NT

Lect-01. Module-V chapter-05

TWO PORT NETWORK

Port: ⇒ A pair of terminal is known as port.

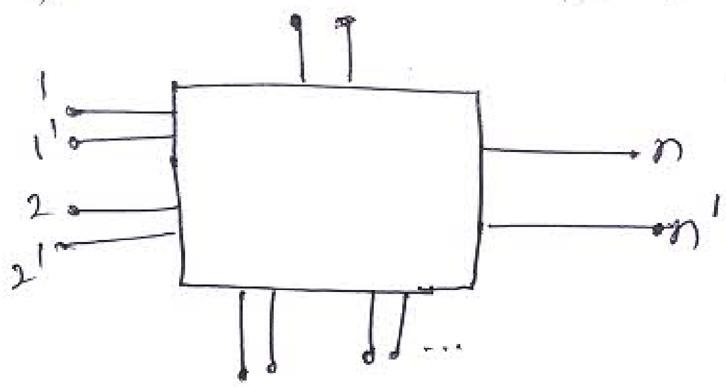
Single port Network → If a n/w consist of one pair of terminal or two terminal is known as single port n/w.

Two port Network → The n/w having two pairs of terminal or four terminals is known as two port n/w.

→ In case of two port network, if one pair act as input port (1-1') and the other pair act as output port (2-2')

n-port Network → If a network having n-pairs of terminals or 2n terminals is known as n-port n/w.

→ In n-port n/w, some of them act as input port and some of them act as output port.

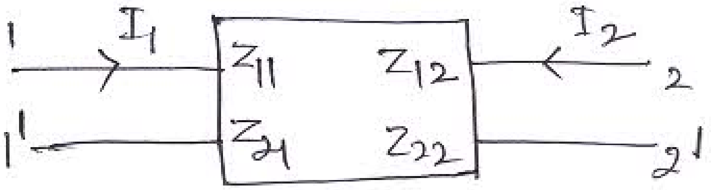


Parameter Representation of Two-port Network →

The relationship between V_1, I_1, V_2, I_2 can be represented in different parameter form i.e.
Z - parameter, Y - parameter.
h - parameter, ABCD (Transmission) parameter.

Representation of Z - parameter (open - ckt parameter):

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



The parameter can be obtained as

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}, \quad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

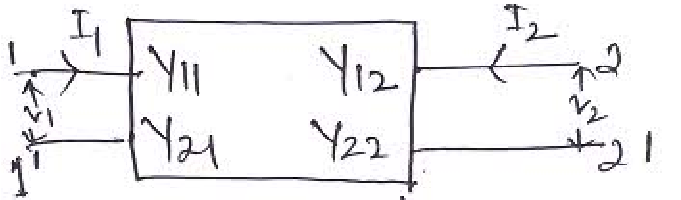
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}, \quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

→ The Z - parameters are also known as open circuit parameters because all the parameters are obtained by opening input port ($I_1 = 0$) or opening the output port ($I_2 = 0$).

Representation of Y-parameters. (short-circuit parameters) :→

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$



The parameters can be obtained as.

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \quad , \quad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

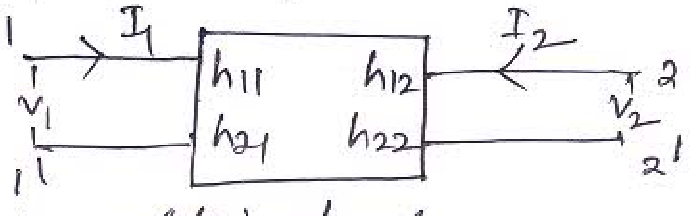
$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} \quad , \quad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

→ The above parameters are also known as short-circuit parameters because we are obtaining all the parameter by short-circuit the input port ($V_1 = 0$) or by short circuiting the output port ($V_2 = 0$).

Representation of h-parameter / Hybrid parameter :→

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$



The parameter can be obtained as.

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \quad , \quad h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

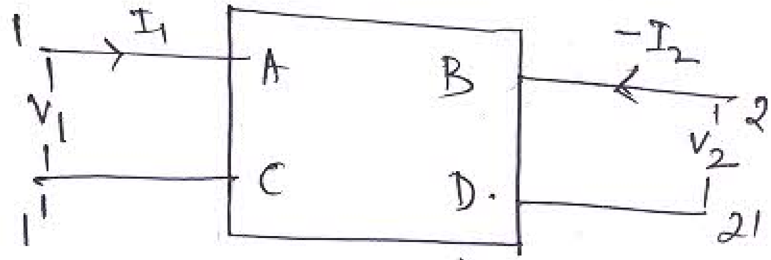
$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} \quad , \quad h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}.$$

The h-parameters are obtained by the combination of Z-parameter and Y-parameter and constant for which, they are called as hybrid parameter.

Representation of ABCD parameter. →

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$



The parameter can be obtained as.

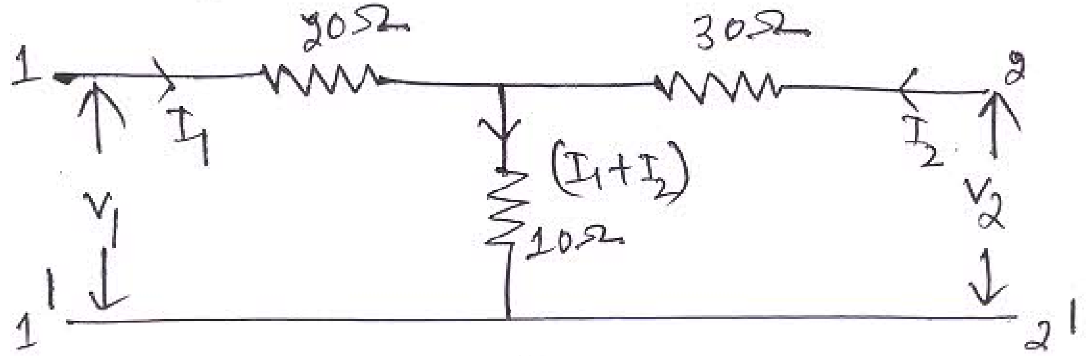
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad , \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad , \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

The above parameters calulated are used in Transmission lines for which they are known as Transmission parameter.

Solved Example.

(1) Determine the Z-parameter for the network shown in figure?



Solution As per the formula.

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{11} = \frac{20 \cdot I_1 + 10 \cdot I_1}{I_1} = \frac{30 I_1}{I_1}$$

$$\Rightarrow \boxed{Z_{11} = 30\Omega}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$= \frac{10 I_1}{I_1} = 10\Omega$$

$$\boxed{Z_{21} = 10\Omega}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$\Rightarrow Z_{22} = \frac{10 I_2 + 30 I_2}{I_1}$$

$$\Rightarrow Z_{22} = \frac{40 I_2}{I_2} = 40\Omega$$

$$\boxed{Z_{22} = 40\Omega}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$= \frac{10 I_2}{I_2} = 10\Omega$$

$$\boxed{Z_{12} = 10\Omega}$$

INTER-RELATIONSHIPS BETWEEN PARAMETERS OF TWO PORT-NETWORK

Case-I: Z parameters in Terms of Y-parameters

Z being the impedance and Y being the admittance.

$$[Z] = [Y]^{-1}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\boxed{\begin{array}{l} Z_{11} = \dfrac{Y_{22}}{\Delta Y}, \quad Z_{12} = -\dfrac{Y_{12}}{\Delta Y} \\ Z_{21} = -\dfrac{Y_{21}}{\Delta Y} \text{ and } Z_{22} = \dfrac{Y_{11}}{\Delta Y} \end{array}}$$

Here $\Delta Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}Y_{21}$

Case-II: Z parameters in Terms of ABCD parameters.

ABCD parameter equations are.

$$V_1 = AV_2 - BI_2 \quad \text{——— (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{——— (2)}$$

From equation (2) we get.

$$V_2 = \left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2 \quad \text{——— (3)}$$

From equation (1) we get.

$$V_1 = A\left[\left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2\right] - BI_2$$

$$\Rightarrow V_1 = \left(\frac{A}{C}\right)I_1 + \left(\frac{AD-BC}{C}\right)I_2 \quad \text{——— (4)}$$

Comparing equ^n (4) and (3) with Z-parameter equation.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

and $V_2 = Z_{21} I_1 + Z_{22} I_2$, we get.

$$\boxed{\begin{aligned} Z_{11} &= \frac{A}{C}, \quad Z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C} \\ Z_{21} &= \frac{1}{C}, \quad Z_{22} = \frac{D}{C}. \end{aligned}}$$

Case - III Z-parameter interms of Hybrid parameter.

h-parameter equations are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——(1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——(2)}$$

From second equation.

$$V_2 = \left(-\frac{h_{21}}{h_{22}}\right) I_1 + \left(\frac{1}{h_{22}}\right) I_2 \quad \text{——(3)}$$

From first equation.

$$V_1 = h_{11} I_1 + h_{12}\left[\left(-\frac{h_{21}}{h_{22}}\right) I_1 + \left(\frac{1}{h_{22}}\right) I_2\right]$$

$$\Rightarrow V_1 = \left(\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right) I_1 + \left(\frac{h_{12}}{h_{22}}\right) I_2 \quad \text{——(4)}$$

Compairing equn (4) and (3) with. z-parameter equation.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

and $V_2 = Z_{21} I_1 + Z_{22} I_2$, we get.

$$\boxed{\begin{aligned} Z_{11} &= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} \\ Z_{12} &= \frac{h_{12}}{h_{22}}, \quad Z_{21} = \frac{-h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}} \end{aligned}}$$

Problem calculate the z-parameters, if the values of other parameters are given below.

(a) Given $A = 2$, $B = -1$, $C = 3$ and $D = -2$.

Solution. $Z_{11} = \frac{A}{C} = \frac{2}{3}\,\Omega$

$$Z_{12} = \frac{\Delta T}{C} = \frac{AD - BC}{C} = \frac{-4+3}{3} = \frac{-1}{3}\,\Omega$$

$$Z_{21} = \frac{1}{C} = \frac{1}{3}\,\Omega$$

$$Z_{22} = \frac{D}{C} = -\frac{2}{3}\,\Omega$$

(b) given $h_{11} = 1$, $h_{12} = -2$, $h_{21} = -3$, $h_{22} = 2$.

Solun

$$Z_{11} = \frac{\Delta h}{h_{22}} = \frac{h_{11}h_{22} - h_{21}h_{12}}{h_{22}} = \frac{2-6}{2} = -2\,\Omega$$

$$Z_{12} = \frac{h_{12}}{h_{22}} = -1\,\Omega$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} = \frac{3}{2}\,\Omega$$

$$Z_{22} = \frac{1}{h_{22}} = \frac{1}{2}\,\Omega$$

(c) given $Y_{11} = \frac{1}{3}$, $Y_{12} = 2/3$, $Y_{21} = -\frac{1}{3}$, $Y_{22} = \frac{1}{6}$

Solun

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1}{6} \times \frac{18}{5} = \frac{3}{5}\,\Omega$$

Because $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = \frac{1}{18} + \frac{2}{9} = \frac{5}{18}$

$$Z_{12} = \frac{-Y_{12}}{\Delta Y} = -2/3 \times \frac{18}{5} = -\frac{12}{5}\,\Omega$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5}\,\Omega$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5}\,\Omega$$

Lect-03

CONDITION FOR SYMMETRY :→

→ A two port network is said to be symmetrical if the ports can be interchanged without changing port voltages or currents.

(a) In terms of Z-parameter.

$$\boxed{Z_{11} = Z_{22}}$$

(b) In terms of Y-parameter.

$$\boxed{Y_{11} = Y_{22}}$$

(c) In terms of 'h' parameter.

$$\boxed{\Delta h = 1} \text{ or } \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1.$$

(d) In terms of ABCD parameter.

$$\boxed{A = D.}$$

CONDITION FOR RECIPROCITY :→

→ A network is said to be reciprocal if the ratio of the response to the excitation remains same to an interchange of the positions of the excitations and response in the network.

(a) In terms of Z-parameter.

$$\boxed{Z_{12} = Z_{21}.}$$

(b) In terms of Y-parameter.

$$\boxed{Y_{12} = Y_{21}}$$

(c) In terms of 'h' parameter.

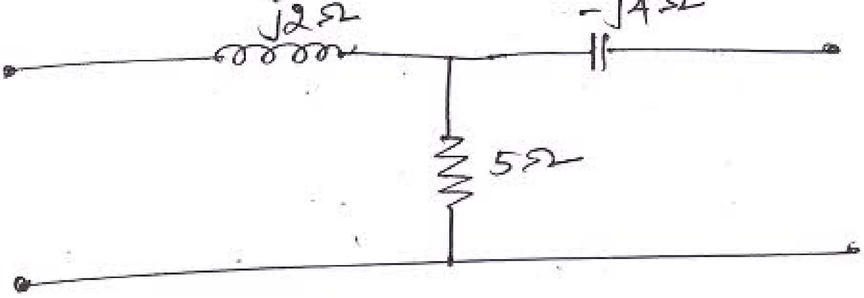
$$\boxed{h_{12} = -h_{21}.}$$

(d) In terms of ABCD parameter.

$$\boxed{\Delta T = 1.} \quad \text{or} \quad \boxed{AD - BC = 1.}$$

$$\text{or} \quad \begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1.$$

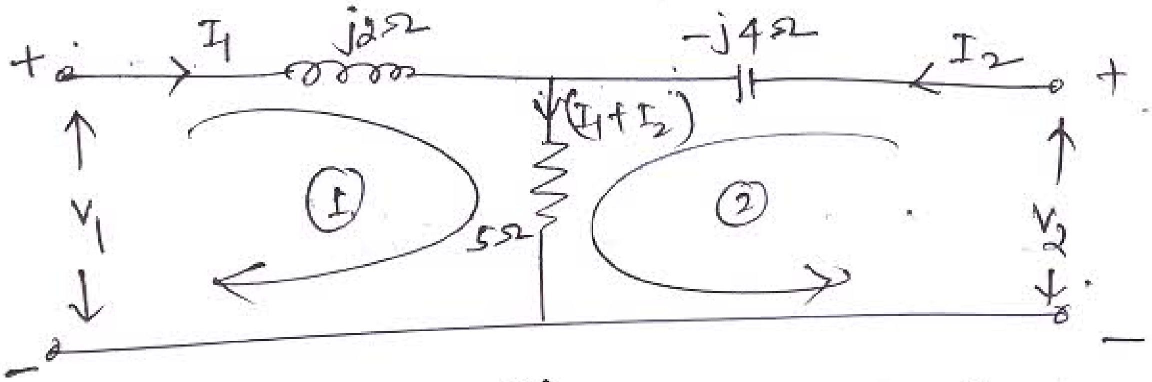
Example: For the network shown in figure calculate T-parameters.



Soln T-parameters equations are.

$$V_1 = AV_2 - BI_2 \quad \text{---} (1)$$

$$I_1 = CV_2 - DI_2 \quad \text{---} (2)$$



By KVL in Loop-(1) $-j2 \times I_1 - 5(I_1+I_2) + V_1 = 0$

$$\Rightarrow V_1 = (5+j2)I_1 + 5I_2 \quad \text{---} (3)$$

By KVL in Loop-(2) $+j4I_2 - 5(I_1+I_2) + V_2 = 0$.

$$\Rightarrow V_2 = 5I_1 + (5-j4)I_2 \quad \text{---} (4)$$

putting I_1 from equation (4) in equation (3) we get.

$$V_1 = (5+j2)\left[\frac{V_2 - I_2(5-j4)}{5}\right] + 5I_2$$

$$\Rightarrow V_1 = \left(\frac{5+j2}{5}\right)V_2 - \left[\frac{(5+j2)(5-j4)}{5} - 5\right]I_2 \quad \text{---} (5)$$

For making as equation (2), rearrange equation (4) we get.

$$I_1 = \frac{1}{5} V_2 - \left(\frac{5-j4}{5}\right) I_2 \text{ ——— } (6)$$

To obtain T- parameters.

Compairing equ^n (1) and (5) we get.

$$A = \left(\frac{5+j2}{5}\right), \quad B = \frac{(5+j2)(5-j4)}{5} - 5$$

$$= \left(\frac{8-j10}{5}\right)$$

Compairing equations (2) and (6) we get.

$$C = \left(\frac{1}{5}\right), \quad D = \left(\frac{5-j4}{5}\right)$$