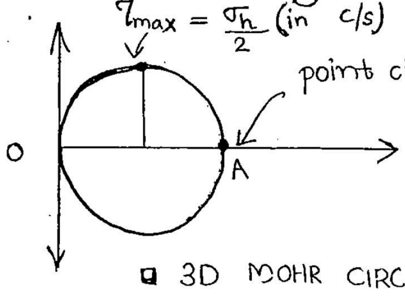
* Shear stress on the surface of thin sphere,

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_h - \sigma_h}{2} = 0$$



On the surface of thin sphere, there is no shear stress.
In all directions, there will be only hoop stress.





$\sigma_1 = \sigma_h = OA$

$\sigma_2 = \sigma_h = OA.$

$\sigma_3 = \sigma_R = 0.$

▫ 3D MOHR CIRCLE

On the surface of a thin sphere, only normal hoop stress is acting in all directions causing isotropic condition. ∴ no shear stress on the surface and only hoop stress in all directions.

→ Strains.

1. Hoop Strain.

$$\epsilon_h = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_h = \frac{\sigma_h}{E} - \mu\frac{\sigma_h}{E}.$$

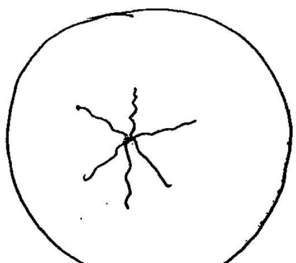
$$\boxed{\epsilon_h = \frac{\sigma_h(1-\mu)}{E}.}$$

2. Volumetric Strain.

$$\boxed{\frac{\partial V}{V} = \epsilon_V = 3\epsilon_h.}$$

→ Failure Criteria.

For a thin sphere, cracks develops in all directions from a weak point.



-73.

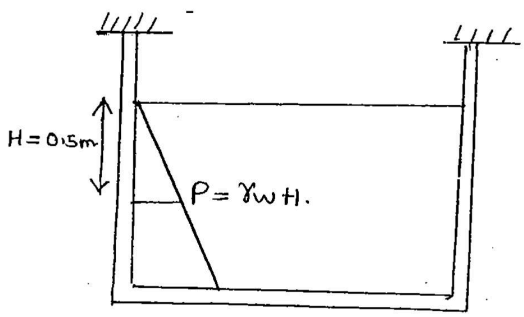
5.

$P = \gamma_w H.$

$= 10\ kN/m^3 \times 0.5m.$

$= 5\ kN/m^2 = \frac{5\times10^3}{10^6}$

$= \underline{\underline{5\times10^{-3}\ MPa.}}$



Given $D = 1000\ mm,\ t = 1\ mm$

$$\sigma_h = \frac{PD}{2t} = \frac{5\times10^{-3}\times 1000}{2\times 1000\ 1} = 2.5\ \text{MPa}.$$

$$\sigma_l = \frac{PD}{4t} = \underline{\underline{1.25\ \text{MPa}}}$$

06. $$\epsilon_h = \frac{\sigma_h}{E} - \mu\frac{\sigma_l}{E}$$

$$= \frac{2.5}{100\times10^3} - \frac{0.3\times1.25}{100\times10^3} = \underline{\underline{2.125\times10^{-5}}}$$

$$\epsilon_l = \frac{\sigma_l}{E} - \mu\frac{\sigma_h}{E} = \frac{1.25-0.3\times2.5}{100\times10^3} = \underline{\underline{0.5\times10^{-5}}}$$

03. $$\partial v = 50\ \text{cc}$$

$$= 50\times10^3\ \text{mm}^3.$$

$\frac{d_v}{v} = 1.546\times10^{-3} =$

$K =$

257.66

$$\sigma_h = \frac{PD}{4t} = \frac{P\times800}{4\times4} = 50P.$$

$$\epsilon_h = \frac{\sigma_h}{E} - \mu\frac{\sigma_h}{E} = \frac{50P(1-0.3)}{2\times10^5} =$$

$$\epsilon_v = 3\epsilon_h$$

$$\frac{dv}{v} = 3\epsilon_h.$$

$$\frac{50\times10^3}{\frac{4}{3}\pi\times\left(\frac{800}{2}\right)^3} = 3\left(\frac{50P}{2\times10^5}(1-0.3)\right)$$

$$P = 0.355\ \text{MPa}$$

04. $$\sigma_h = 50P = 50\times0.355$$

$$= \underline{\underline{17.75\ \text{MPa}}}$$

25th Oct,
SATURDAY

10. COLUMNS & STRUTS

COLUMNS

→ Classification :

1. Short Columns :-

   – Fails suddenly by crushing

$$P_c = \sigma A$$

P_c → crushing load / ultimate load on the column.

A → c/s area.

σ → allowable stress on the column.

$$\text{Safe load, } P = \frac{P_c}{FOS.}$$

* RCC (LSM)

$$P_u = P_c + P_{sc}$$
$$= (0.4\, f_{ck})(A_c) + (0.67\, f_y)(A_{sc}).$$

$$\text{Safe or working load, } P = \frac{P_u}{FOS.}$$

2. Long Columns / Slender Columns.

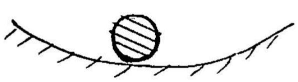
   – Fails gradually by buckling.

* Equilibrium Conditions (Stability conditions).

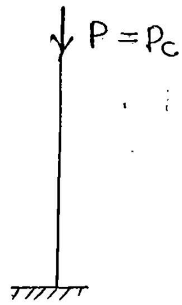
Stability is an important factor for a column in the design.

(i) Stable Equilibrium Condition.

$P < P_c$

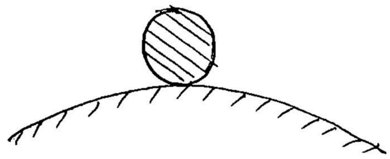


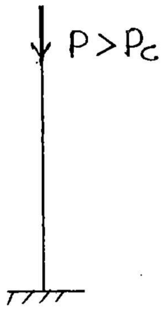
(ii) Neutral Equilibrium Condition.



It is the condition just before failure.

(iii) Unstable Equilibrium.





This condition is not possible over any member.

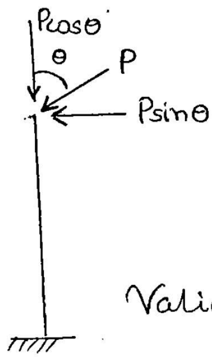
Stable equilibrium is the best condition for design.

→ Euler's Theory

- only for <u>long columns</u>

* Assumptions :-

(i)



$P\cos\theta$: causes buckling

$P\sin\theta$: causes bending.

Valid only for vertical loads.

(ii) $l >>> b$ & D (long columns)

Length of column very much greater than lateral dimensions.

(iii) $\sigma = \frac{P}{A}$ → causes crushing

$f = \frac{M}{Z}$ → causes buckling.

$\sigma <<< f$; only buckling in long columns.

(iv) Self wt. is ignored.

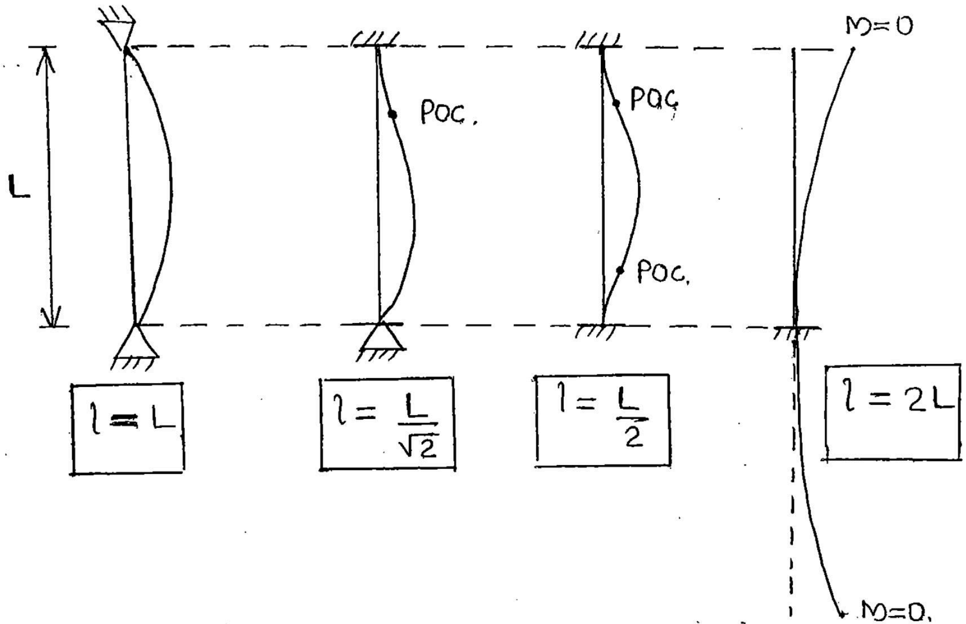
$$\boxed{P_e = \frac{\pi^2 EI}{l^2}}$$

where $I = I_{min}$; minimum moment of inertia.

l = effective length. (distance b/w two successive zero BM points)

Zero BM points may be hinges, rollers, free ends, point of contraflexures etc.

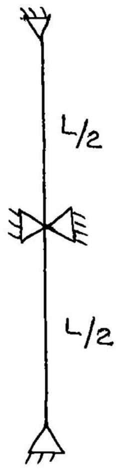
* Safe Load, $P = \frac{P_e}{FOS.}$



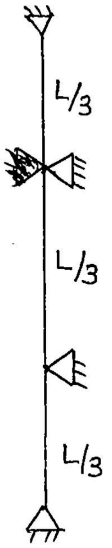
$$P_e \propto \frac{1}{l^2}$$

$$\frac{P_{fix-free}}{P_{fix-fix}} = \left(\frac{l_{fix-fix}}{l_{fix-free}}\right)^2 = \left(\frac{L/2}{2L}\right)^2 = \underline{\underline{\frac{1}{16}}}$$

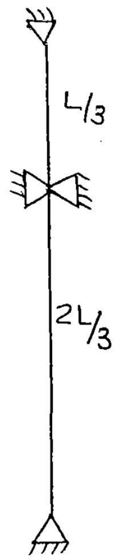
27th Oct, Special Cases:
MONDAY



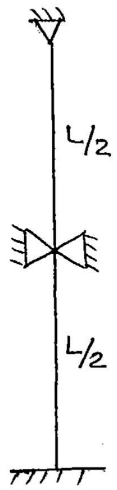
$l = \frac{L}{2}$



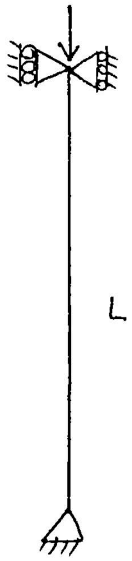
$l = \frac{L}{3}$



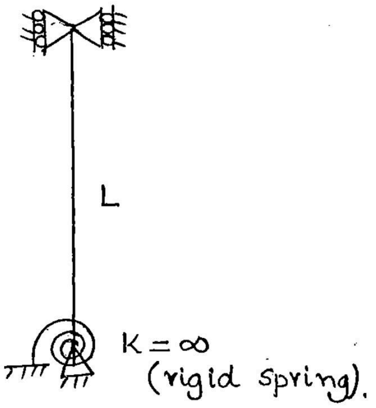
$l = \frac{2L}{3}$ (max of 'l' value)



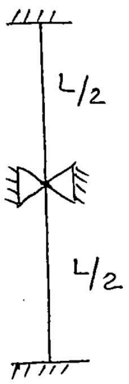
$l = \frac{L}{2}$



$l = L$



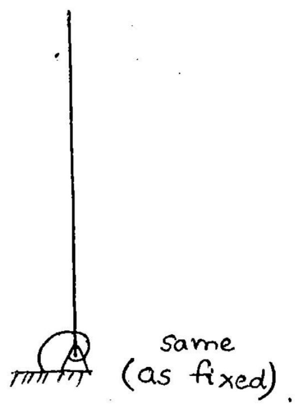
$l = \frac{L}{\sqrt{2}}$



$l = \frac{L}{2\sqrt{2}}$

$l = \infty$

($P = 0$
$\Rightarrow l = \infty$).



$l = 2L$

* Slenderness Ratio (λ)

$$\frac{P_e}{A} = \frac{\pi^2}{l^2} E \frac{I_{min}}{A}.$$

$$\sigma_e = \frac{\pi^2}{l^2} E (r)^2$$

$$\sigma_e = \frac{\pi^2 E}{(l/r)^2} = \frac{\pi^2 E}{\lambda^2}$$

$\lambda \rightarrow$ slenderness ratio.

From the above equation, λ at which short column changes to a long column can be assessed. If strength of material and E are known.

Eg: For mild steel, $\sigma_e = f_y = 250$ MPa

$E = 2 \times 10^5$ MPa.

$$\sigma_e = \frac{\pi^2 E}{\lambda^2} \Rightarrow \lambda = 88 \text{ (limiting slenderness ratio)}.$$

ie $\lambda \leq 88 \Rightarrow$ Short Column (Euler's theory invalid)

$\lambda > 88 \Rightarrow$ Long column. (Euler's theory valid).

For M20 grade concrete,

$\sigma_e = f_{ck} = 20$ MPa.

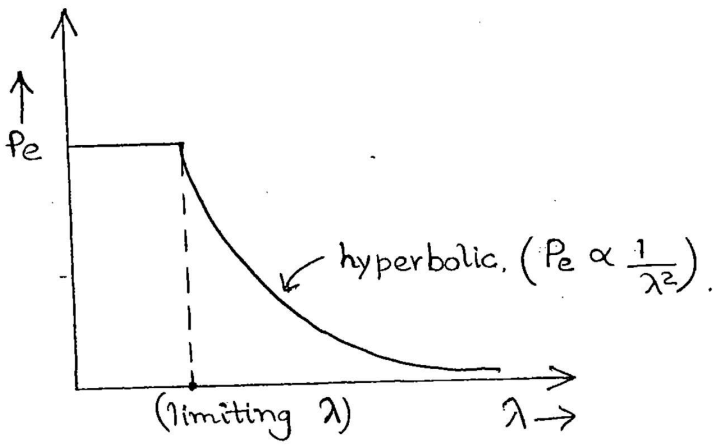
$E_c = 5000\sqrt{20}$

$\lambda = 105$

$\lambda \leq 105 \Rightarrow$ short column.

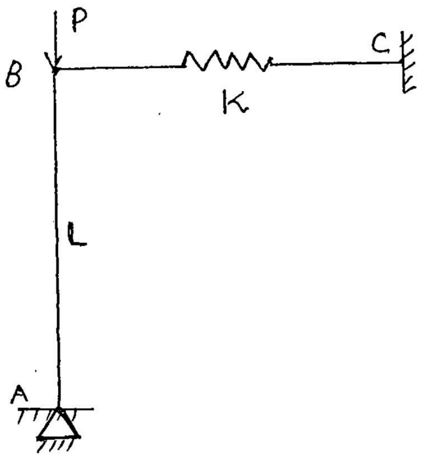
$\lambda > 105 \Rightarrow$ long column.

* Failure Envelope.



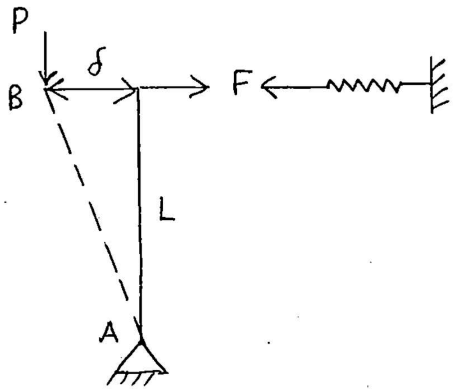
Load carrying capacity of short columns remains the same upto limiting λ. But for long columns, as λ increases, P_e decreases. $\therefore$ short columns are always preferred.

→ Unstable Struts Connected by Springs.

Q. 

A strut AB is hinged at A and connected by a spring at B of stiffness K. Calculate load P at collapse.

For the members shown in fig, euler's formula is not valid
Use eqbm conditions.



$$K = \frac{F}{\delta}$$

$$F = K\delta$$

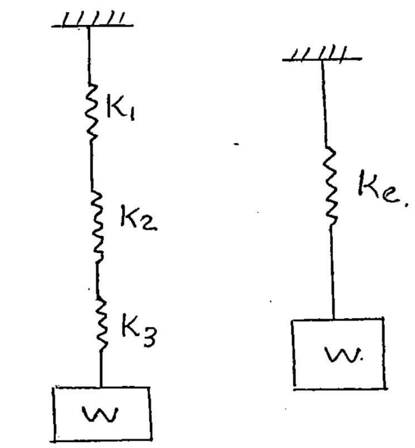
$\Sigma M_A = 0$ (FBD of AB).

$$F \times L = P \times \delta$$

$$K\delta \times L = P \times \delta$$

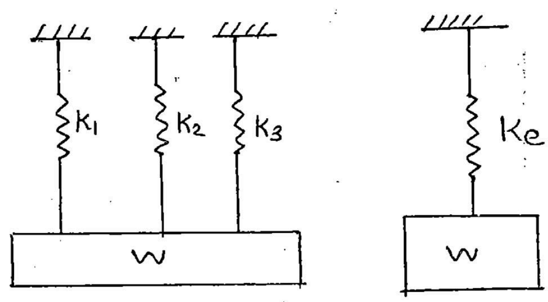
$$\Rightarrow \underline{\underline{P = KL}}$$

* Springs in Series.



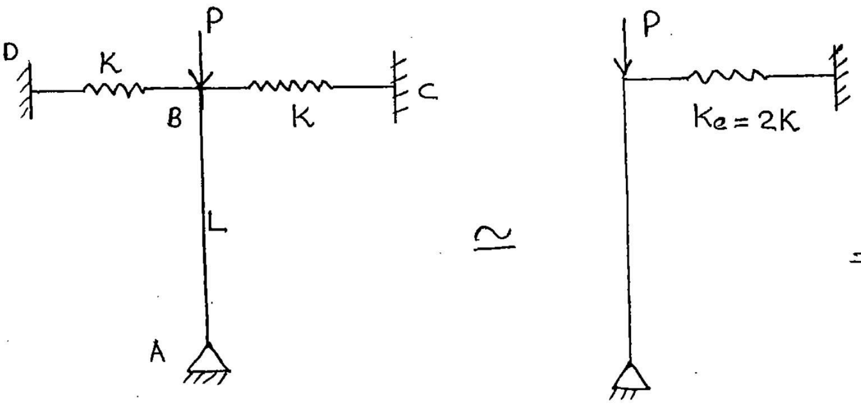
$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

* Springs in Parallel.



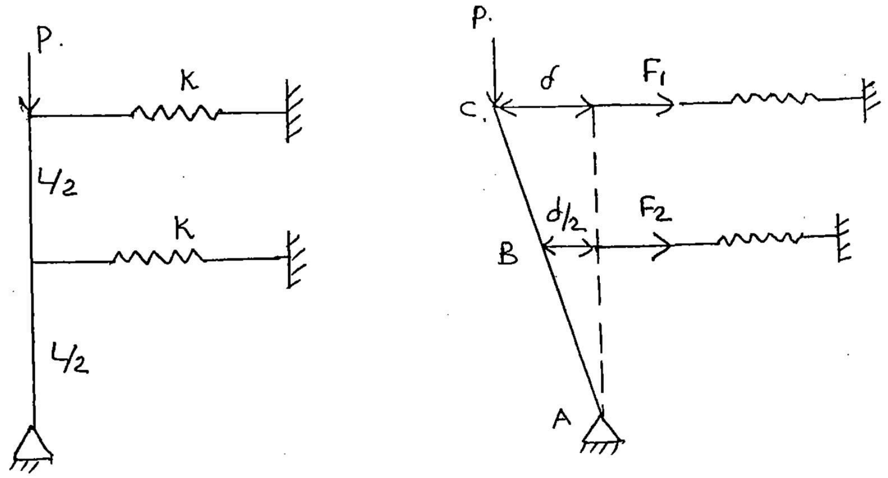
$$K_e = K_1 + K_2 + K_3$$

Q.



$\Rightarrow P = 2KL$

Q.



$F \times \delta \perp$

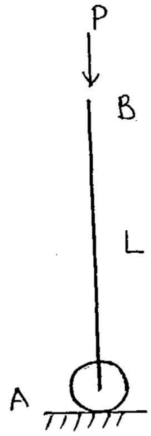
$\Sigma M_A = 0.$

$F_1 \times L + F_2 \times \frac{L}{2} = P \times \delta.$

$L \times K\delta + K\frac{\delta}{2} \times \frac{L}{2} = P\delta.$

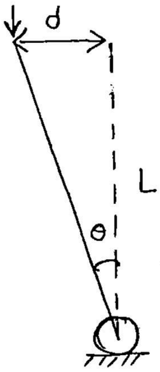
$$P = \frac{5KL}{4}$$

Q.



A strut AB is held by a torsion spring of stiffness (K_T) at A and subjected to axial force of P at B. Determine load at collapse

Torsional stiffness = torsion required to produce unit angular twist in radians.



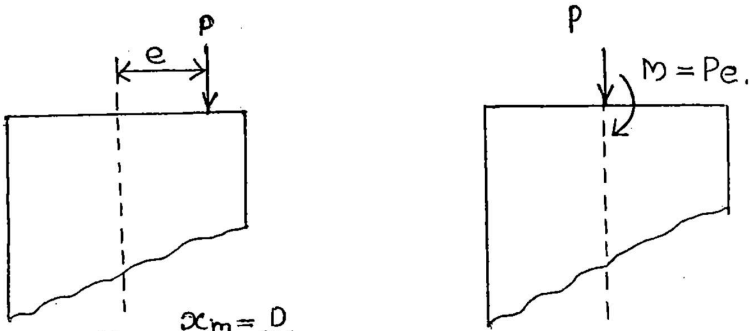
$$P\delta = T \qquad K_T = \frac{T}{\theta}$$

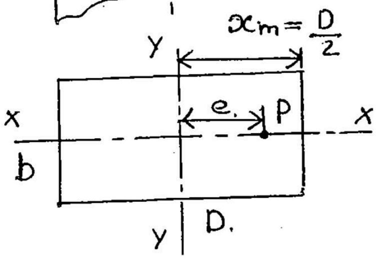
$$P\delta = K_T \frac{\delta}{L} \qquad T = K_T\theta.$$

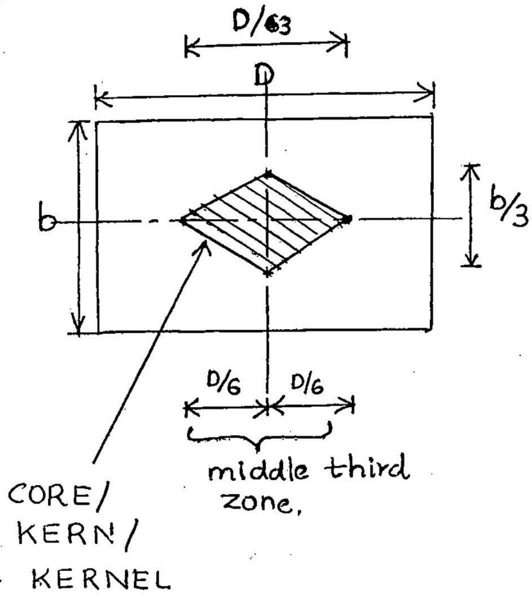
$$\Rightarrow \quad P = \frac{K_T}{L} \qquad \tan\theta = \theta = \frac{\delta}{L}.$$

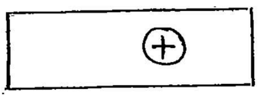
→ Short Column with Eccentric Load.

- combined stresses :- axial + bending stresses.

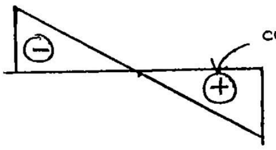








Direct Stress $\quad \sigma_D = \frac{P}{A}$



compression on load side.

Bending Stress $\quad f = \frac{M}{I}y = \frac{M}{Z}$

Load on x-axis :

$$\sigma_R = \sigma_D \pm f$$

$$= \frac{P}{A} \pm \frac{M}{I_y}(x_{max}).$$

$$= \frac{P}{A} \pm \frac{M}{Z_y}$$

$$I_y = \frac{bD^3}{12} \quad ; \quad x_{max} = \frac{D}{2} \Rightarrow Z_y = \frac{bD^2}{6}$$

If eccentricity of loading is along x-axis, the c/s bends wrt y-axis. ∴ I_y should be used. From y axis, extreme fibre distance is $x_{max} = \frac{D}{2}$.

In general, columns are made of brittle material which may fail suddenly if tension develops. ∴ the eccentricity of the load can be limited to have no tension.

For no tension,

$$\sigma_R = 0.$$

$$\Rightarrow \frac{P}{A} - \frac{Pe}{Z_y} = 0.$$

$$\frac{P}{bd} - \frac{Pe}{\frac{bd^2}{6}} = 0.$$

$$\Rightarrow e = \frac{d}{6}$$

* Limiting (max) eccentricity for no tension,

$\boxed{e_{max} = \frac{d}{6}.}$ (eccentricity along x-axis).

$\boxed{e_{max} = \frac{b}{6}}$ (eccentricity along y-axis).

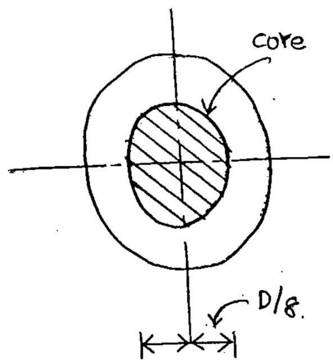
Middle Third Rule:

As long as the column load is in the middle third zone, there is no tension to column c/s. This is applicable for square & rectangle only.

Area of core, $A_c = 2\left\{\frac{1}{2} \times \frac{D}{6} \times \frac{B}{3}\right\} = \underline{\underline{\frac{BD}{18}}}$

$$A_c = \underline{\underline{\frac{A_g}{18}}} \quad (5.55\,\%)$$

* For circular section :



For no tension,

$$\sigma_{min} = 0 = \frac{P}{A} - \frac{Pe}{Z}.$$

$$\frac{P}{\frac{\pi}{4}D^2} - \frac{Pe}{\frac{\pi D^3}{32}} = 0.$$

$$\boxed{e_{max} = \frac{D}{8}}$$

In case of solid circular section, middle fourth rule is applicable for no tension in c/s.

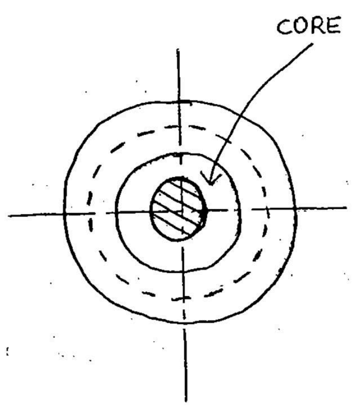
$$\left.\begin{matrix}\text{Area of}\\ \text{core}\end{matrix}\right\}, \; A_c = \frac{\pi}{4}\left(\frac{D}{4}\right)^2 = \frac{1}{16}\left(\frac{\pi}{4}D^2\right).$$

$$\Rightarrow A_c = \underline{\underline{\frac{A_g}{16}}} \; (6.25\%)$$

⊙ For isolated columns subjected to loading better to have circular cross sections.

⊙ For columns in a framed structure with heavy moment due to unequal spans, better is rectangle.

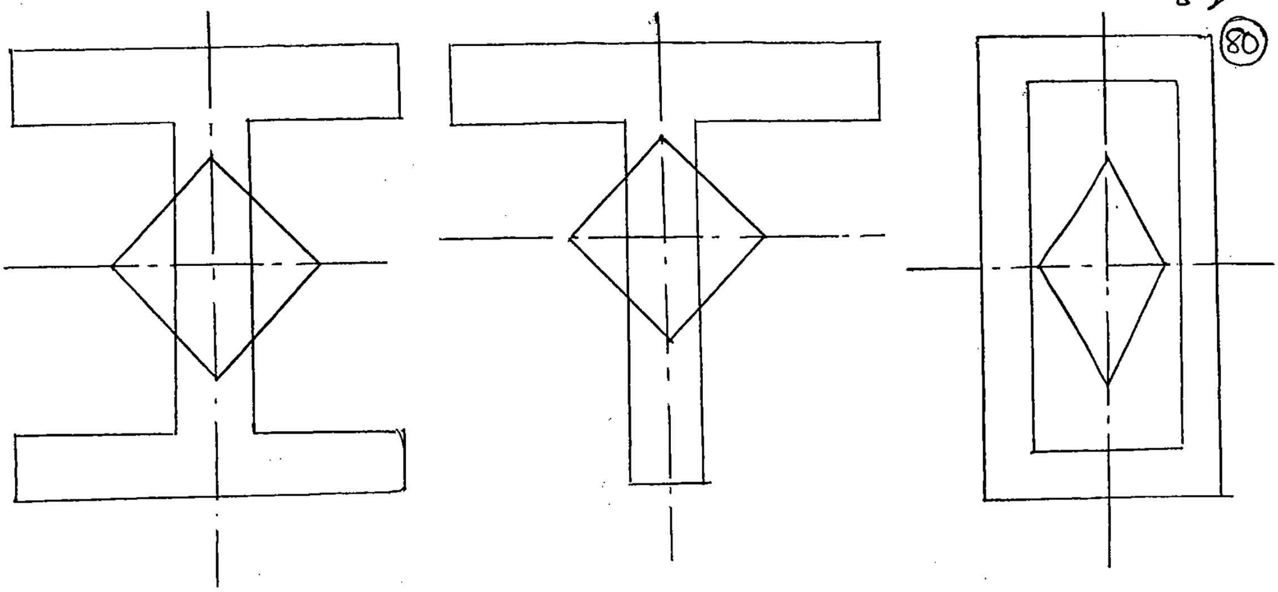
* For hollow circular.



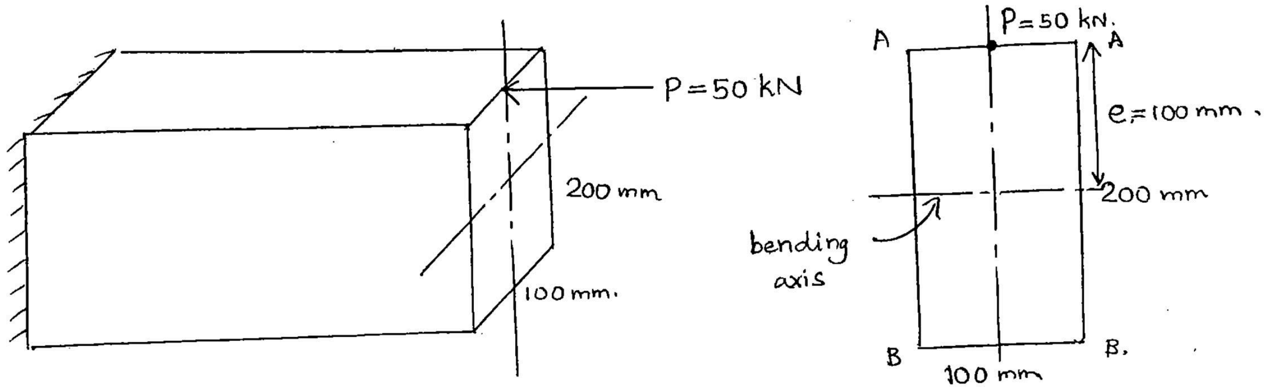
$$\frac{P}{\frac{\pi}{4}(D^2-d^2)} - \frac{Pe}{\frac{\pi(D^4-d^4)}{32D}} = 0.$$

$$e = \frac{D^4-d^4}{8D(D^2-d^2)} = \frac{D^2+d^2}{8D\cancel{(D^2-d^2)}}$$

$$\boxed{e_{max} = \frac{D^2+d^2}{8D}}$$



Q.

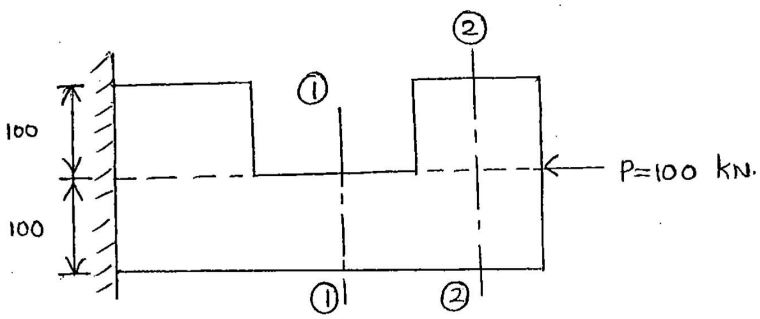


$$\left.\begin{array}{l}\sigma_{max}\\ @\text{ top layer AA}\end{array}\right\} = \frac{50\times10^3}{100\times200} + \frac{50\times10^3\times100}{\frac{100\times200^2}{6}}$$

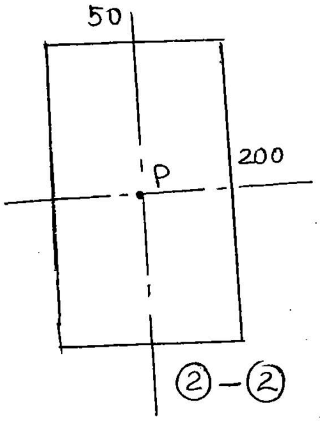
$$= 10 \text{ MPa (compression)}$$

$$\left.\begin{array}{l}\sigma_{min}\\ @\text{ bottom layer BB}\end{array}\right\} = \frac{50\times10^3}{100\times200} - \frac{50\times10^3\times100}{\frac{100\times200^2}{6}} = -5 \text{ MPa (tension)}.$$

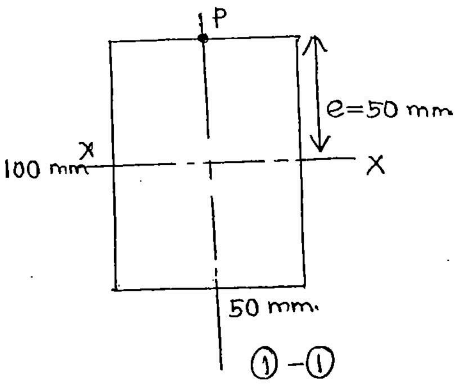
Q.



A stepped bar shown in fig of const. thickness 50 mm. ($\perp^r$ to paper), is subjected to axial force at free shown in fig. Determine max. and min. stresses at the sections ① & ② seperately.



$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{P}{A} = \frac{100\times10^3}{50\times200.} = 10\ MPa$$



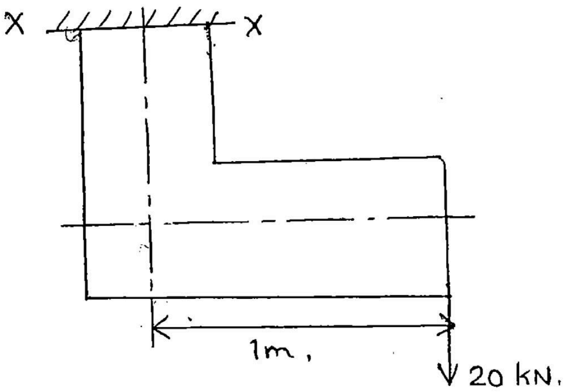
$$\frac{\sigma_{max}}{min} = \frac{P}{A} \pm \frac{Pe}{Z_x.}$$

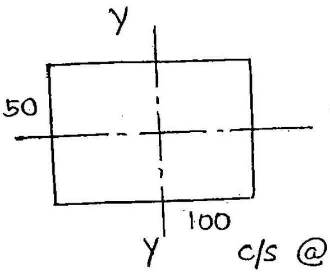
$$= \frac{100\times10^3}{50\times100} \pm \frac{100\times10^3\times50}{\frac{50\times100^2}{6.}}$$

$$\sigma_{max} = 20+60 = \underline{\underline{80\ MPa}}.$$

$$\sigma_{min} = 20-60 = \underline{\underline{-40\ MPa}}$$

Q.





c/s @ fixed end.

Calculate max & min stresses @ fixed end. (neglecting bending effect of vertical part).

$$\sigma_{max/min} = -\frac{P}{A} \pm \frac{Pe}{Z_y.} \quad \text{(bending about y-y axis).}$$

$$= \frac{20\times10^3}{50\times100} + \frac{20\times10^3\times1000}{\frac{50\times100^2}{6}}$$

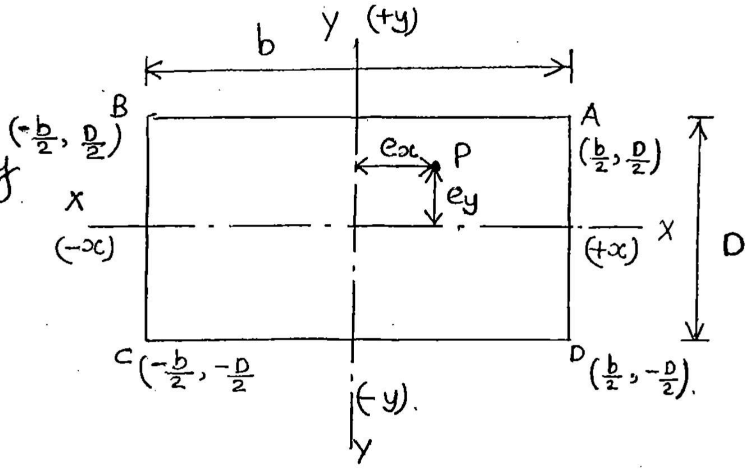
$$\sigma_{max} = 244\ MPa\ (T).$$

$$\sigma_{min} = \underline{\underline{-236\ MPa.}}\ (C)$$

29th Oct,
WEDNESDAY → Column with Bi-axial Bending Moment.

$M_x = P.e_y$

$M_y = P.e_x$

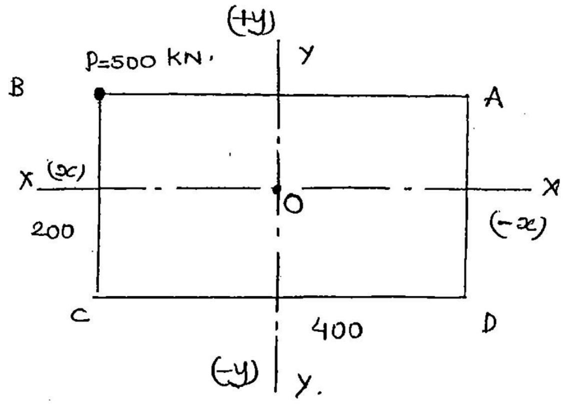


* Resultant stress at any point (x, y) in c/s,

$$\sigma_R = \frac{P}{A} + \frac{M_x}{I_x}(y) + \frac{M_y}{I_y}(x)$$

$$I_x = \frac{bD^3}{12} \quad \& \quad I_y = \frac{Db^3}{12}$$

Q. A rectangular column 200 mm x 400 mm. is subjected to a compressive load of 500 kN as shown. Determine resultant stresses at all corners.



Load P acts through corner B.

$$I_x = \frac{400 \times 200^3}{12} = 266.67 \times 10^6 \ mm^4$$

$$I_y = \frac{200 \times 400^3}{12} = 1066.667 \times 10^6 \ mm^4.$$

$M_x = 500 \times 100 \times 10^3 = 50 \times 10^6$ Nmm.

$M_y = 500 \times 200. = 100 \times 10^6$ Nmm

$$\sigma_B = \frac{P}{A} + \frac{M_x}{I_x}y + \frac{M_y}{I_y}x = \frac{500 \times 1000}{400 \times 200} = 6.25 \ MPa + 18.75 \times 2 = \underline{43.75 \ MPa}$$

Quadrant through which load is acting is having both x & y positive.

$$\sigma_O = \frac{P}{A} = \underline{6.25 \ MPa}$$

$$\sigma_C = \frac{P}{A} - \frac{M_x}{I_x} \times 100 + \frac{M_y}{I_y} \times 200$$

$$= 6.25 - \frac{50 \times 10^6}{266.67 \times 10^6} \times 100 + \frac{100 \times 10^6}{1066.67 \times 10^6} \times 200$$

$$= \underline{\underline{6.2502}} \text{ MPa}$$

$$\sigma_A = \frac{P}{A} + \frac{M_x}{I_x} \times \overset{1}{\cancel{2}}00 - \frac{M_y}{I_y} \times 200$$

$$= 6.25 + 18.75 - 18.75 = \underline{\underline{6.25}} \text{ MPa}$$

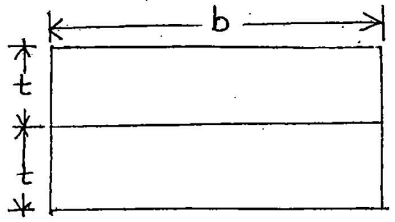
$$\sigma_D = \frac{P}{A} - 18.75 - 18.75 = \underline{\underline{-31.25}} \text{ MPa}$$

P-81

07. $P_e = \dfrac{\pi^2 E I}{l^2} \Rightarrow P \propto I.$

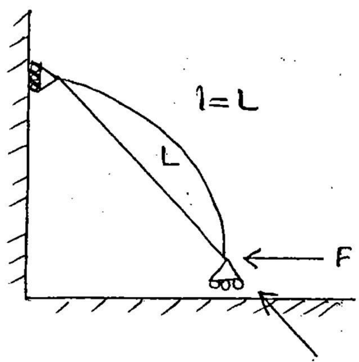
$$\frac{P_x}{P_y} = \frac{I_x}{I_y} = \underline{\underline{1.85}}$$

09. $\dfrac{P_{bonded}}{P_{no\ bond}} = \dfrac{I_{bonded}}{2\left(I_{of\ each\ slice}\right)}$



$$= \frac{b\,(2t)^3/12}{2\left(bt^3/12\right)} = \underline{\underline{4}}$$

10.

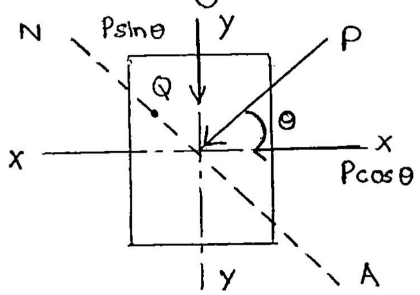


29th Oct,
WEDNESDAY

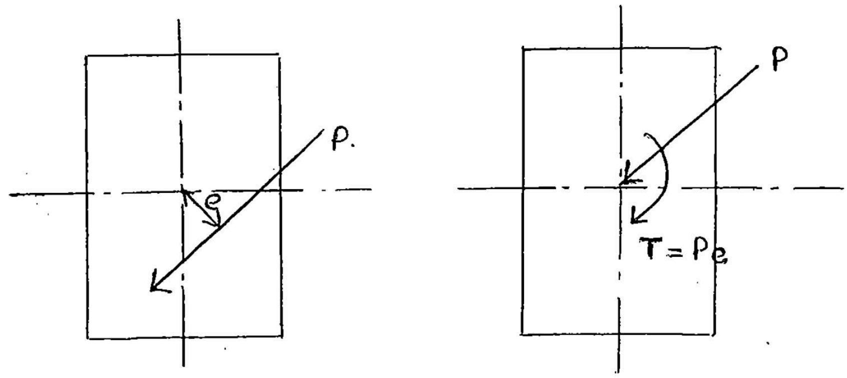
13 SHEAR CENTRE & UNSYMMETRICAL BENDING

→ Unsymmetrical Bending (or) Bi-axial (or) Skew Bending.

If a member is subjected to a load not passing through symmetrical axis but passes through centroid caused unsymmetrical bending.



⊙ The load or force is not passing through the centroid of c/s causes torsion.



P sinθ : causes bending about x-axis.

P cosθ : causes bending about y-axis.

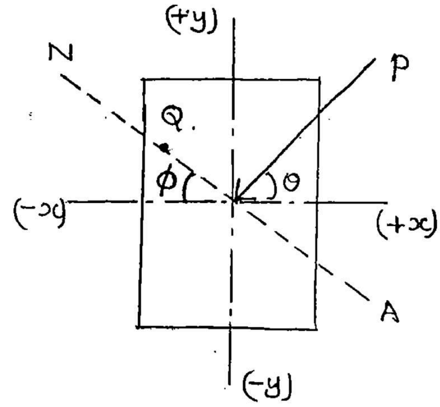
* Resultant stress @ any point (x, y) in the c/s :

$$\sigma_R = \frac{M_x}{I_x}(y) + \frac{M_y}{I_y}(x).$$

* To locate neutral axis (NA) :

Assume a point Q on NA,

$(\sigma_R)_Q = 0.$



* Sign convention :

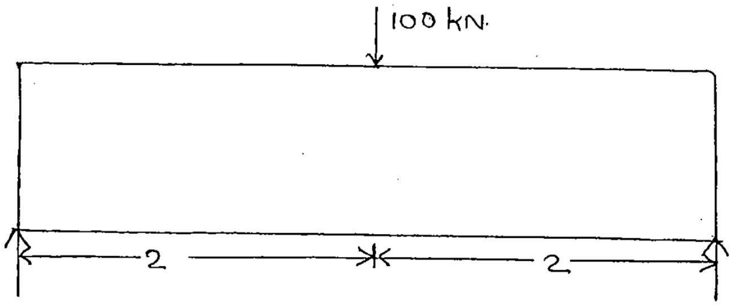
- Point through which load is applied will have both x & y are positive.

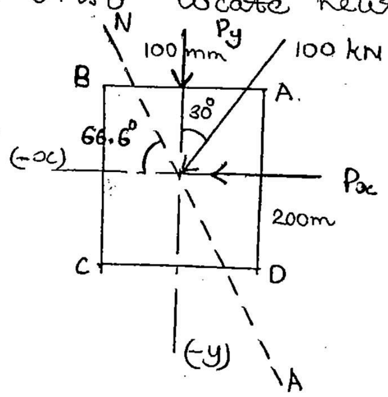
Use $Q(-x, +y)$.

$$(\sigma_R)_Q = \frac{M_x}{I_x}(+y) + \frac{M_y}{I_y}(-x).$$

$$\tan\phi = \frac{y}{x} = \frac{M_y}{M_x} \cdot \frac{I_x}{I_y}.$$

Q. A rectangular beam 4m span, simply supported at ends is subjected to a conc. point load of 100 kN, which is passing through centroid of c/s but inclined at 30° vertical axis. The beam is laterally supported against lateral bending. Determine stresses at all corners. Also locate neutral axis





$P_x = P\sin 30 = 50 \text{ kN}.$

$P_y = P\cos 30 = 86.602 \text{ kN}.$

$$M_x = \frac{P_y l}{4} = \frac{86.602 \times 10^3 \times 4}{4} = 86.602 \text{ kNm}$$

$$M_y = \frac{P_x l}{4} = \frac{50 \times 4}{4} = 50 \text{ kNm}.$$

$$I_x = \frac{100 \times 200^3}{12} = 66.67 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{200 \times 100^3}{12} = 16.67 \times 10^6 \text{ mm}^4.$$

$$\sigma_A = \frac{M_x}{I_x} \cdot y + \frac{M_y}{I_y} \cdot x.$$

$$= \frac{86.606 \times 10^6}{66.67 \times 10^6} \times 100 + \frac{50 \times 10^6}{16.67 \times 10^6} \times 50$$

$$= 129.896 + 149.97 = \underline{\underline{279.86}} \text{ MPa} \quad (C)$$

$$\underset{(-x, y)}{\sigma_B} = 129.896 - 149.97 = \underline{\underline{-20.074}} \text{ MPa} \quad (T)$$

$$\sigma_{C(-x,-y)} = -129.896 - 149.97 = \underline{\underline{-279.86}} \text{ MPa} \quad (T)$$

$$\sigma_{D(x,-y)} = -129.896 + 149.97 = \underline{\underline{20.074}} \text{ MPa} \quad (C)$$

$$\tan\phi = \frac{y}{x} = \frac{M_y}{M_x} \cdot \frac{I_x}{I_y}$$

$$= \frac{50}{86.606} \times \frac{66.67}{16.67} = 2.309$$

$$\Rightarrow \phi = \underline{\underline{66.58^\circ}}$$

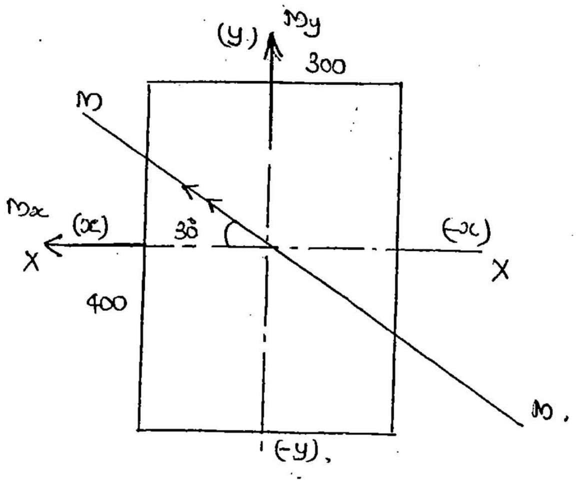
So NA is inclined at an angle of 66.58° with the horizontal (B & C are under tension and are on the same side of NA).

02. $M_x = M \cos 30$

$$= 2000 \cos 30 = 1.73 \text{ kNm}.$$

$$M_y = M \sin 30$$

$$= 2000 \sin 30 = 1 \text{ kNm}.$$



$$I_{xx} = \frac{300 \times 400^3}{12} = 1600 \times 10^6$$

$$I_{yy} = \frac{400 \times 300^3}{12} = 900 \times 10^6$$

$$\sigma_{max\ comp.} = \frac{M_x}{I_x} y + \frac{M_y}{I_y} x.$$

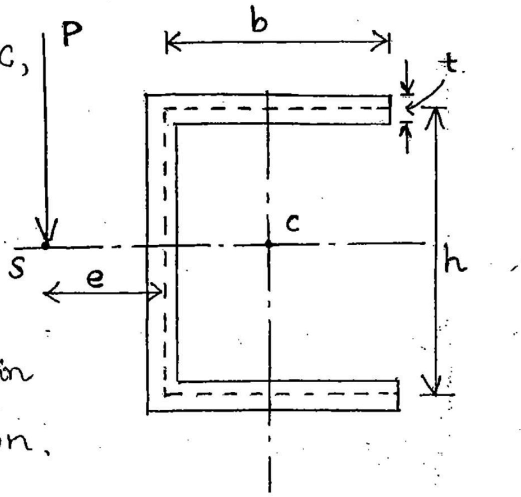
$$= \frac{1.73}{1600} \times 200 + \frac{1}{900} \times 150 = \underline{\underline{0.383\ MPa}}$$

$$\sigma_{max\ tensile} = \underline{\underline{0.383\ MPa}}.$$

$$\phi = \tan^{-1}\left(\frac{1 \times 1600}{1.73 \times 900}\right) = \underline{\underline{45.78^\circ}}$$

→ Shear Centre. (SC)

In case of unsymmetrical sections wrt loading axis, torsion develops apart from shear force and BM, even though the load pass through the centroid.



If the load is applied through SC, there will be no torsion in the c/s.

However BM and shear force will be acting over the section.

Shear centre is applicable for thin walled sections or light gauge section.

$$\boxed{e = \frac{b^2 h^2 t}{4I}}$$

⊙ Refer P-95,96 for more sections

11. STRAIN ENERGY

RESILIENCE

Strain Energy:

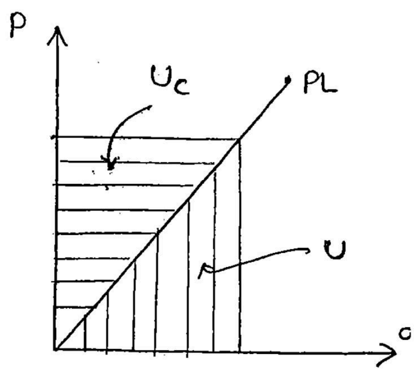
The internal energy stored due to external work done is strain energy.

Energy is a scalar quantity with a unit Nm, J.

Resilience: (U)

Strain energy stored in a member upto proportionality limit is resilience.

Area under load-deformation (P-δ) curve within PL is resilience.



$$\boxed{U = \frac{1}{2} P \delta}$$

But $\sigma = \frac{P}{A} \Rightarrow P = \sigma A.$

$\epsilon = \frac{\delta}{l} \Rightarrow \delta = \epsilon l.$

$$\therefore \quad U = \frac{1}{2}(\sigma A)(\epsilon l).$$

$$\boxed{U = \frac{1}{2}\sigma \epsilon V.}$$; $V \rightarrow$ volume $(= Al)$.

$$U = \frac{1}{2}\sigma \cdot \left(\frac{\sigma}{E}\right) V.$$

$$\boxed{U = \frac{\sigma^2}{2E} V.}$$

$$U = \frac{1}{2} P \left(\frac{PL}{AE}\right)$$

$$\boxed{U = \frac{P^2 L}{2AE}}$$

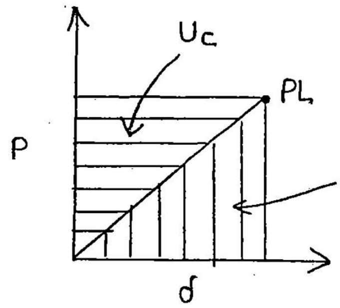
- Area above the curve is complementary strain energy

Due to complementary energy, member can regain back to normal position.

Proof Resilience: (U_p)

The maximum resilience in a member is proof resilience which can be obtained by loading the member upto PL.

$$\boxed{U = U_c}$$

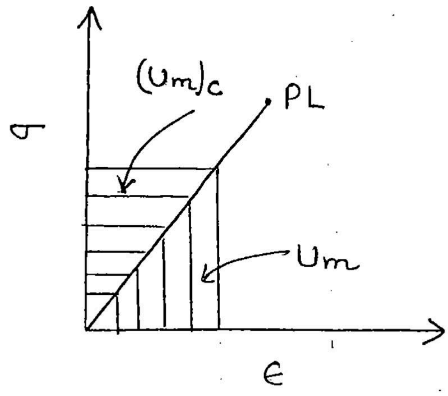


Modulus of Resilience: (U_m)

Resilience per unit volume is modulus of resilience.

$$U = \frac{U_m}{V}$$

$$\boxed{U_m = \frac{1}{2} \sigma \epsilon}$$



Area under stress-strain curve upto PL is modulus of resilience.

$$\boxed{U_m = \frac{\sigma^2}{2E}}$$

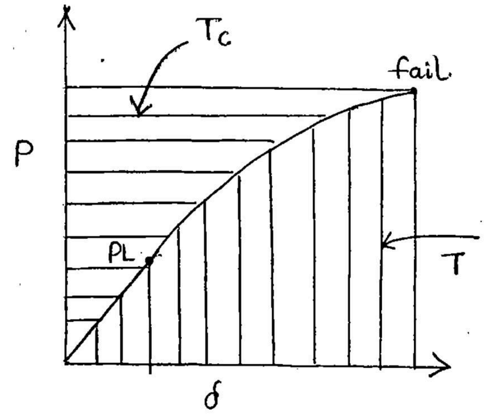
Unit: N/m^2 (stress unit).

⊙ Um is a material property, constant for a given material irrespective of volume and other parameters; similar to E, G & K.

⊙ Um is a non zero positive value.

Toughness : (T)

The max. strain energy absorbed by a member ~~upto~~ at failure is toughness.



Area under P–δ curve upto failure is toughness.

⊙ Usually, ductile material are tough material and can absorb a lot of energy before failure.

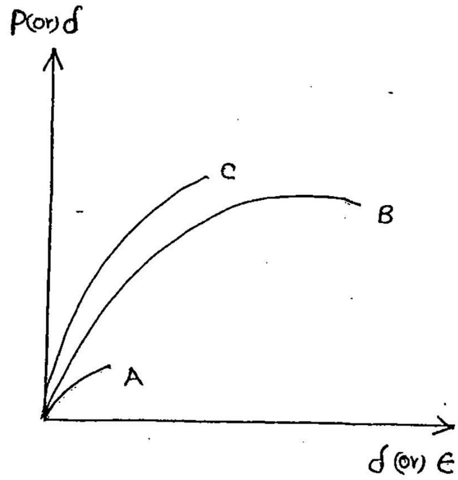
⊙ Loading beyond proportionality limit gives lesser complementary energy. ∴ the member may not regain back to original size and shape. Then permanent set (or) plastic deformation (or). residual strain occurs which cannot be removed from the member.

↑ tough ⇒ ↑ area under curve (B)

↑ ductility ⇒ ↑ x component. (B).

↑ strong ⇒ ↑ y component (C)

↑ brittle ⇒ ↓ x component (A).



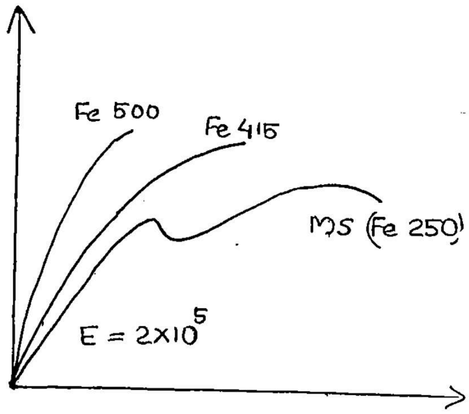
↑ tough ⇒ Fe 250

↑ ductility ⇒ Fe 250

↑ strong ⇒ Fe 500

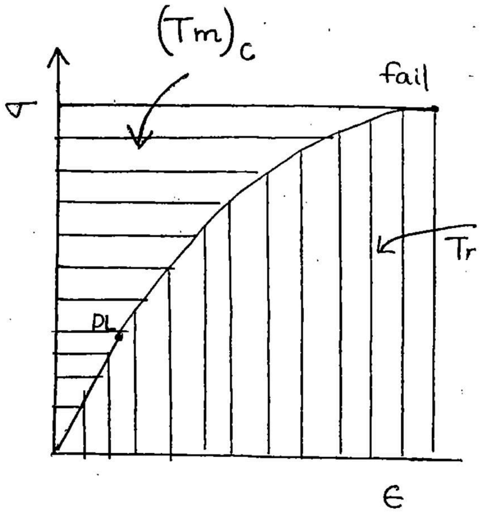
↑ brittle ⇒ Fe 500

↑ resilient ⇒ area under curve upto PL (all are same).



Modulus of Toughness : (Tm)

Toughness per unit volume or area under $\sigma - \epsilon$ curve upto failure is called modulus of toughness.



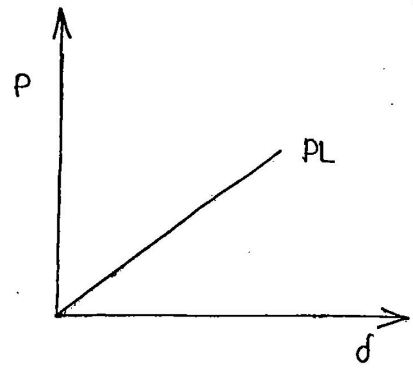
→ Type of Loading

(i) Gradual Loading.

Axial Force :-

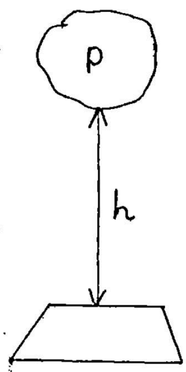
$$\boxed{\sigma = \frac{P}{A}}$$

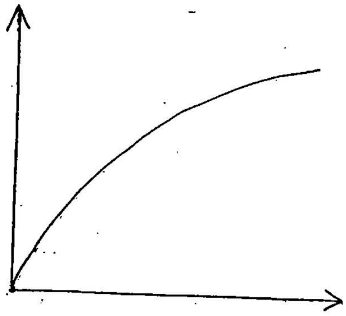
$$\boxed{\delta = \frac{Pl}{AE}}$$



(ii) Impact Loading

Work done = strain energy stored.

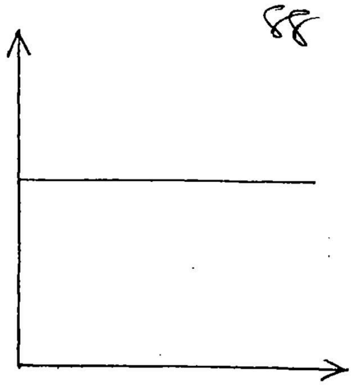




$$Ph = \frac{\sigma^2}{2E} V.$$

$$\boxed{\sigma_{(impact)} = \sqrt{\frac{2PhE}{V}}}$$

(iii) Sudden Load. (Imaginary Load).



$$\boxed{\sigma_{sudden} = 2\,\sigma_{grad} = \frac{2P}{A}}$$

$$\boxed{\delta_{sudden} = 2\,\delta_{grad} = \frac{2PL}{AE}}$$

→ Various forms of Strain Energy

* Bending

$$\boxed{U = \int \frac{M^2 dx}{2EI} = \frac{f^2}{2E}\cdot \text{Volume}}$$

* Shear force

$$\boxed{U = \frac{\tau^2}{2G}\cdot \text{Volume},}$$

τ → flexural shear stress in beams.

* Torsion

$$\boxed{U = \frac{1}{2} T\theta}$$

But $\frac{T}{J} = \frac{G\theta}{l}$ ⇒ $\theta = \frac{Tl}{GJ}$.

$$\boxed{U = \frac{T^2 L}{2GJ}}$$

$$\boxed{U = \frac{\tau^2}{4G}\cdot \text{volume.}}$$

τ → torsional shear stress

* Volumetric Stress (σ)

$$\boxed{U = \frac{\sigma^2}{2K} \cdot \text{volume}}$$