20/08/20

Structural Analysis - II

Module - I

-> Analysis of Continuous Beam and plane frames
-> By slope deflection method
-> By Moment distribution method.

Introduction :-

Structure :- Structure refers to anything that is built from different interrelated parts with a fixed location on the ground.

ex:- Aqueducts, Bridges, Buildings.

-> A structure is defined as a system of interconnected members assembled in a stable configuration and used to support a load or combination of loads under equilibrium of various external forces and internal reactions.
-> The load can have vertical or lateral effects on the components.
-> The structural members are connected together by providing different types of joints or supports.

Supports -> Pin jointed or hinged
-> Roller
-> Fixed

Structural Analysis :-

-> Structural analysis is the determination of the effects of loads on physical structure and their components.

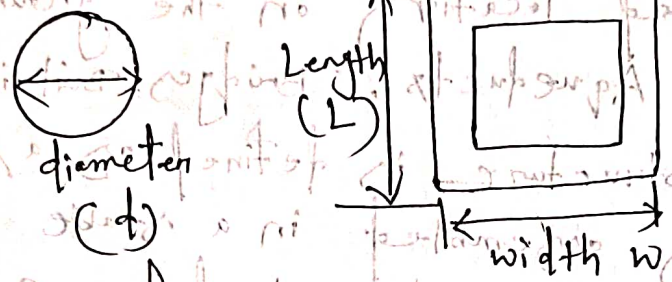
Structural design

→ Structural design means to define or ~~design~~ decide structural member dimensions, shape and their materials properties.

ex:- If i want to design a building structure then i have to

→ calculate dimensions of members.

- Depth of beam, depth of foundations.



→ shape of member.

- I shape beam
- Rectangular
- T-beam

→ Material properties

- Quantities of Concrete & reinforcing steel.
- Quality i-e grade of concrete & steel.

Civil Engineering

→ In civil Engineering the engineers do the job of dealing with the planning of design, construction and as well as maintanance.

Structural Engineering

→ In structural engg, the engineers mainly focus only on the structural aspects of a building or any man made str.

Structures

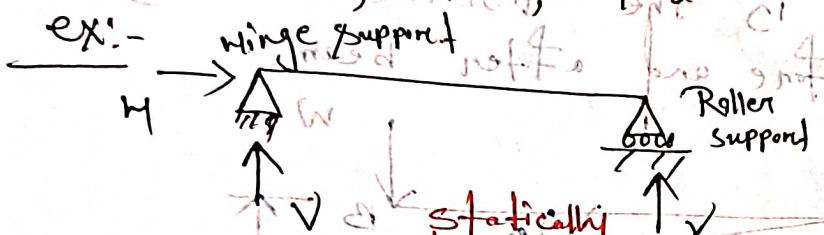
Determinate

→ The structure for which the all reaction & force can be analysis or solve or found out by using 3 equⁿ of equilibrium

→ 3 equation of static equilibrium

$\Sigma H = 0, \Sigma V = 0, \Sigma M = 0$

ex:-



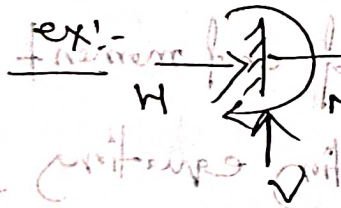
statically determinate str.

Unknown for this structure = 3

equⁿ of equilibrium = 3

3 - 3 = 0

ex:-



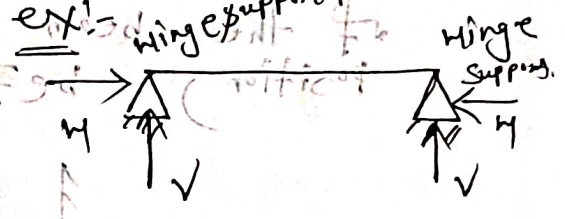
unknown = 3

equilibrium = 3

Indeterminate

→ The structure for which all reaction & forces can not be resolve or analysis or found out by using 3 equation of equilibrium

ex:-



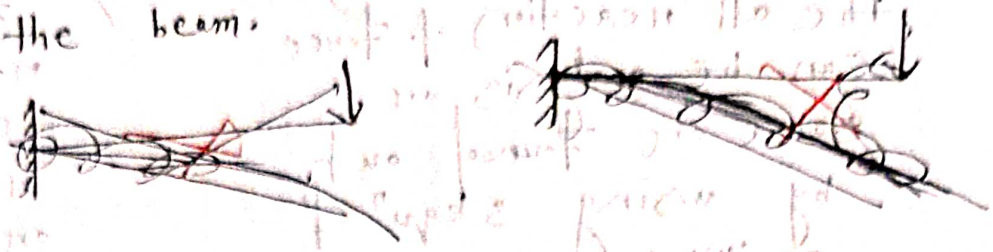
A unknown forces = 4

equilibrium = 3

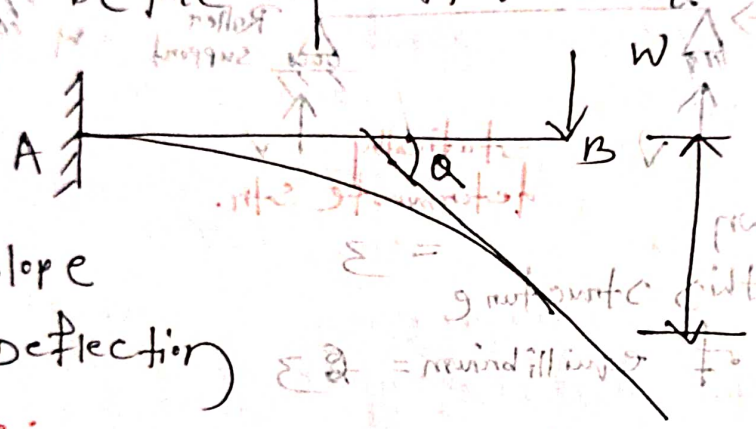
4 - 3 = 1

⇒ Slope deflection method

slope:- slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.



Deflection:- Deflection at any point on the axis of the beam is the distance between its position before and after beam.



θ → slope

y → Deflection

Procedure:-

step-1 :- calculation of bending moment

step-2. calculation of fixed end moment

step-3. Apply slope deflection equations.

step-4. Apply equilibrium equations.

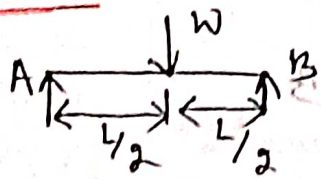
step-5. Calculate final moment

step-6 Draw B.M.D.

25/08/23

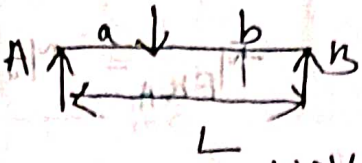
Bending moment

Case-1



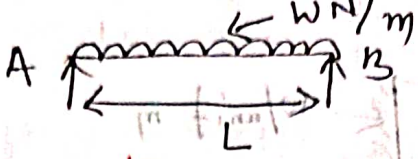
$= \frac{WL}{4}$

Case-2



$= \frac{Wab}{L}$

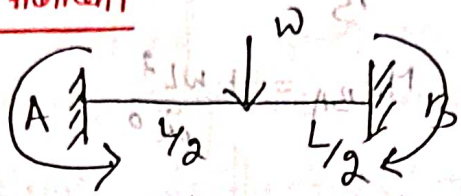
Case-3



$= \frac{WL^2}{8}$

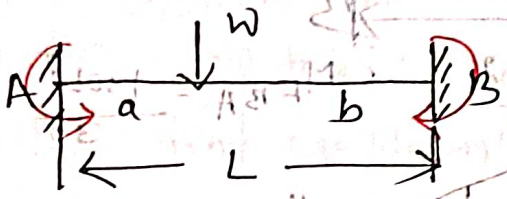
Fixed end moment

Case-1



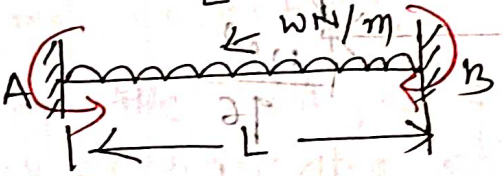
$M_{F\,AB} = \frac{-WL}{8}$ $\quad M_{FBA} = \frac{+WL}{8}$

Case-2



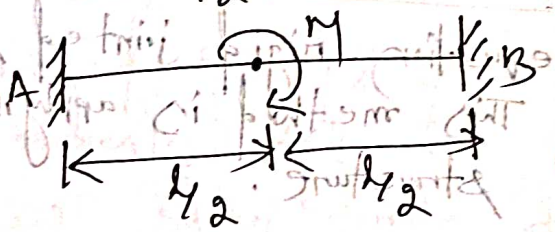
$M_{FAB} = \frac{-Wab^2}{L^2}$ $\quad M_{FBA} = \frac{+Wa^2b}{L^2}$

Case-3



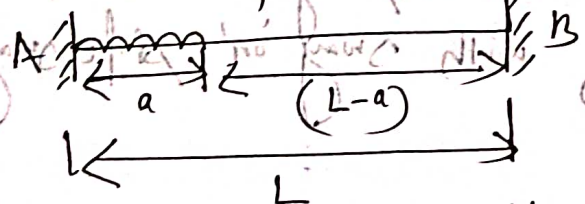
$M_{FAB} = \frac{-WL^2}{12}$ $\quad M_{FBA} = \frac{+WL^2}{12}$

Case-4



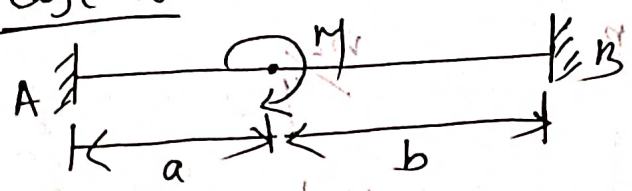
$M_{FAB} = \frac{+M}{4}$ $\quad M_{FBA} = \frac{+M}{4}$

Case-5



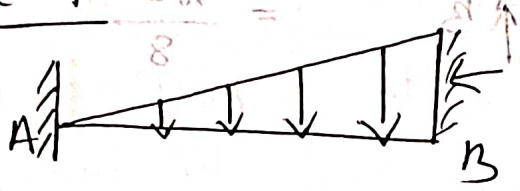
$M_{FAB} = \frac{-Wa^2}{12L^2}(6l^2 - 8la + 3a^2)$ $\quad M_{FBA} = \frac{+Wa^3}{12l^2}(4l - 3a)$

Case-6

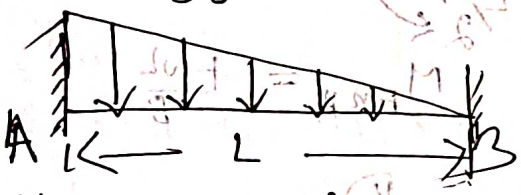


$$M_{FAB} = +\frac{Mb(3a-l)}{l^2} \qquad M_{FBA} = \frac{Ma(3b-l)}{l^2}$$

Case-7



$$M_{FAB} = \frac{-wL^2}{30} \qquad M_{FBA} = +\frac{wL^2}{20}$$



$$M_{FAB} = \frac{-wL^2}{20} \qquad M_{FBA} = \frac{+wL^2}{30}$$

Case-8



$$\frac{-5wl^2}{96} \qquad +\frac{wl^2}{96}$$

=> Application of slope deflection equation

slope deflection equation rigid jointed structure can be analysed, This method is applying to the following types of structure.

1) Continuous beam.
2) Frames without sway or side sway.
3) Frames with sway or side sway.

(i) Continuous beam :-

Steps

1) Each span of continuous beam is taken as fixed beam and fixed end moments are noted down. (FEM). The standard expression $\frac{wa^2b}{l^2}$, $\frac{wab^2}{l^2}$, $\frac{wl^2}{12}$ etc. for finding fixed end moments of fixed beam can be used. clockwise end moments are noted down as positive moment and anticlockwise as negative moment.

2) Using slope deflection equation, write down all the end moments. On this equation some of rotation and deflection will be known.

3) Write down the joint equilibrium equation.

4) Solve the joint equilibrium equation to get unknown rotation and deflection.

5) Substituting the values of unknown in slope deflection equation, determine the end moments.

6) Treating each member of the continuous beam a simply supported beam, subjected to a given loading and end moments, determine the end reaction and draw shear force & bending moment diagram.

→ Let AB be any indeterminate span of continuous beam subjected to an external load system.

Let θ_A & θ_B be the slope at end A & B.

'δ' be the downward transverse deflection of the right end 'B' with respect to the left end and

M_{AB}, M_{BA} be the final end moments at A & B.

M_{FAB} & M_{FBA} be the fixed end moment at A & B respectively.

Analysis :-

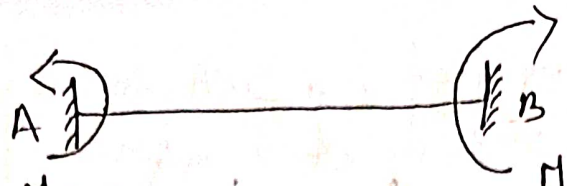
1) Consider AB is a fixed beam carrying the given load system, Calculate M_{FAB} & M_{FBA} at A & B.

2) Release the fixity at A & maintain the fixity at 'B'. Apply a moment $\frac{4EI}{l}\theta_A$ at 'A' produce a slope θ_A at 'A'. This will induce a moment $\frac{2EI}{l}\theta_A$ at 'B'

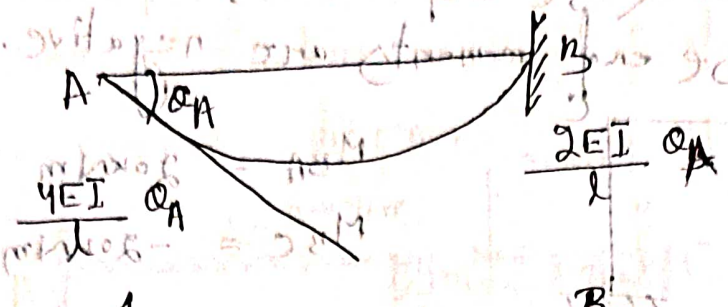
3) Maintain fixity at 'A' release the fixed at 'B' Apply a moment $\frac{4EI}{l}\theta_B$ at 'B' produce a slope θ_B at 'B'. This will induce a moment $\frac{2EI}{l}\theta_B$ at 'A'

4) Maintaining fixity at A & B. provide a vertical deflection 'δ'. For the end 'B' w.r.t the end 'A'. For this condition the moment at each end will be $\frac{6EI\delta}{l^2}$.

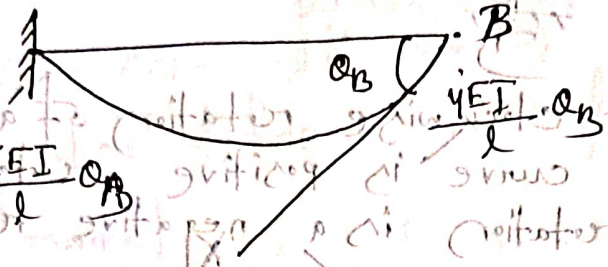
(i)



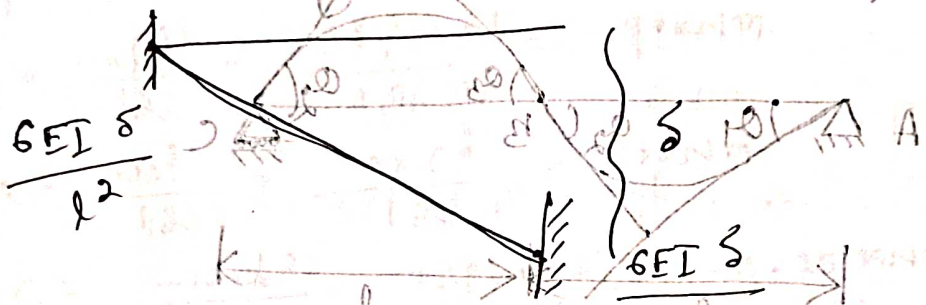
(ii)



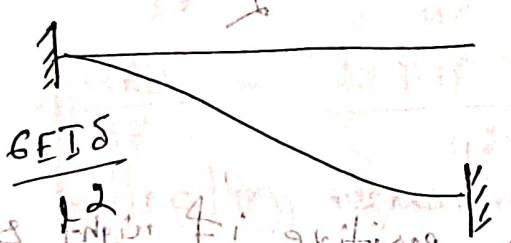
(iii)



(iv)



(or)



Final moment at the end A:

$$M_{AB} = M_{FAB} + \frac{4EI}{l}\theta_A + \frac{2EI}{l}\theta_B - \frac{6EI\delta}{l^2}$$

$$= M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B) - \frac{6EI\delta}{l^2}$$

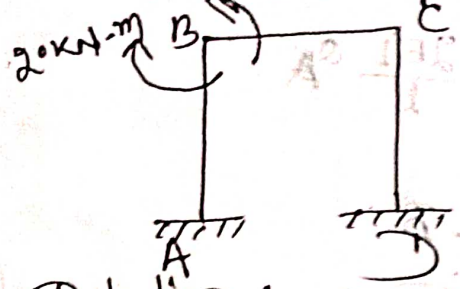
$$\boxed{M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)}$$

$$\boxed{M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\delta}{l}\right)}$$

This is known as slope deflection deflection.

=> Sign Convention :-

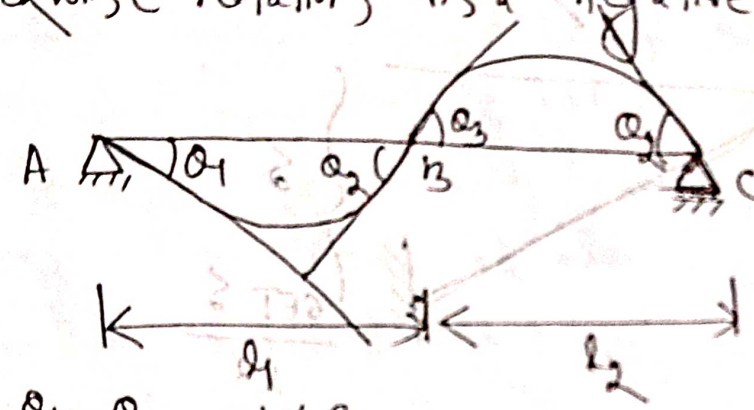
1) Moment :- clockwise end moments are positive & anticlockwise end moments are negative.



M_{BA} = 20KNm

M_{BC} = -20KNm

2) Rotation :- clockwise rotation of a tangent of an elastic curve is positive rotation and anticlockwise rotation is a negative rotation.

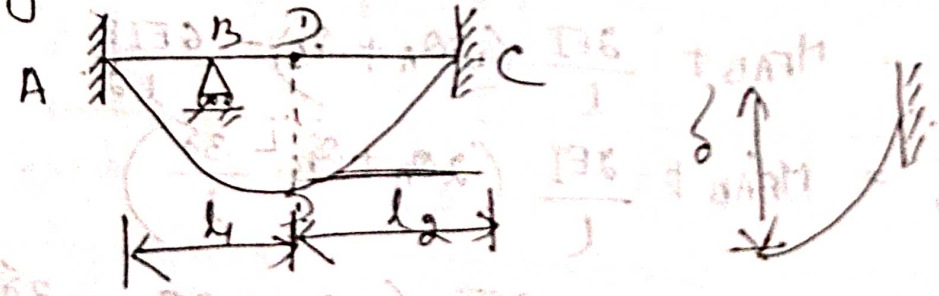


θ_1, θ_3 = +ve

θ_2 = -ve

3) Settlement :-

Settlement (δ) is positive if right sight support is below the left sight support. & (δ) is negative if left sight support is below the right-sight support.

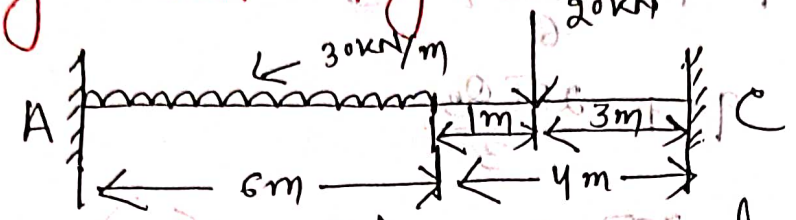


For beam AB, δ is +ve

& for beam BC, δ is -ve.

Q.) A fixed beam ABC has two spans AB = 6m & BC = 4m. A u/l of 30KN/m acts on span AB, on BC a point of 20KN acts at 1m from 'B'. Analyse the beam using slope deflection method & also draw the bending moment diagram.

Sol?:-



By using slope deflection method calculated fixed end moment.

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-30 \times 6^2}{12} = -90 \text{ KNm}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{30 \times 6^2}{12} = 90 \text{ KNm}$$

$$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-20 \times 1 \times 3^2}{4^2} = -11.25 \text{ KNm}$$

$$M_{FCB} = +\frac{wa^2b}{l^2} = \frac{20 \times 1^2 \times 3}{4^2} = 3.75 \text{ KNm}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -90 + \frac{2 \times EI}{6}\left(2 \times 0 + \theta_B - \frac{3\delta}{l}\right)$$

(Since A is fixed rotation at A = 0)

$$= -90 + \frac{2EI}{6}\left(\theta_B - \frac{3\delta}{l}\right) \quad (\delta = 0)$$

$$= -90 + \frac{2EI}{6}\theta_B$$

$$= -90 + \frac{EI}{3}\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\delta}{l}\right)$$

$$= 90 + \frac{2EI}{6}\left(0 + 2\theta_B - 3\delta/l\right)$$

$$= 90 + \frac{2EI}{6} \times 2\theta_B$$

$$= 90 + \frac{2EI\theta_B}{3}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -11.25 + \frac{2EI}{4} \times 2\theta_B$$

$$= -11.25 + EI\theta_B$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(\theta_B + 2\theta_C - 3\delta/l\right)$$

$$= 3.75 + \frac{2EI}{4}\theta_B$$

$$= 3.75 + \frac{EI\theta_B}{2}$$

Joint equillibrium equation:

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow 90 + \frac{2EI\theta_B}{3} - 11.25 + EI\theta_B = 0$$

$$\Rightarrow \boxed{EI\theta_B = -47.25}$$

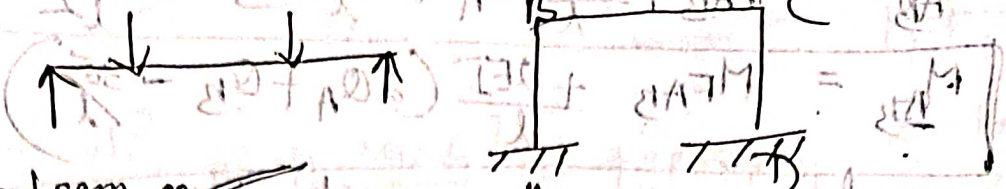
Final moments

$$M_{AB} = -90 + \frac{EI\theta_B}{3}$$

$$= -90 - \frac{47.25}{3} = -105.75 \text{ kNm}$$

Sub:- SA-II Date:- 27/08/20 Time:- 2.50-3.30

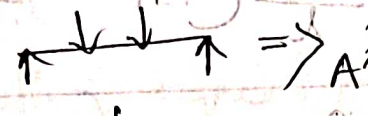
Application of slope deflection equation:-

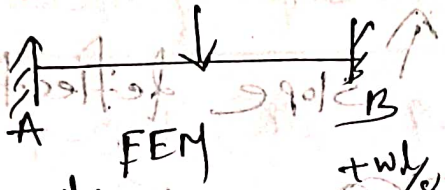


-> Continuous beam
-> frames with sway
-> frames without sway

Continuous beam

1) FEM





FEM

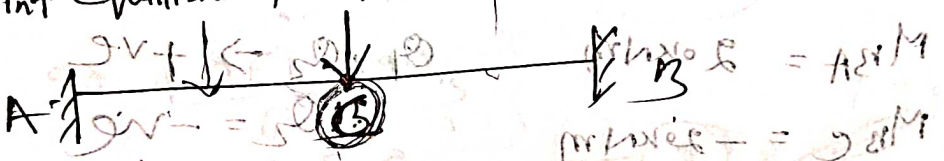
$-wl/8$ $+wl/8$

clockwise end moments → +ve

Anticlockwise end moments → -ve

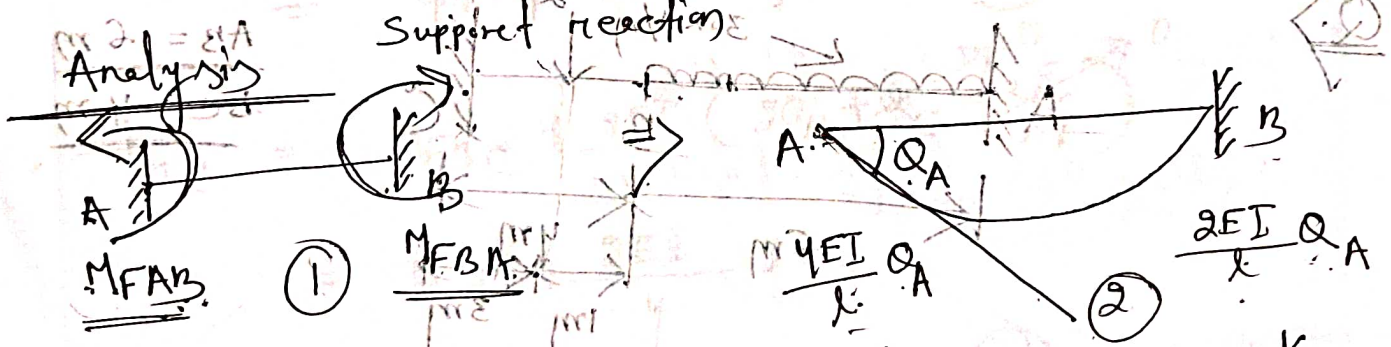
2) Slope deflection equation.

3) Joint equilibrium eqn.



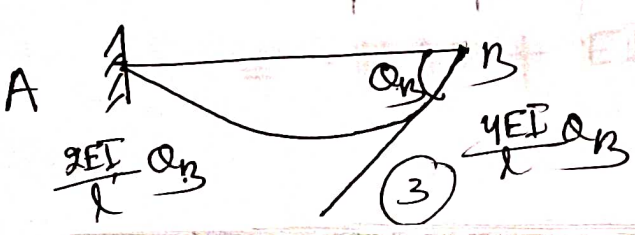
Analysis

Support reaction

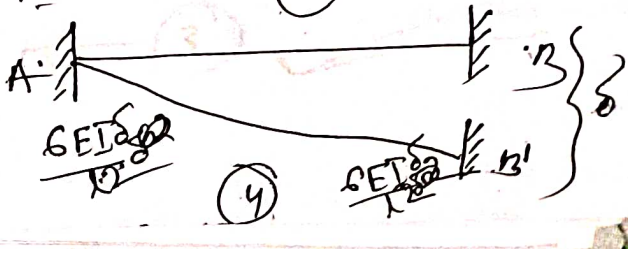


M_{FAB} ① M_{FBA}

$\frac{4EI}{L}\theta_A$ ② $\frac{2EI}{L}\theta_A$



$\frac{2EI}{L}\theta_B$ ③ $\frac{4EI}{L}\theta_B$



$\frac{6EI\delta}{L^2}$ ④ $\frac{6EI\delta}{L^2}$

Final moment at the end 'A' =

$$M_{AB} = M_{FAB} + \frac{4EI}{l}\theta_A + \frac{2EI}{l}\theta_B - \frac{6EI\delta}{l^2}$$

$$\boxed{M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)}$$

Final moment at the end 'B' =

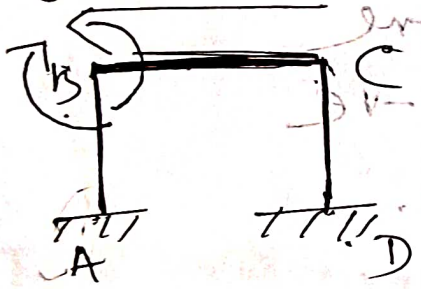
$$M_{BA} = M_{FBA} + \frac{2EI}{l}\theta_A + \frac{4EI}{l}\theta_B - \frac{6EI\delta}{l^2}$$

$$\boxed{M_{BA} = M_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\delta}{l}\right)}$$

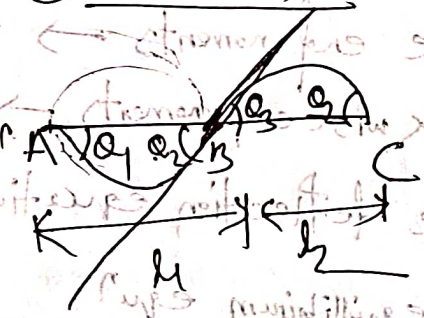
↑ Slope deflection eqn.

Sign Convention

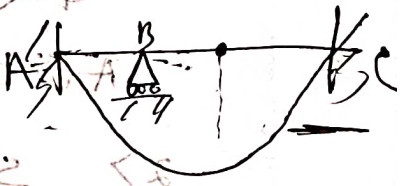
① Moment



② Rotation



③ Settlement



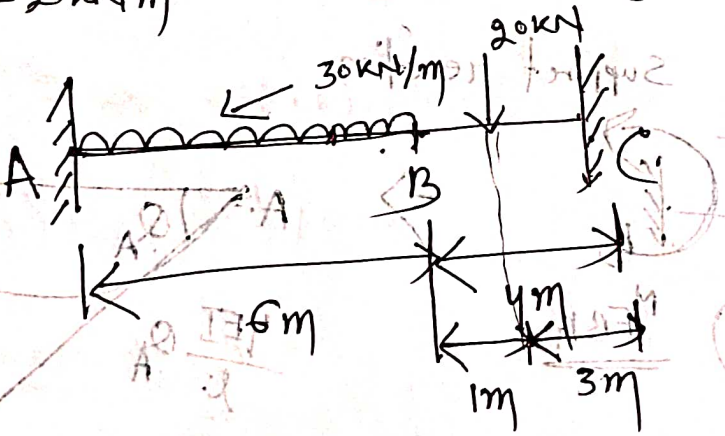
$M_{BA} = 20\,kN\,m$

$M_{BC} = -20\,kN\,m$

$\theta_1, \theta_2 \rightarrow +ve$

$\theta_2 = -ve$

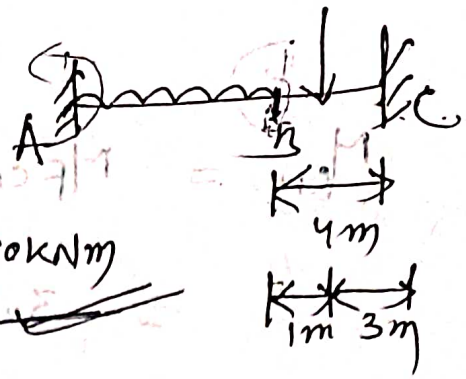
Q.



$AB = 6\,m$

$BC = 4\,m$

Calculate FEM



$$M_{FAB} = \frac{-wl^2}{12} = \frac{-30 \times 6^2}{12} = -90 \text{ kNm}$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{+30 \times 6^2}{12} = +90 \text{ kNm}$$

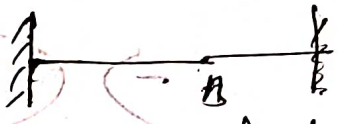
$a = 1m$, $b = 3m$

$$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-20 \times 1 \times 3^2}{4^2} = -11.25 \text{ kNm}$$

$$M_{FCB} = \frac{+wa^2b}{l^2} = \frac{+20 \times 1^2 \times 3}{4^2} = 3.75 \text{ kNm}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$$



At fixed end rotation = 0

$\theta_A = 0$

$\theta_C = 0$

$$= -90 + \frac{2EI}{6}(2 \times 0 + \theta_B)$$

$$= -90 + \frac{EI}{3}\theta_B \quad \text{---(1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A - \frac{3\delta}{l})$$

$$= 90 + \frac{2EI}{6} 2\theta_B$$

$$= 90 + \frac{2}{3} EI\theta_B \quad \text{---(2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$= -11.25 + \frac{2EI}{4} 2\theta_B$$

$$= -11.25 + EI\theta_B \quad \text{---(3)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\psi}{l}\right)$$

$$= 3.75 + \frac{2EI}{4}\theta_B$$

$$= 3.75 + \frac{1}{2}EI\theta_B \quad \text{(4)}$$

Joint equilibrium equation

$$\sum M_B = 0$$



$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow 90 + \frac{2EI\theta_B}{3} - 11.25 + EI\theta_B = 0$$

$$\Rightarrow EI\theta_B = -47.25 \quad \text{(1)}$$

Final moments

$$M_{AB} = -90 + \frac{EI\theta_B}{3}$$

$$= -90 + \frac{-47.25}{3} = -105.75\ kNm$$

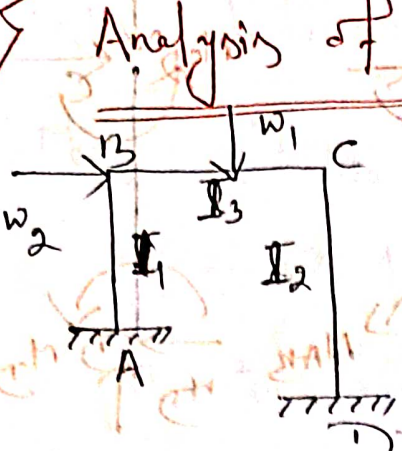
$$M_{BA} = 58.5\ kNm \quad \text{(3)}$$

$$M_{BC} = -58.5\ kNm$$

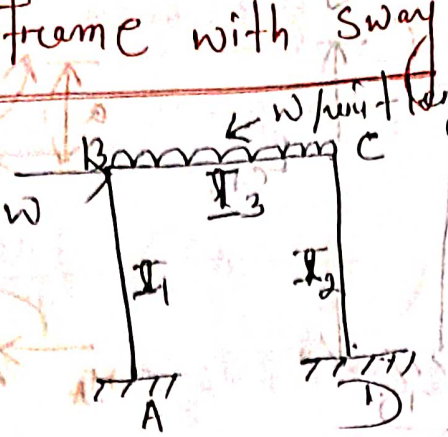
$$M_{CB} = -19.875\ kNm$$

27/08/20

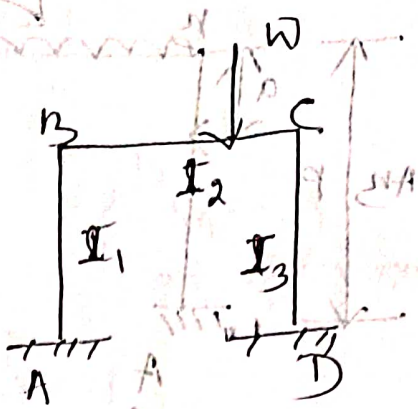
Analysis of frame with sway :-



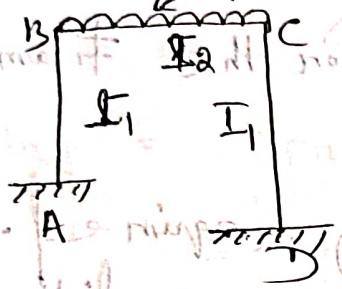
(sway due to horizontal force of unsymmetry)



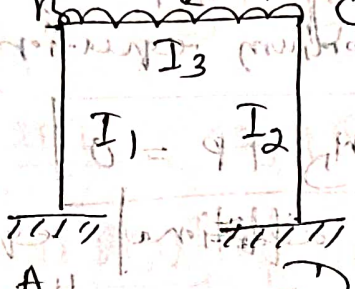
(Sway due to horizontal force)



(Sway due to unsymmetry of load)



(Sway due to unsymmetry of geometry)



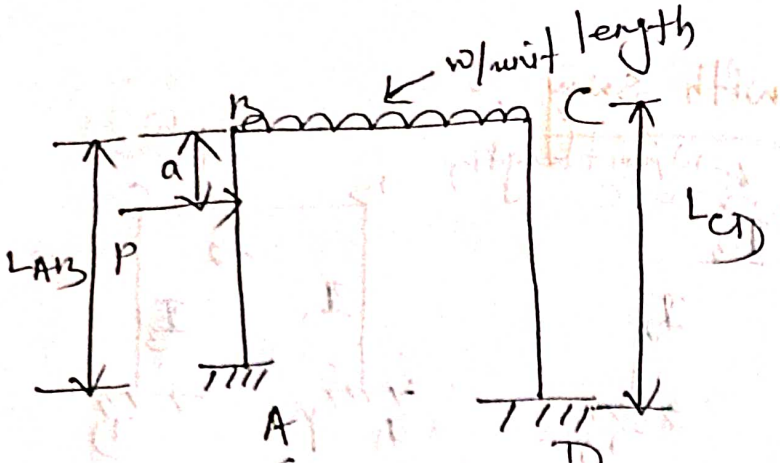
(Sway due to unsymmetry of properties of columns)

→ Let Δ be the sway, while writing the slope deflection equations. It may be noted that due to sway the ends are at different levels.

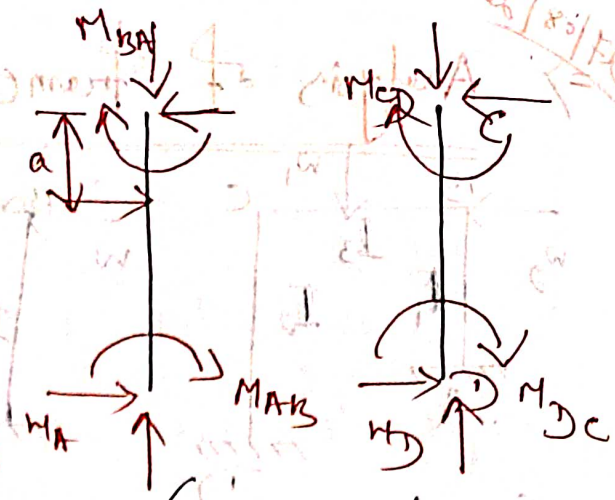
→ Taking moment about top joints, expression for horizontal shear at supports A and B will be

$$H_A = \frac{M_{AB} + M_{BA} - P.a}{L_{AB}}$$

$$H_D = \frac{M_{DC} + M_{CD}}{L_{CD}}$$



(Typical frame with sway)



(Free body diagram of Column)

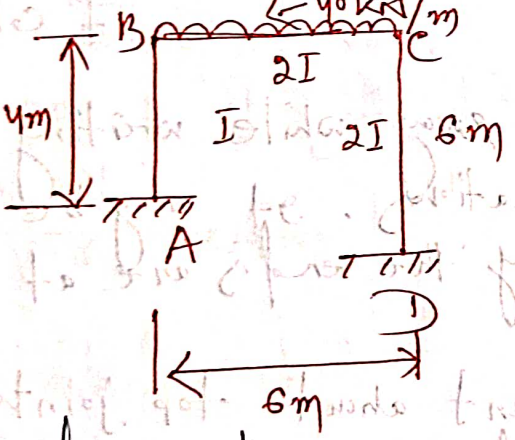
After knowing H_A & H_D we can write - Horizontal equilibrium equation for the frame

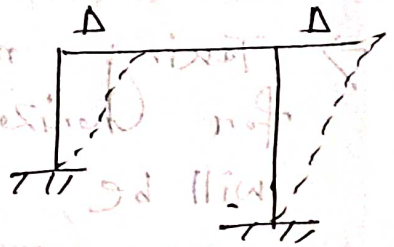
$$H_A + H_D + P = 0$$

This gives additional equation required. It is called shear equation / shear condition.

Hence, Q_A, Q_B and Δ can be found out.

Q.) Analyse the frame shown in fig. and draw B.M.D





Sol?

Fixed end moment

$M_{AB} = 0$ $\quad M_{CD} = 0$

$M_{BA} = 0$ $\quad M_{DC} = 0$

$$M_{BC} = \frac{-wl^2}{12} = \frac{-40 \times 6^2}{12} = -120 \text{ kN m}$$

$$M_{CB} = +wl^2/12 = \frac{+40 \times 6^2}{12} = +120 \text{ kNm}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

($\theta_A = 0$ at 'A' is fixed)

$$= 0 + \frac{2EI}{4}\left(\theta_B - \frac{3\delta}{4}\right)$$

$$= \frac{EI\theta_B}{2} - \frac{3}{8}EI\delta \quad \text{——— (1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - 3\delta/l\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_B - 3\delta/l\right)$$

$$= EI\theta_B - \frac{3}{8}EI\delta \quad \text{——— (2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - 3\delta/l\right)$$

$$= -120 + \frac{2E(2I)}{6}\left(2\theta_B + \theta_C\right)$$

$$= -120 + \frac{4EI}{3}\theta_B + \frac{2EI\theta_C}{3} \quad \text{——— (3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - 3\delta/l\right)$$

$$= 120 + \frac{2E(2I)}{6}\left(\theta_B + 2\theta_C\right)$$

$$= 120 + \frac{4EI\theta_C}{3} + \frac{2EI\theta_B}{3} \quad \text{——— (4)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - 3\delta/l\right)$$

$$= 0 + \frac{2E(2I)}{6}\left(2\theta_C + \theta_D - 3\delta/6\right)$$

$$= \frac{4EI\theta_C}{3} - \frac{EI\delta}{3} \quad \text{——— (5)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - 3\delta/l\right)$$

$$= 0 + \frac{2E(2I)}{6}\left(\theta_C - 3\delta/6\right)$$

$$= \frac{2EI\theta_C}{3} - \frac{EI\delta}{3} \quad \text{——— (6)}$$

Joint equilibrium equation :-

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow EI\theta_B - \frac{3}{8}EI\delta + \left(-120 + \frac{4EI\theta_B}{3} + \frac{2}{3}EI\theta_C\right) = 0$$

$$\Rightarrow \boxed{2.333\,EI\theta_B + 0.667\,EI\theta_C - 0.375\,\delta = 120} \quad (7)$$

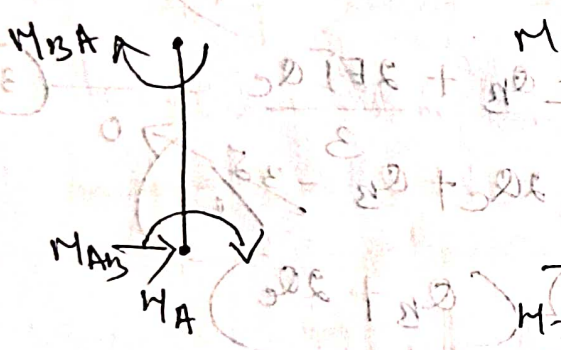
$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$120 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B + \frac{4}{3}EI\theta_C - \frac{EI\delta}{3} = 0$$

$$\Rightarrow \boxed{0.667\,EI\theta_B + 2.667\,EI\theta_C - 0.333\,\delta = -120} \quad (8)$$

Considering free body diagram of columns and taking moment about top joints, we get



$$H_A = \frac{M_{AB} + M_{BA}}{l_{AB}} \qquad H_D = \frac{M_{CD} + M_{DC}}{l_{CD}}$$

Now Considering horizontal equilibrium of a frame we get,

$$H_A + H_D = 0$$

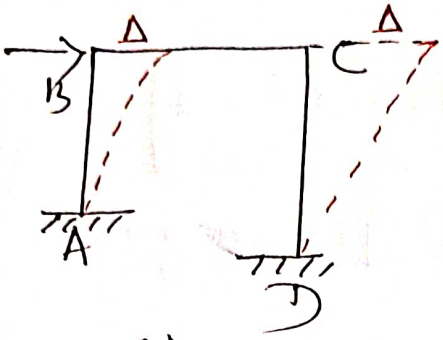
$$\frac{M_{AB} + M_{BA}}{4} + \frac{M_{CD} + M_{DC}}{6} = 0$$

=> 29/08/20

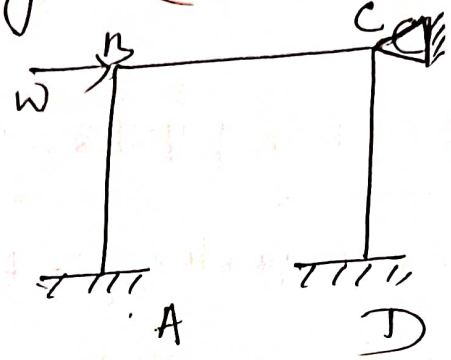
29/08/20

Analysis of frames without sway :-

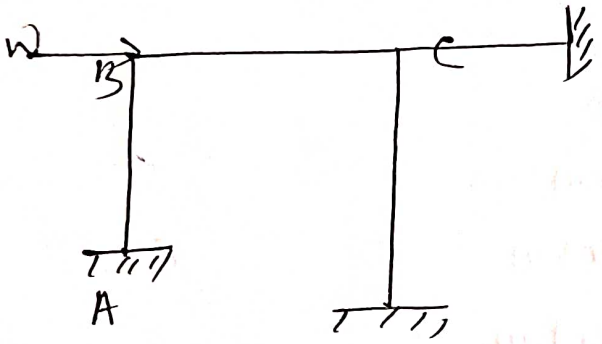
→ The side movement of the end column in a frame is called sway. (Δ or δ)



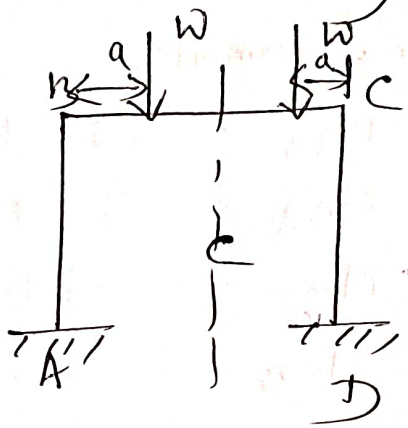
(frame with sway)



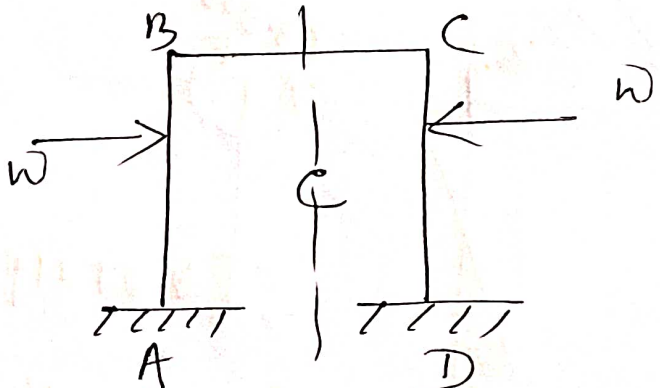
(sway prevented by unyielding support)



(Sway prevented by unyielding support)



(No sway due to symmetry)



(No sway due to symmetry)

$$\Rightarrow 3M_{AB} + 3M_{BA} + 2M_{CB} + 2M_{DC} = 0$$

$$\Rightarrow 3(0.5EI\theta_B - 0.375EI\delta + EI\theta_B - 0.375EI\delta) + 2(1.333EI\theta_C - 0.333EI\delta + 0.667EI\theta_C - 0.333EI\delta) = 0$$

$$\Rightarrow \boxed{4.5EI\theta_B + 4EI\theta_C - 3.583EI\delta = 0} \quad \text{——— (9)}$$

Solving equation (7), (8), (9) we get

$$\begin{cases} EI\delta = 23.842 \checkmark \\ EI\theta_C = -60.127 \checkmark \\ EI\theta_B = 72.43 \checkmark \end{cases}$$

Final moments

$M_{AB} = 22.274\ KNm$

$M_{BA} = 63.489\ KNm$

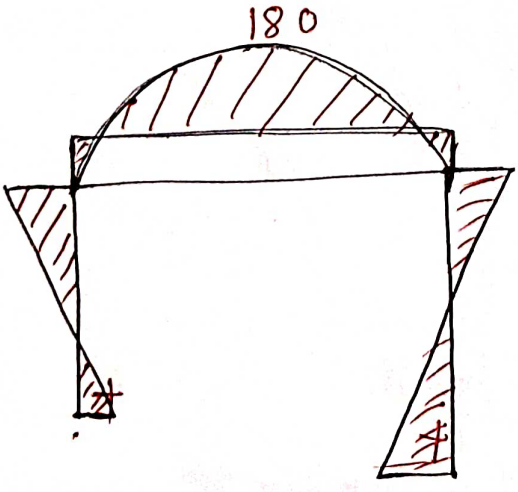
$M_{BC} = -63.556\ KNm$

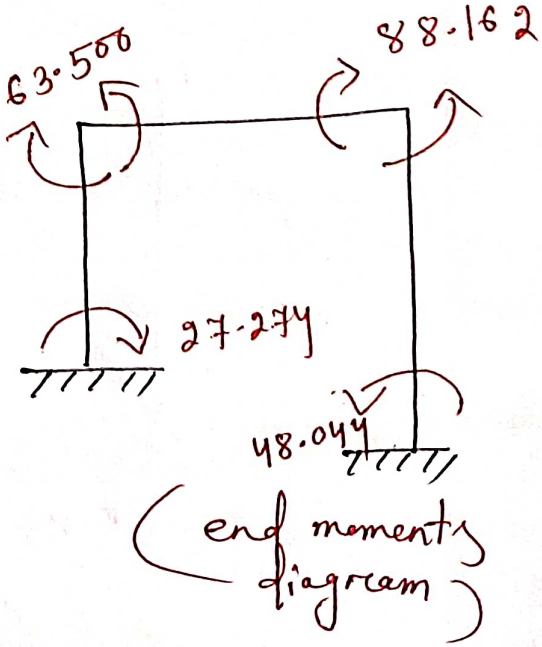
$M_{CB} = 88.162\ KNm$

$M_{CD} = -88.162\ KNm$

$M_{DC} = -48.044 KNm.$

B.M.D



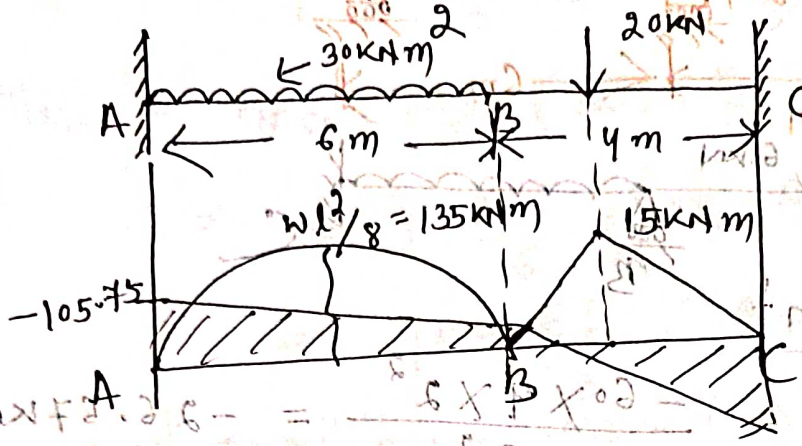


(end moments diagram)

$$M_{BA} = 90 + \frac{2EI\theta_B}{3} = 90 - \frac{2 \times 47.25}{3} = 58.5 \text{ kNm}$$

$$M_{BC} = -11.25 + EI\theta_B = -11.25 - 47.25 = -58.5 \text{ kNm}$$

$$M_{CB} = 3.75 + \frac{EI\theta_B}{2} = 3.75 - \frac{47.25}{2} = -19.875 \text{ kNm}$$



For span AB, $B.M._{(max)} = \frac{wl^2}{8}$

$$= \frac{30 \times 6^2}{8} = 135 \text{ kNm}$$

For span BC, $B.M._{(max)} = \frac{wab}{l}$

$$= \frac{20 \times 1 \times 3}{4}$$

$$= 15 \text{ kNm}$$

Reaction at the support :

Taking moment about point 'B'

$$\Sigma M_B = 0$$

$$R_a \times 6 + \frac{30 \times 6 \times 6}{2} - 105.75 + 58.5 = 0$$

$$\Rightarrow \boxed{R_a = 97.875 \text{ kN}}$$

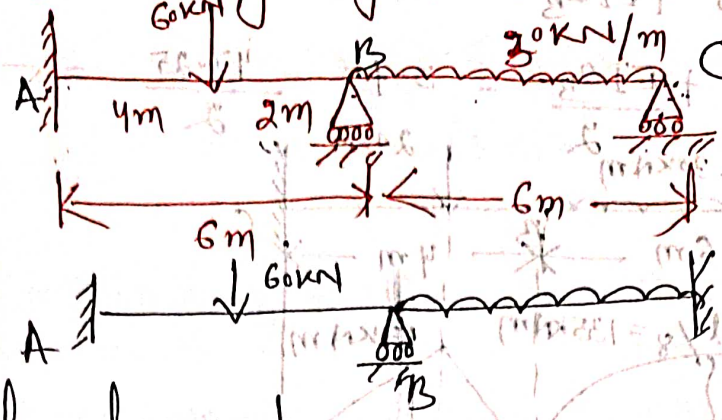
$$\Sigma M_C = 0$$

$$R_c \times 4 - 20 \times 1 - 19.875 - 58.5 = 0$$

$$\boxed{R_c = -4.65 \text{ kN}}$$

24/08/20

Q.) Analyse the continuous beam. Draw the B.M.D as shown in fig. by slope deflection method.



Soln:-

Fixed end moment

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-60\times4\times2^2}{6^2} = -26.67\text{KNm}$$

$$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{+60\times4^2\times2}{6^2} = 53.33\text{KNm}$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{30\times6^2}{12} = -90\text{KNm}$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{+30\times6^2}{12} = +90\text{KNm}.$$

Using slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$$

($\theta_A = 0$ as fixed end)

$$= -26.67 + \frac{2EI}{6}\theta_B$$

$$= -26.67 + \frac{EI\theta_B}{3} \quad \text{---(1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A - \frac{3\delta}{l})$$

$$= 53.33 + \frac{2EI}{6}\times2\theta_B$$

$$= 53.33 + \frac{2EI\theta_B}{3} \quad \text{---(2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -90 + \frac{2EI}{6}\left(2\theta_B + \theta_C\right)$$

$$= -90 + \frac{EI}{3}\left(2\theta_B + \theta_C\right) \quad \text{——— (3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 90 + \frac{2EI}{6}\left(2\theta_C + \theta_B\right)$$

$$= 90 + \frac{EI}{3}\left(2\theta_C + \theta_B\right) \quad \text{——— (4)}$$

Joint equillibrium equation

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0 \quad \text{——— (A)}$$

$$\Rightarrow 53.33 + \frac{2EI\theta_B}{3} - 90 + \frac{EI}{3}\left(2\theta_B + \theta_C\right) = 0$$

$$\Rightarrow \boxed{53.33 + \frac{2}{3}EI\theta_B - 90 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C = 0}$$

$$\Sigma M_C = 0$$

$$M_{CB} = 0$$

$$\Rightarrow \boxed{90 + \frac{EI}{3}\left(2\theta_C + \theta_B\right) = 0}$$

$$\Rightarrow 90 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B = 0$$

$$\Rightarrow 90 + \frac{EI\theta_B + 2EI\theta_C}{3} = 0$$

$$\Rightarrow \frac{EI\theta_B + 2EI\theta_C}{3} = -90$$

$$\Rightarrow EI\theta_B + 2EI\theta_C = -90 \times 3$$

$$\boxed{EI\theta_B + 2EI\theta_C = -270} \quad \text{——— (5)}$$

$2x + 3y = 0$

$5x + 4y = 0$

$x = ?$

$y = ?$

$$53.33 + \frac{2EI}{3}\theta_B - 90 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C = 0$$

$$\Rightarrow -36.67 + \frac{4}{3}EI\theta_B + \frac{1}{3}EI\theta_C = 0$$

$$\Rightarrow -36.67 + \frac{4EI\theta_B + EI\theta_C}{3} = 0$$

$$\Rightarrow 4EI\theta_B + EI\theta_C = 36.67 \times 3$$

$$\Rightarrow \boxed{4EI\theta_B + EI\theta_C = 110.01} \quad \text{— (6)}$$

Solving the eqn (5) & (6)

$$2\times\{4EI\theta_B + EI\theta_C = 110.01 \times 2$$

$$EI\theta_B + 2EI\theta_C = -270$$

$$7EI\theta_B = 490$$

$$\Rightarrow \boxed{EI\theta_B = 70}$$

Putting the value of $EI\theta_B$ in eq (6)

$$4\times 70 + EI\theta_C = 110.01$$

$$\Rightarrow \boxed{EI\theta_C = -170}$$

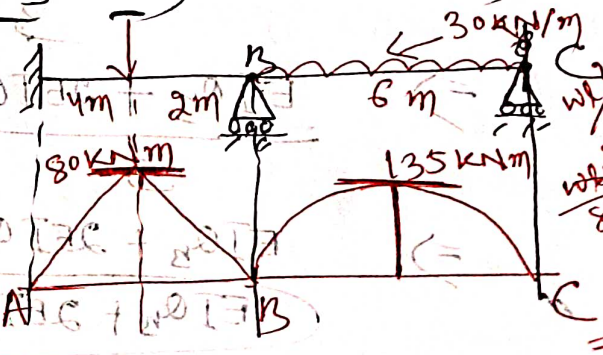
Final moment

$$M_{AB} = -26.67 + \frac{EI\theta_B}{3} = -26.67 + \frac{70}{3} = -3.33 \text{ kNm}$$

$$M_{BA} = 53.33 + \frac{2EI\theta_B}{3} = 100 \text{ kNm}$$

$$M_{BC} = -90 + \frac{EI}{3}(2\theta_B + \theta_C) = -100 \text{ kNm}$$

$$M_{CB} = 0$$



$\frac{10}{60}\times 4\times 2 \quad \frac{wab}{L} = \frac{6}{6}$

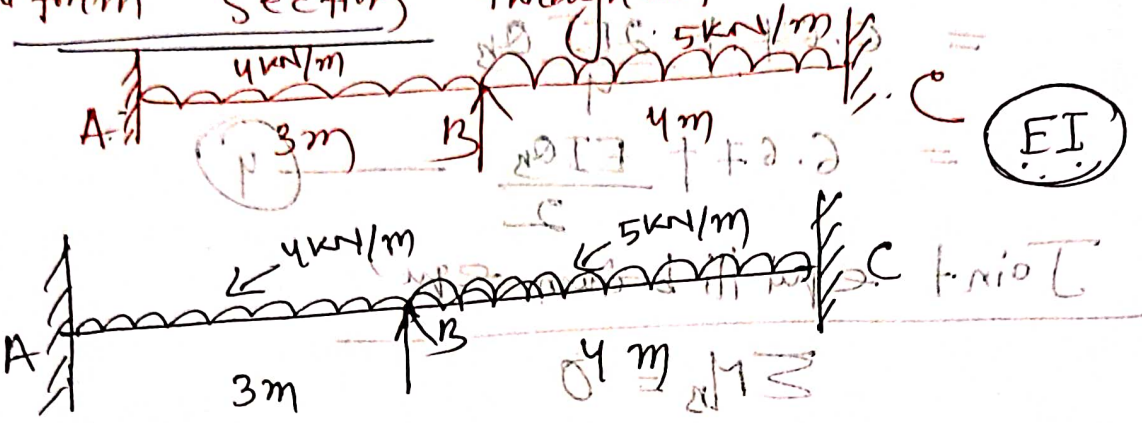
$\frac{30\times 6^2}{8} \quad \frac{wL}{8}\,6$

$\frac{wL^2}{8} = \frac{30\times 36}{8}$

$= 135$ kNm

01/08/20

Q. A continuous beam ABC consists of spans AB = 3m and BC = 4m, the ends A and C being fixed. AB & BC carry UDL of intensity 4kN/m and 5kN/m respectively. Find the support moments and draw B.M.D for the beam. The beam is of uniform section throughout.



FE.M

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-4 \times 3^2}{12} = -3\ kNm$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{+4 \times 3^2}{12} = +3\ kNm$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-5 \times 4^2}{12} = -6.67\ kNm$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{+5 \times 4^2}{12} = +6.67\ kNm$$

Slope deflection eqn

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - 3\delta/l)$$

$\theta_A = 0$ (A is a fixed support)

$$= -3 + \frac{2EI}{3}\theta_B \quad \text{---(1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A - 3\delta/l)$$

$$= +3 + \frac{2EI}{3}2\theta_B$$

$$= 3 + \frac{4}{3}EI\theta_B \quad \text{---(2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -6.67 + \frac{2EI}{4} 2\theta_B$$

$$= -6.67 + EI\theta_B \quad \text{---(3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 6.67 + \frac{2EI}{4}\theta_B$$

$$= 6.67 + \frac{EI\theta_B}{2} \quad \text{---(4)}$$

Joint equilibrium eqn

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$3 + \tfrac{1}{2}EI\theta_B + (-6.67 + EI\theta_B) = 0$$

$$\Rightarrow \boxed{EI\theta_B = 1.5728.6}$$

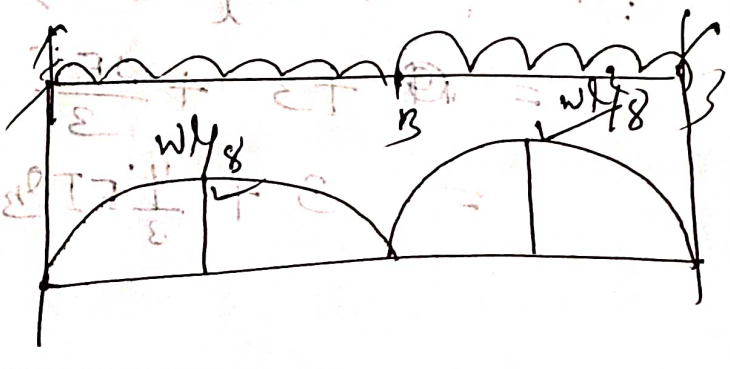
Final moment

$$M_{AB} = -1.95 \text{ kNm}$$

$$M_{BA} = +5.10 \text{ kNm}$$

$$M_{BC} = -5.10 \text{ kNm}$$

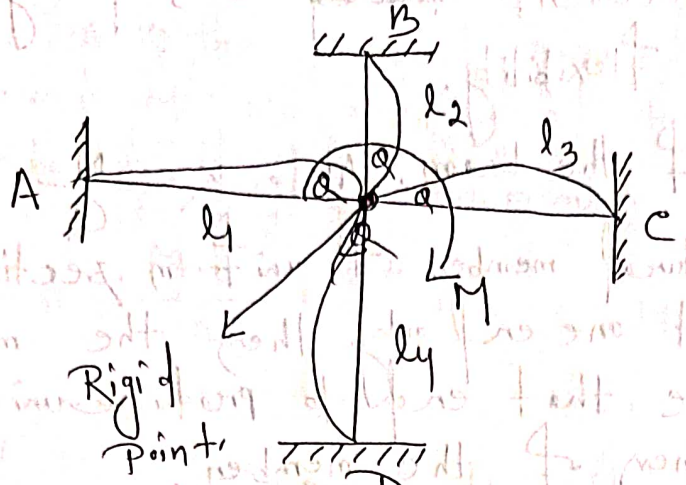
$$M_{CB} = +17.46 \text{ kNm}$$



Moment distribution method :-

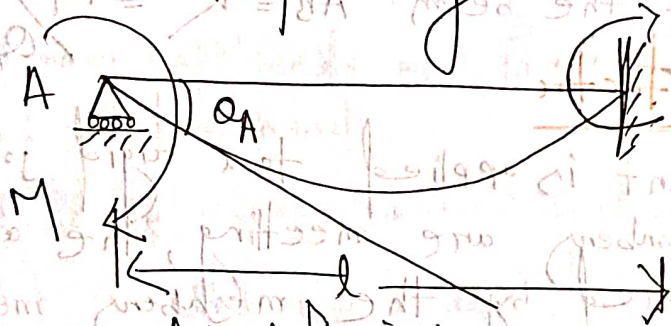
-> This method is used for the analysis of indeterminate structure.

-> Distribute the moment in to the individual moment.



Carry over moment :- (COM)

-> When a moment is applied at one end of a member allowing rotation of that end & fixing the far end, some moment develop at the far end also. This moment is called Carry over moment.



In this beam AB if 'M' is the applied moment at end 'A'. allowing rotation of 'A' & M' is the moment developed at B, then M' is called as Carry over moment.

Carry over factor : (COF)

-> The ratio of Carry over moment to applied moment is called carry over factor.

$$COF = \frac{M'}{M} = \frac{\text{Far end moment}}{\text{Near end moment.}}$$

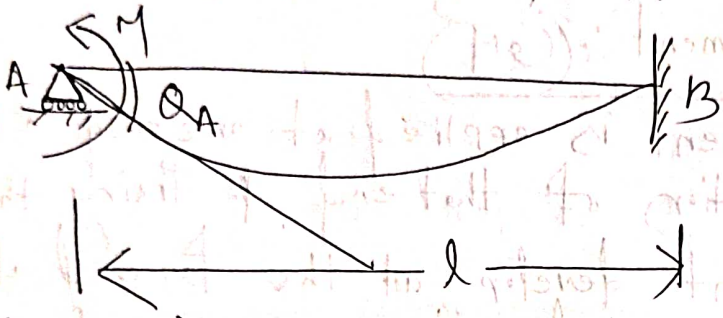
Stiffness:-

→ The force or moment required per unit displacement is called stiffness.

→ The displacement ~~Force~~ caused by unit force is called flexibility.

→ stiffness of the beam, $AB = K = \frac{M}{\theta}$

→ when structural member of uniform section is subjected to a moment at one end only then the moment required so as to rotate that end to produce unit slope is called stiffness of the member.



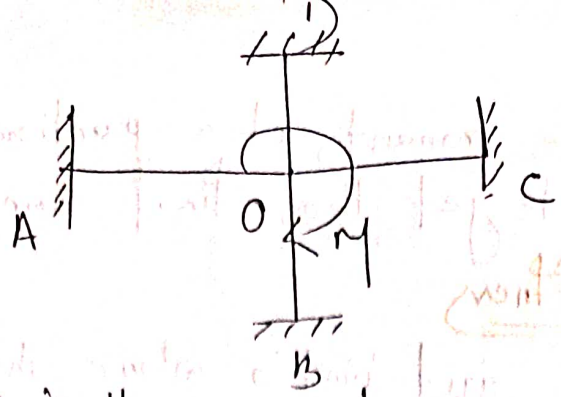
Stiffness of the beam $AB = K = M/\theta_A$

Distribution factor

→ when a moment is applied to a rigid joint where number of members are meeting, the applied moment is shared by the members meeting at that joint.

→ The ratio of the moment shared by a member to the applied at that joint is called the distribution factor of that member.

→ The distribution factor for a member at a joint is the ratio of the stiffness of the member to the total stiffness at all the members meeting at that joint.



Let M_{OA} is the moment shared by a member OA, when moment 'M' is applied at that point 'O'. Then the distribution factor for member OA is

$$DF_{OA} = \frac{M_{OA}}{M}$$

$$D.F = \frac{K}{\Sigma K}$$

Application of moment distribution method to Continuous beam with fixed end

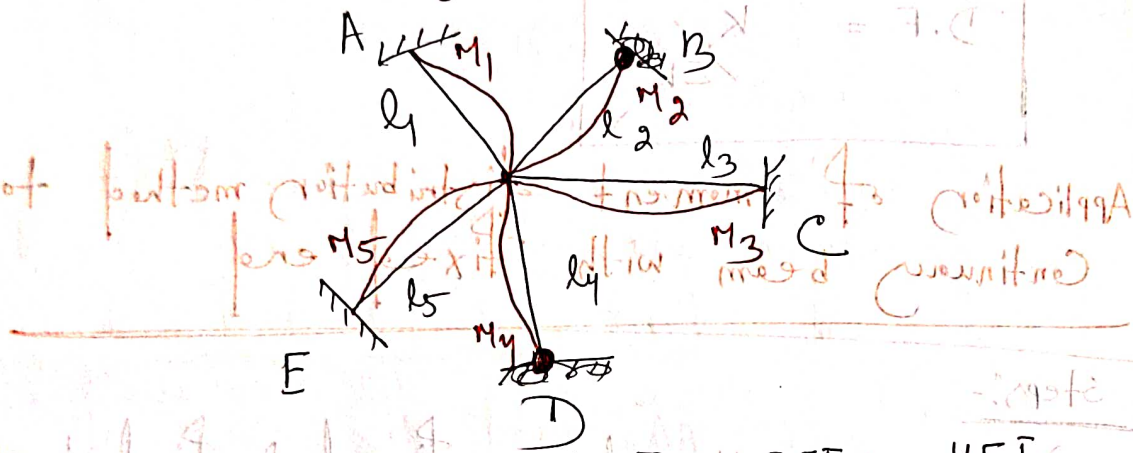
Steps:-

1) Assuming all ends are fixed & find out the fixed end moment.
2) Calculate distribution factor for all the members meeting at a joint.
3) Balance a joint by distributing balancing moment to various members meeting at that joint proportional to their distribution factor. Do similar exercise for all joints.
4) Carry over half the distribution moment to the far end of the member.
5) Repeat step 3 & 4 still distributed moments are negligible.

6) sum of all the moments at a particular end of the member to get the final moment.

Relative stiffness

→ Consider the rigid joint 'o' where the member AO, BO, CO, DO & EO meet at that point. If 'M' is applied at that rigid joint 'o'. we know it gets distributed among the member into the components M_1, M_2, M_3, M_4 and M_5 in proportion of their stiffness.



$$M_1 : M_2 : M_3 : M_4 : M_5 = \frac{4EI_1}{l_1} : \frac{3EI_2}{l_2} : \frac{4EI_3}{l_3} : \frac{3EI_4}{l_4} : \frac{4EI_5}{l_5}$$

Divide 4E

$$= \frac{I_1}{l_1} : \frac{3}{4}\frac{I_2}{l_2} : \frac{I_3}{l_3} : \frac{3}{4}\frac{I_4}{l_4} : \frac{I_5}{l_5}$$

Note:-

1) The relative stiffness of a member at a joint whose farther end is fixed is I/l or $4EI/l$

2) The relative stiffness of a member at a joint whose farther end is hinged or simply supported is $3EI/l$ (or) $\frac{3EI}{4l}$

=> Continuous beam with simply supported end :-

Method-1

1) To start with simply supported end is also treated as fixed end and fixed end moments are calculated.

2) In successive iteration also carry over moments are coming to this end, if moment exist at this end, it means end is not balanced.

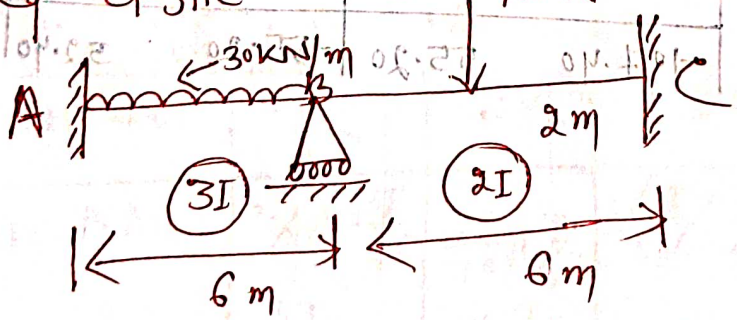
3) Since at this joint only one member exist distribution factor is 1. Rest of the procedure is same and discussed for the fixed end Continuous beam.

Method-2

1) Carry over moment is calculated at other end. For further calculation modified stiffness is used for the span which is having simply supported end and carry over factor is taken as '0'.

2) In this method also fixed end moments are calculated. At simply supported end then joint balance is required by distribution factor is 1.

Q.) Analyse the Continuous beam as shown in fig. by moment distribution method and draw B.M.D, S.F.D and elastic curve.



Sol:- **Fixed end moment**

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-30 \times 6^2}{12} = -90\ kNm$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{30 \times 6^2}{12} = 90\ kNm$$

$$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-72 \times 4 \times 2^2}{6^2} = -32\ kNm$$

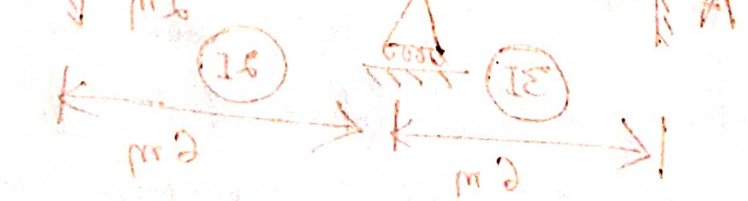
$$M_{FCB} = \frac{wa^2b}{l^2} = \frac{72 \times 4^2 \times 2}{6^2} = 64\ kNm$$

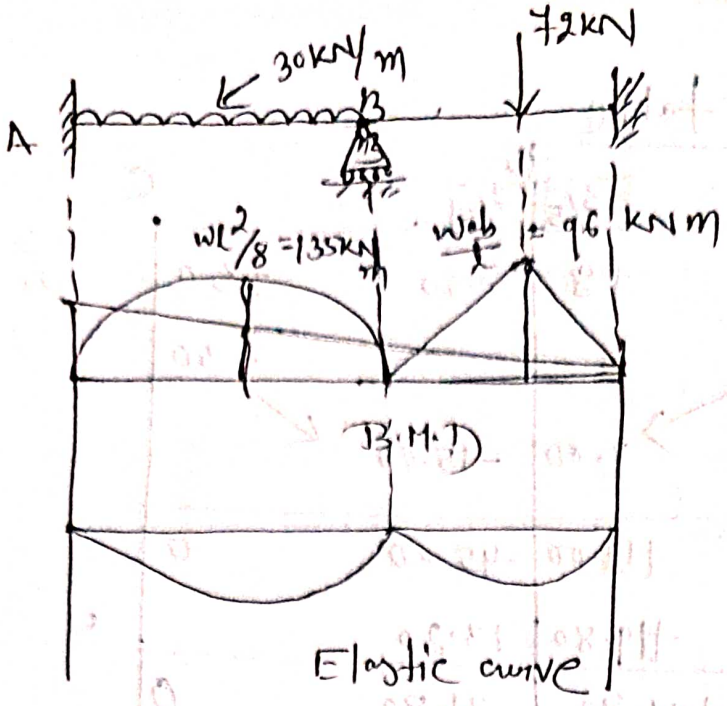
Distribution factor

Joint	Member	Relative stiffness (K)	ΣK	$D.F = (K/\Sigma K)$
B	BA	$I/l = 3I/6 = I/2$	$I/2 + I/3$	$(I/2)/(5I/6) = 3/5$
	BC	$I/l = 2I/6 = I/3$	$= 5I/6$	$(I/3)/(5I/6) = 2/5$

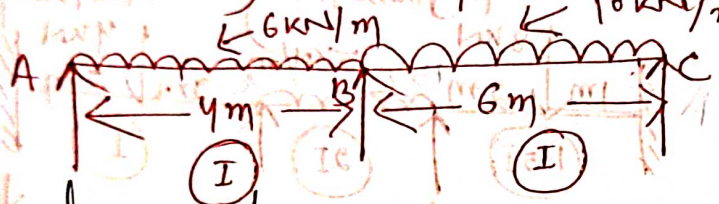
Distribution table

	A	B		C
		3/5	2/5	
FEM	−90	+90	−32	+64
Balancing		−34.8	−23.20	
		−17.40		−11.60
	−107.40	55.20	−55.20	52.40





Q) Analyse the continuous beam as shown in fig. by moment distribution method.



Sol?- Fixed end moment

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-6\times4^2}{12} = -8\ kN\text{-}m$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{6\times4^2}{12} = 8\ kN\text{-}m$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-10\times6^2}{12} = -30\ kN\text{-}m$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{10\times6^2}{12} = 30\ kN\text{-}m$$

Distribution factor

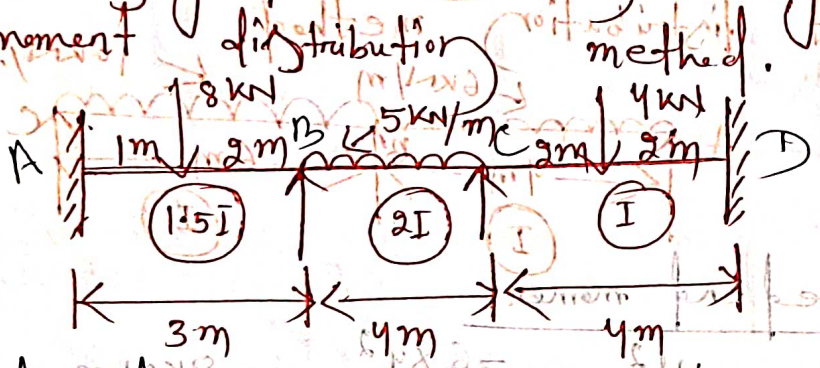
Joint	Member	K	ΣK	D.F = (K/ΣK)
B	BA	$3I/4l = 3I/16$	$\frac{3I}{16} + \frac{I}{8}$	$(3I/16)/(5I/16) = 3/5$
	BC	$\frac{3I}{4}/l = I/8$	$= 5I/16$	$(I/8)/(5I/16) = 2/5$

Distribution table

	A	B		C
		3/5	2/5	
FEM	−8	+8	−30	+30
Balancing	+8.00			+30
COM (½ × balancing value)		4.00	−15.00	
	0	+12.00	−45.00	0
Balancing		+19.80	13.20	
	0	+31.80	−31.80	0

08/09/20

Q. Determine the support moment at A, B, C & D the continuous girder as shown in fig. by using moment distribution method.



Soln:- Fixed end moment

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-8\times1\times2^2}{3^2} = -3.56\ \text{kNm}$$

$$M_{FBA} = \frac{wa^2b}{l^2} = \frac{8\times1^2\times2}{3^2} = 1.78\ \text{kNm}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-5\times4^2}{12} = -6.67\ \text{kNm}$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{+5\times4^2}{12} = 6.67\ \text{kNm}$$

$$M_{FCD} = -wl/8 = \frac{-4\times4}{8} = -2\ \text{kNm}$$

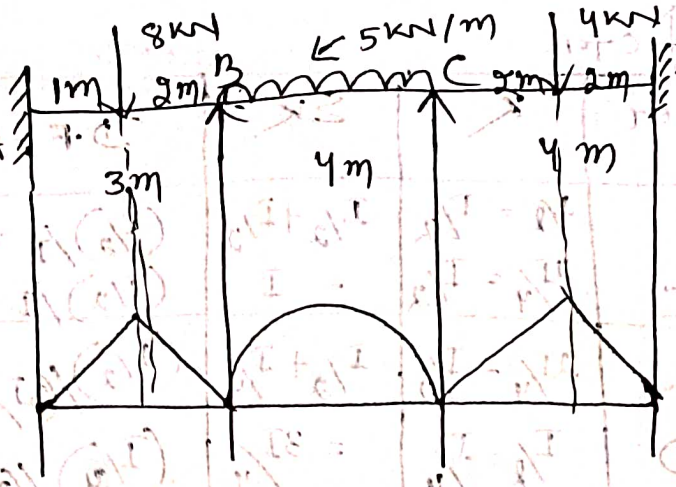
$$M_{FDC} = +wl/8 = \frac{4\times4}{8} = 2\ \text{kNm}$$

Distribution factor

Joint	Member	K	ΣK	D.F = (K/ΣK)
B	BA	$I/\ell = I/2$	$I/2 + I/2$	$(I/2)/I = 1/2$
	BC	$2I/4 = I/2$	$= I$	$(I/2)/I = 1/2$
C	CB	$2I/4 = I/2$	$I/2 + I/4$	$(I/2)/(3I/4) = 2/3$
	CD	$I/\ell = I/4$	$= 3I/4$	$(I/4)/(3I/4) = 1/3$

Distribution table

	A	B		C		D
		1/2	1/2	2/3	1/3	
FEM	-3.56	1.78	-6.67	+6.67	-2	2
Balancing		2.45	2.45	-3.11	-1.56	
COM	1.22		-1.56	1.23		-0.78
Balancing		+0.78	+0.78	-0.82	-0.41	
	0.39		-0.41	0.39		-0.20
		+0.20	+0.20	-0.26	-0.13	
	0.10		-0.13	0.10		-0.10
		+0.07	+0.07	-0.07	-0.03	
	0.04		-0.03	+0.03		-0.02
		0.02	0.02	-0.02	-0.01	
	-1.81	5.3	-5.3	4.14	-4.14	0.93



For Span AB, Maximum B.M $= \frac{Wab}{l} = \frac{8 \times 1 \times 2}{3} = \frac{16}{3}$
$= 5.33 \text{ KNm}$

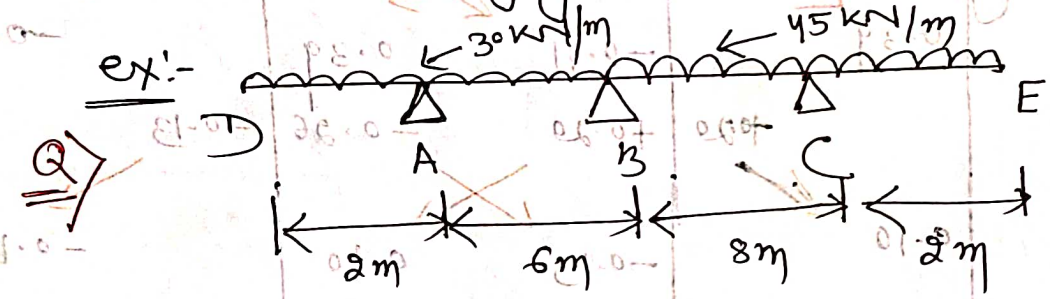
For Span BC, Maximum B.M $= Wl^2/8 = \frac{5 \times 4^2 \times 4}{8}$
$= 10 \text{ KNm}$

For Span CD, Maximum B.M $= Wl/4 = \frac{4 \times 4}{4}$
$= 4 \text{ KNm}$

⇒ In case of overhanging beam :-

1) Calculate the fixed end moment.
2) 1st enter the ~~over hanging~~ moment. (fixed end moment)

ex:-



Q)

$M_{FCE} = -90 \text{ KNm}$

$M_{FAD} = +60 \text{ KNm}$

3) Then applying Correction i-e +90KNm at 'C' the member CB and −90KNm at 'A' for the member AB.

4) Now using carry over moment (COM) at point 'B' for the member BA $= -60/2 = -30 \text{ KNm}$.

COM at point 'B' for the member BC $= +90/2 = +45 \text{ KNm}$.

5) Then apply a correction. At point 'C' for member CB = −240 kNm. At point 'B' for member AB = +90 kNm.

6) Now using carry over moment.

7) Then add the result and find out the final moment.

Solve the above example by moment distribution method

Sol:-

Fixed end moment

$$M_{FAB} = -wl^2/12 = \frac{-30\times6^2}{12} = -90\text{ kNm}$$

$$M_{FBA} = wl^2/12 = \frac{30\times6^2}{12} = 90\text{ kNm}$$

$$M_{FBC} = -wl^2/12 = \frac{-45\times8^2}{12} = -240\text{ kNm}$$

$$M_{FCB} = +wl^2/12 = +240\text{ kNm}$$

$$M_{FAD} = +wl^2/2 = +\frac{30\times2^2}{2} = +60\text{ kNm}$$

$$M_{FDA} = -wl^2/2 = -\frac{30\times2^2}{2} = -60\text{ kNm}$$

$$M_{FCE} = -wl^2/2 = \frac{-45\times2^2}{2} = -90\text{ kNm}$$

$$M_{FEC} = +\frac{wl^2}{2} = \frac{+45\times2^2}{2} = 90\text{ kNm}$$

Note

In case of overhanging the point load is applied then fixed end moment = wl

In case of udl, then fixed end moment = $w\times l\times l/2 = wl^2/2$.

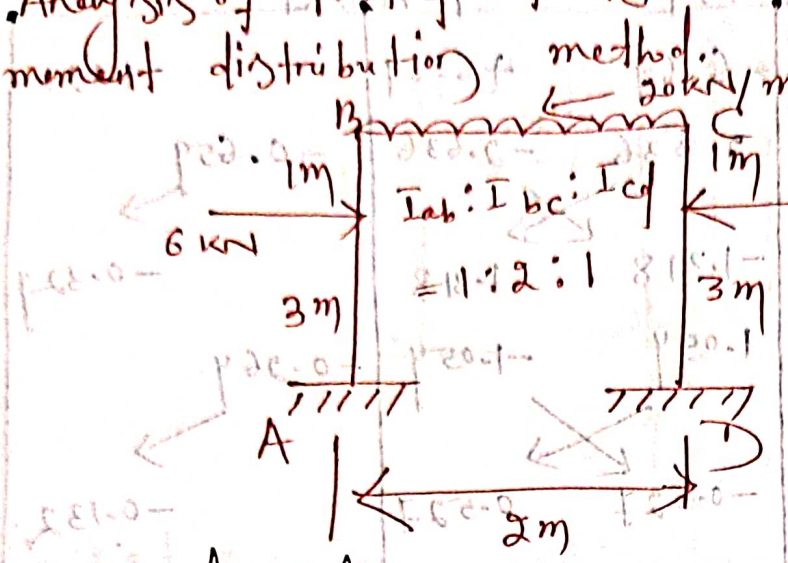
Distribution factors

Joint	Member	K	ΣK	D.F (K/ΣK)
B	BA	$\frac{3}{4} \times I/6 = I/8$	$I/8 + 3I/32 = 7I/32$	$I/8 \times 32/7I = 4/7$
	BC	$\frac{3}{4} \times I/8 = 3I/32$		$3I/32 \times 32/7I = 3/7$

Distribution table

	A	B		C	
		4/7	3/7		
FEM	+60	-90	+90	-240	+240	-90
		-60 →	-30	-45	← +90	
		+90 →	45	-120	← -240	
	+60	-60	+105	-315	+90	-90
			+120	+90		
	+60	-60	+225	-225	+90	-90

Q.) Analysis of portal frame shown in fig. by using moment distribution method.



Sol?

Fixed end moment

$M_{FAB} = -wab^2/l^2 = \frac{-6 \times 3 \times 1^2}{1^2} = -1.125 kNm$

$M_{FBA} = wa^2b/l^2 = \frac{6 \times 3^2 \times 1}{4^2} = 3.375 kNm$

$M_{FBC} = -wl^2/12 = \frac{-20 \times 2^2}{12} = -6.67 kNm$

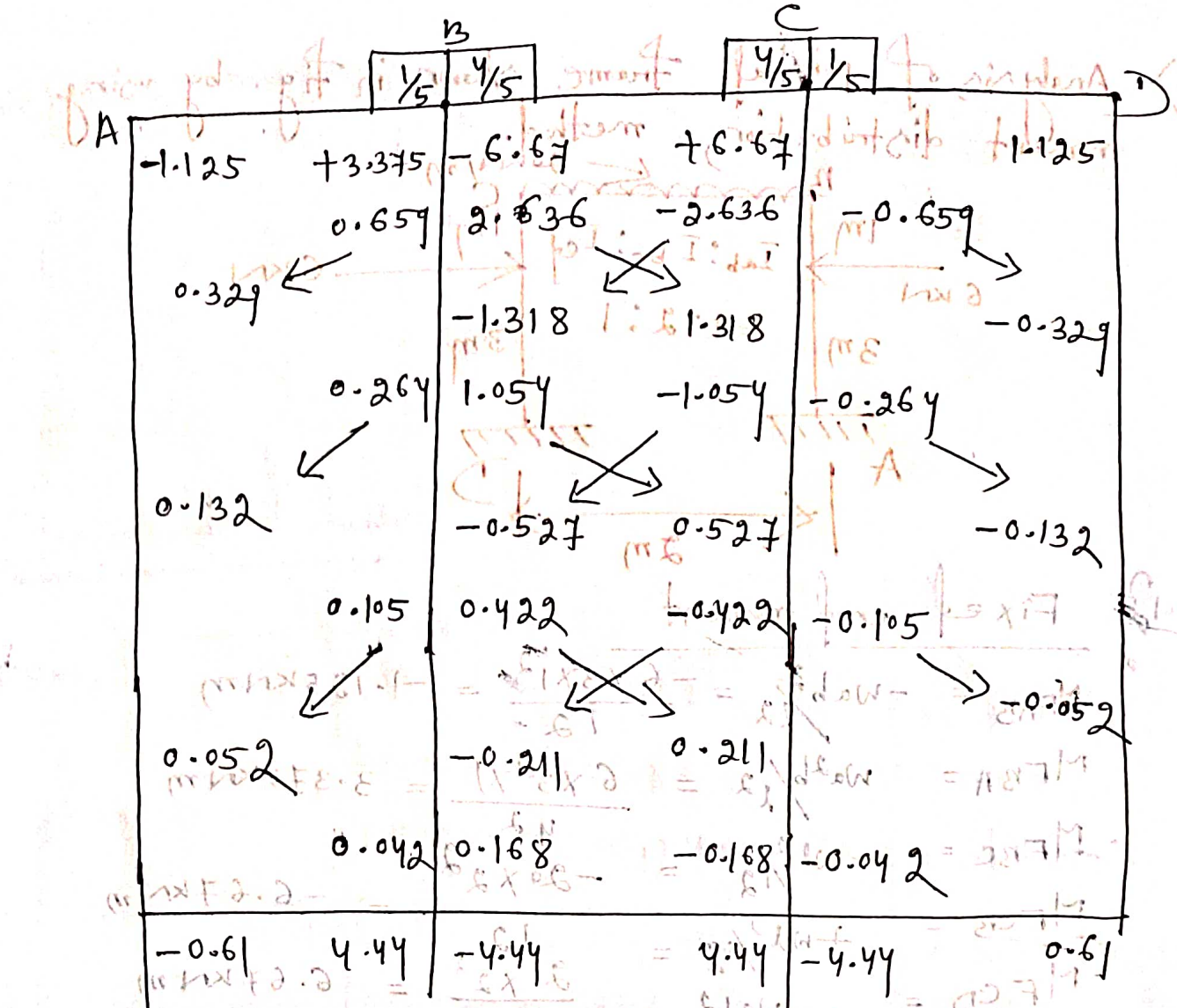
$M_{FCB} = +wl^2/12 = \frac{20 \times 2^2}{12} = 6.67 kNm$

$M_{FCD} = \frac{-wab^2}{l^2} = \frac{-6 \times 1 \times 3^2}{4^2} = -3.375 kNm$

$M_{FDC} = \frac{+wa^2b}{l^2} = \frac{6 \times 1^2 \times 3}{4^2} = 1.125 kNm$

Distribution factor

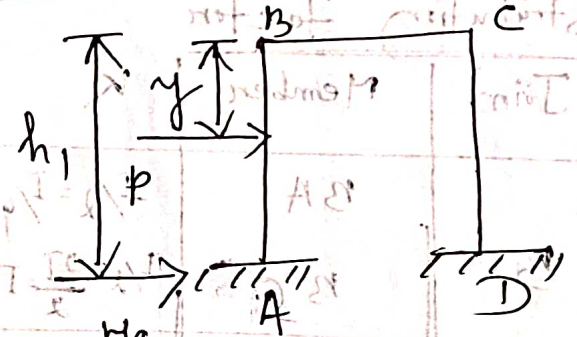
Joint	Member	K	ΣK	D.F. $(K/\Sigma K)$
B	BA	$I/l = I/4$	$I/4 + I = 5I/4$	$I/4 \times 4/5I = 1/5$
	BC	$I/l = 2I/2 = I$		$I \times 4/5I = 4/5$
C	CB	$I/l = 2I/2 = I$	$I + I/4 = 5I/4$	$I \times 4/5I = 4/5$
	CD	$I/l = I/4$		$I/4 \times 4/5I = 1/5$



Note:-

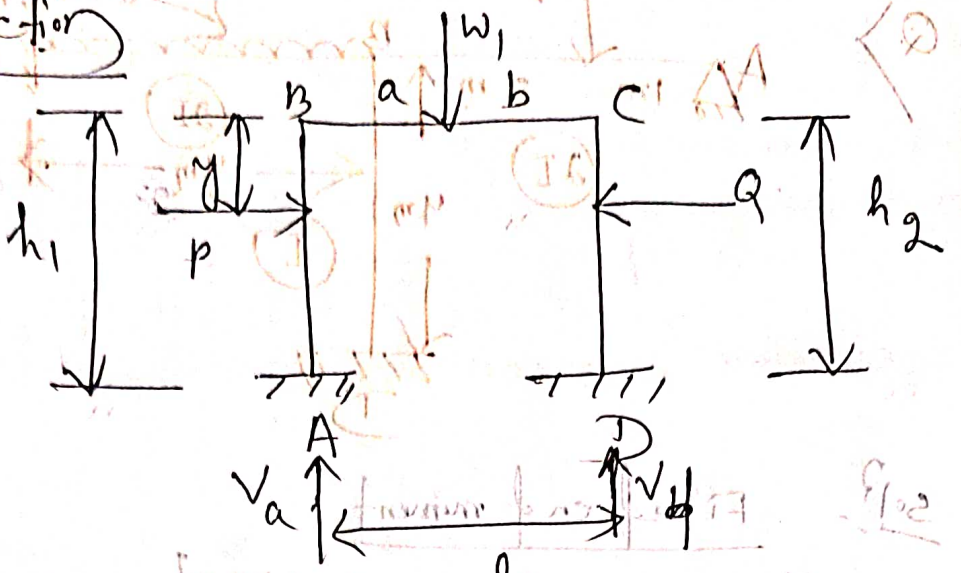
To determine the horizontal thrust for a portal-frame from the fixed moment

$$H_a = \frac{M_{ab} + M_{ba} - P.y}{h_1}$$



$$H_d = \frac{M_{cd} + M_{dc} + \text{Moment of forces on CD about C'}}{h_2}$$

Vertical reaction



B.M at C $M_{Cb} = -M_{bC} - W_1 b + V_a l$

$$V_a = \frac{M_{bC} + M_{Cb} + W_1 b}{l}$$

$$V_d = \frac{M_{bC} + M_{Cb} + \text{moment of forces on BC about B}}{l_{BC}}$$

Continue above problem

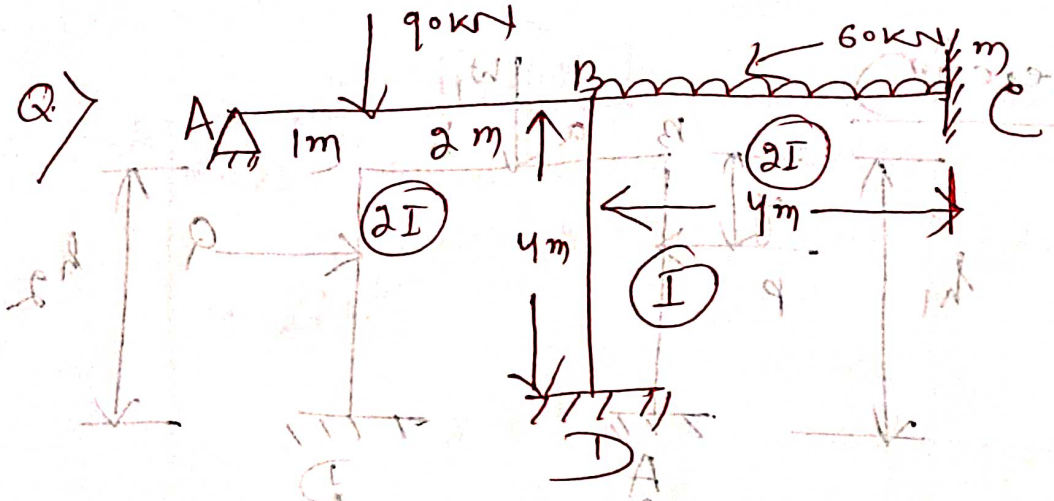
$$M_a = \frac{M_{ab} + M_{ba} - P \cdot y}{h_1} = \frac{-0.61 + 4.44 - 6x1}{4}$$

$$= -0.54 \text{ kN}$$

$$M_d = \frac{M_{cd} + M_{dc} + P \cdot y}{h_2} = \frac{-4.44 + 0.61 + 6x1}{4}$$

$$= 0.54 \text{ kN}$$

Q)



Sol:- Fixed end moment

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-90 \times 1 \times 2^2}{3^2} = -40\ KNm$$

$$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{90 \times 1^2 \times 2}{3^2} = 20\ KNm$$

$$M_{FBC} = -wl^2/12 = \frac{-60 \times 4^2}{12} = -80\ KNm$$

$$M_{FCB} = +wl^2/12 = \frac{60 \times 4^2}{12} = +80\ KNm.$$

$$M_{FBD} = 0$$

$$M_{FDB} = 0$$

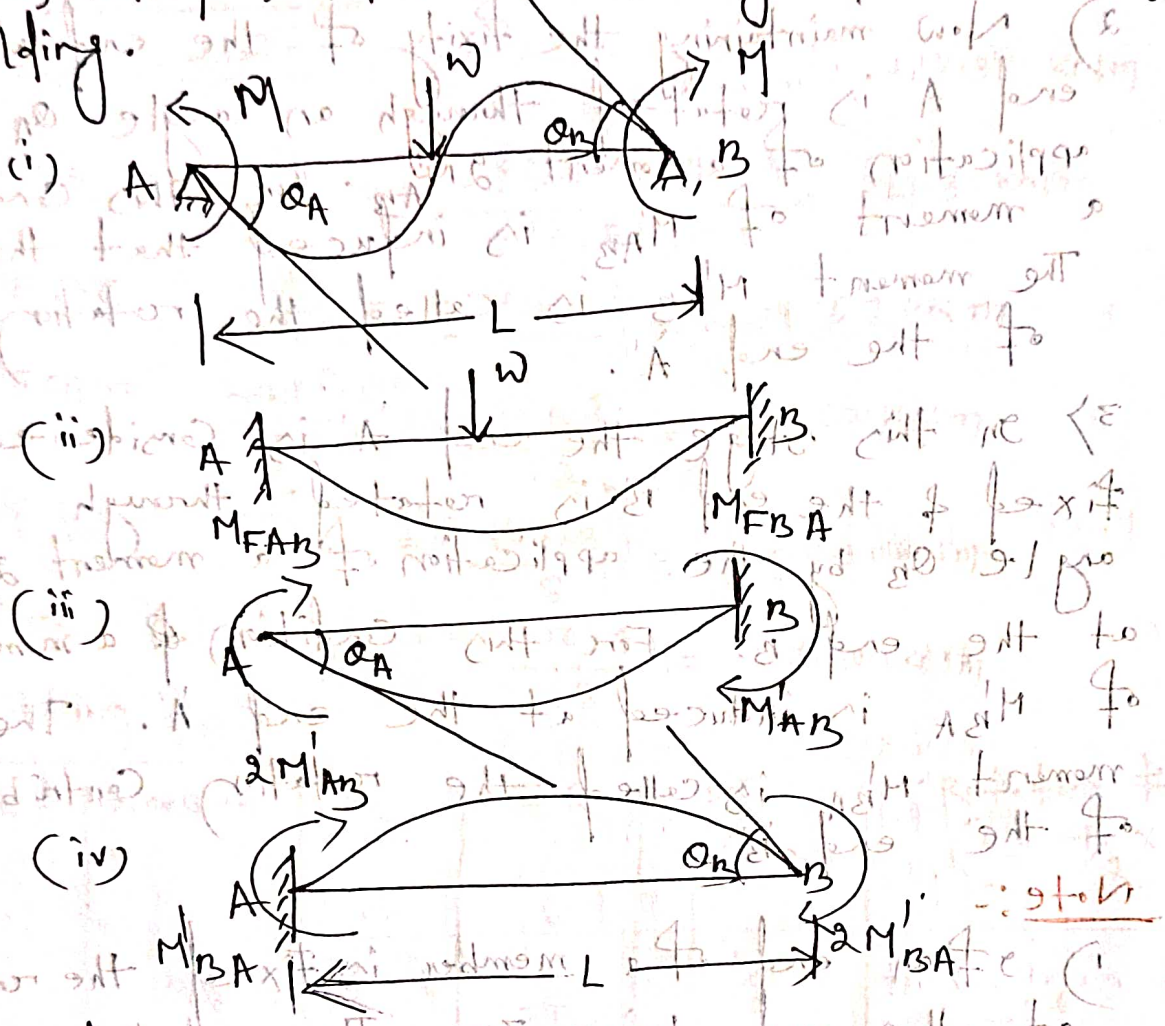
Distribution factor

Joint	Member	K	ΣK	D.F = $(K/\Sigma K)$
B	BA	$3/4 \times \frac{2I}{3} = I/2$	$5I/4$	$I/2 \times \frac{4}{5I} = 2/5$
	BC	$2I/4 = I/2$		$I/2 \times 4/5I = 2/5$
	BD	$I/4$		$I/4 \times 4/5I = 1/5$

⇒ Rotation method (or) Rotation Contribution method :-

→ This method is first explain for structure with fixed end then the modification to handle simply supported and overhanging ends are discussed.

→ This method is useful for the analyse of multistoney-building.



Final moments

$$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}$$

$$M_{BA} = M_{FBA} + 2M'_{BA} + M'_{AB}$$

Here, M'_{AB} & M'_{BA} are the rotation contribution.

Final moment = Fixed end moment + 2 × (rotation contribution of near end) + rotation contribution of far end.

→ The moments are determined by going through the following stages.

1) The ends A & B of the members are first regarded as fixed. Corresponding to this condition the fixed end moments M_{FAB} at 'A' & M_{FBA} at 'B' are determined.

2) Now maintaining the fixity of the end 'B', The end 'A' is rotated through an angle θ_A. By the application of a moment $2M'_{AB}$. For this condition a moment of M'_{AB} is induced that the end B. The moment M'_{AB} is called the rotation Contribution of the end 'A'.

3) In this stage the end 'A' is considered as fixed & the end 'B' is rotated through an angle θ_B by the application of a moment $2M'_{BA}$ at the end 'B'. For this condition a moment of M'_{BA} is induced at the end 'A'. The moment M'_{BA} is called the rotation Contribution of the end 'B'.

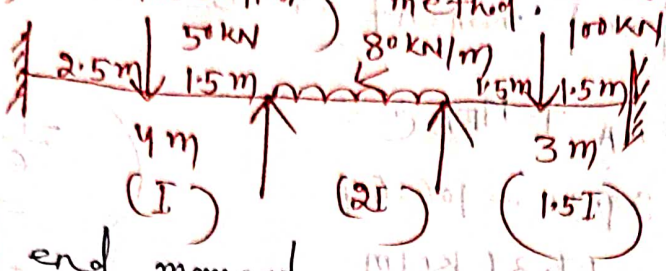
Note:-

1) If an end of a member is fixed the rotation at the end being zero. The rotation Contribution of the end is also zero.

2) If an end of a member is hinged or pinned it will be convient to consider the end at fixed and to take the relative stiffness as $3/4\, I/l$.

$$\left(R.F = -\tfrac{1}{2} \times D.F\right)$$

Q.) Analyse the continuous beam by Kani's method (or) rotation Contribution method.



Sol?- Fixed end moment

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-50 \times 2.5 \times 1.5^2}{4^2} = -17.58 \text{ kNm}$$

$$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{50 \times 2.5^2 \times 1.5}{4^2} = 29.30 \text{ kNm}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-80 \times 4^2}{12} = -106.67 \text{ kNm}$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{80 \times 4^2}{12} = 106.67 \text{ kNm}$$

$$M_{FCD} = \frac{-wl}{8} = \frac{-100 \times 3}{8} = -37.50 \text{ kNm}$$

$$M_{FDC} = \frac{+wl}{8} = \frac{100 \times 3}{8} = 37.50 \text{ kNm}$$

Rotation factor

Joint	Member	K	ΣK	D.F = K/ΣK	Rotation factor ($U = -\frac{1}{2} \times K/\Sigma K$)
B	BA	$I/l = I/4$	$3I/4$	$1/3$	$-1/6$
	BC	$I/l = 2I/4$		$2/3$	$-1/3$
C	CB	$2I/4$	I	$1/2$	$-1/4$
	CD	$1.5I/3$		$1/2$	$-1/4$

Iteration table :-

Sum of fixed end moment at 'B' =

$\Sigma M_{FB} = M_{FBA} + M_{FBC}$

$= 29.30 + 106.67$

$= -77.37$ kNm

Sum of fixed end moment at 'C' :

$\Sigma M_{FC} = M_{FCB} + M_{FCD}$

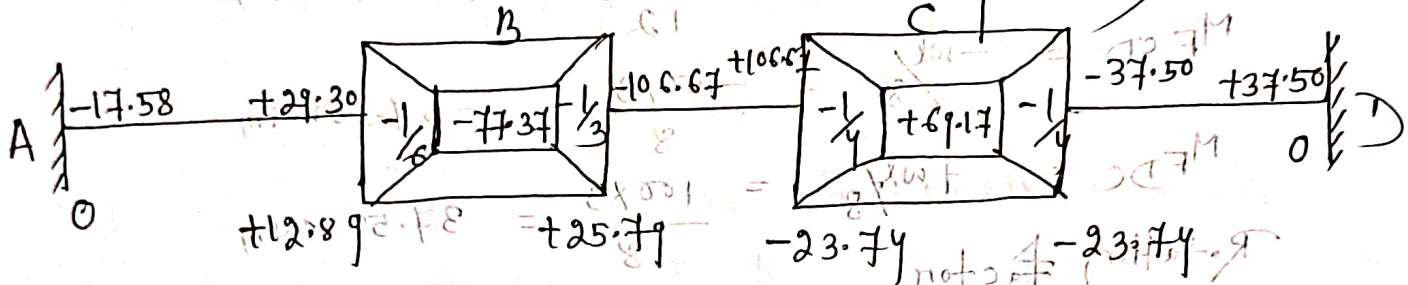
$= 106.67 - 37.50$

$= +69.17$ kNm.

$M'_{BA} = R.F \left(\Sigma \text{ fixed end moment at Joint 'B'} + \Sigma \text{ rotation contribution from far end} \right)$

$\frac{-1}{6}(-77.37 + 0) = 12.89$

$-\frac{1}{4}(69.17 + 25.79 + 0) = -23.74$

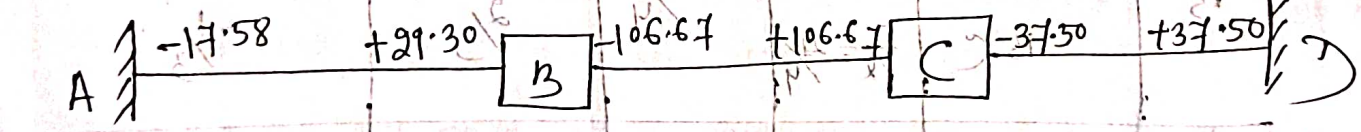


+16.85 +33.70 -25.72 -25.72

+17.18 +34.35 -25.88 -25.88

+17.21	+34.42	-25.90	-25.90

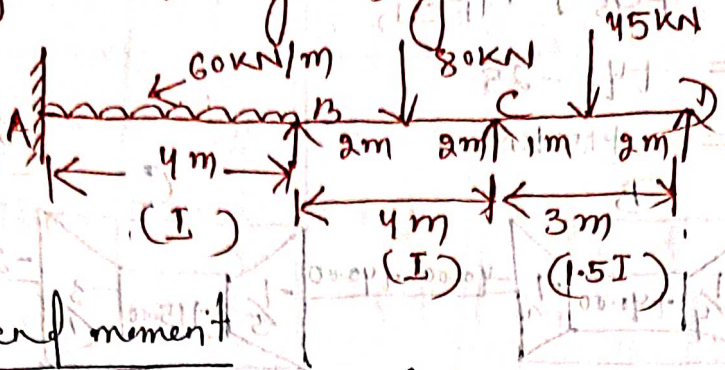
Final moment



Near end Contribution	2X0	2X17.21	2X34.42	2X(-25.90)	2X(-25.90)	2X0
Far end Contribution	17.21	0	-25.90	+34.42	0	-25.90
Final moments	-0.37	+63.72	-63.72	+89.30	-89.30	+11.60

Check for equilibrium $M_{BA} + M_{BC} = 0$ & $M_{CB} + M_{CD} = 0$

Q.) Find the support moment at A, B, C, D for the Continuous beam by using Kani's method.



Sol?- Fixed end moment

$$M_{FAB} = -wl^2/12 = \frac{-60\times4^2}{12} = +80\text{KNm}$$

$$M_{FBA} = +wl^2/12 = \frac{+60\times4^2}{12} = +80\text{KNm}$$

$$M_{FBC} = -wl/8 = \frac{-80\times4}{8} = -40\text{KNm}$$

$$M_{FCB} = +wl/8 = \frac{+80\times4}{8} = +40\text{KNm}$$

$$M_{FCD} = \frac{-wab^2}{l^2} = \frac{-45\times1\times2^2}{3^2} = -20\text{KNm}$$

$$M_{FDC} = \frac{+wa^2b}{l^2} = \frac{+45\times1^2\times2}{3^2} = 10\text{KNm}$$

Rotation factor

Joint	Member	K	ΣK	$\frac{K}{\Sigma K}$	RF = (U) = $-\frac{1}{2}\times\frac{K}{\Sigma K}$
B	BA	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{I}{4}\times\frac{4}{2I}=\frac{1}{2}$	$-\frac{1}{2}\times\frac{1}{2}=-\frac{1}{4}$
	BC	$\frac{I}{4}$		$\frac{I}{4}\times\frac{4}{2I}=\frac{1}{2}$	$-\frac{1}{2}\times\frac{1}{2}=-\frac{1}{4}$
C	CB	$\frac{I}{4}$	$\frac{5I}{8}$	$\frac{I}{4}\times\frac{8}{5I}=\frac{2}{5}$	$-\frac{1}{2}\times\frac{2}{5}=-\frac{1}{5}$
	CD	$\frac{3I}{4l}=\frac{3\times1.5I}{4\times3}=\frac{3I}{8}$		$\frac{3I}{8}\times\frac{8}{5I}=\frac{3}{5}$	$-\frac{1}{2}\times\frac{3}{5}=-\frac{3}{10}$

Iteration table

Sum of fixed end moment at 'B'

$$\Sigma M_{FB} = M_{FBA} + M_{FBC}$$

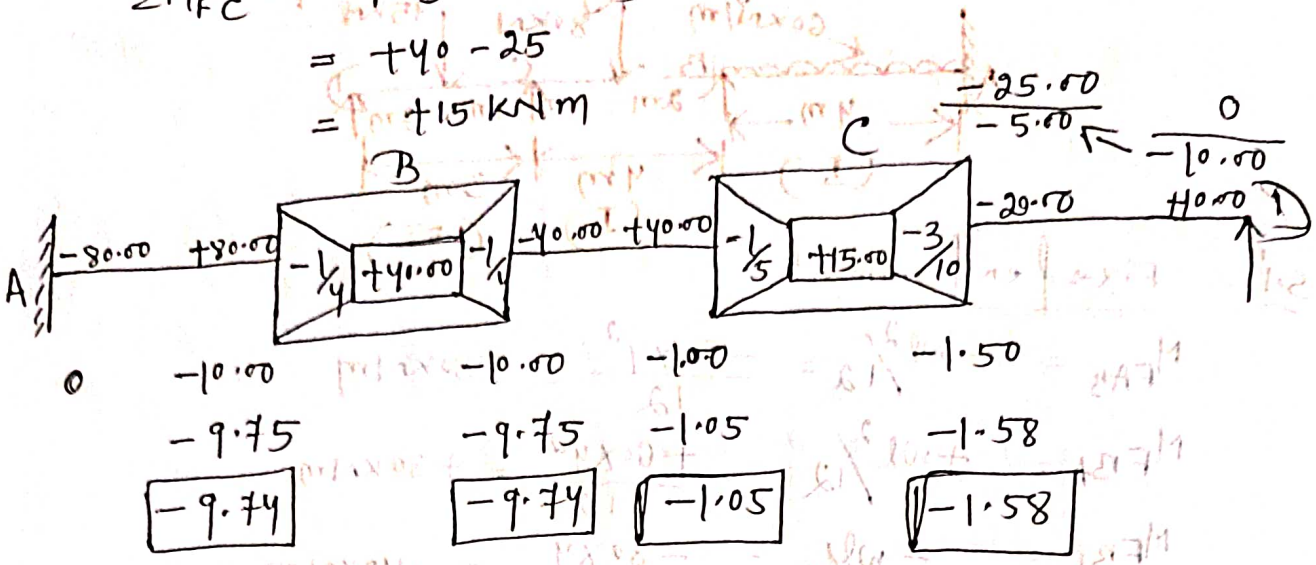
$$= +80 - 40 = 40\text{KNm}.$$

Sum of fixed end moment at 'c'

$\Sigma M_{FC} = M_{FCB} + M_{FCD}$

$= +40 - 25$

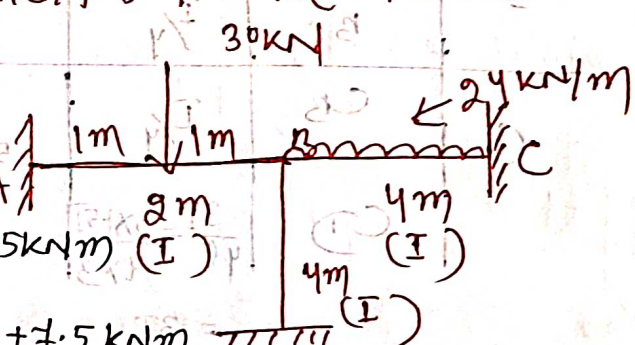
$= +15$ kNm



Final moment

FEM		-80.00	+80.00	-40.00	+40.00	-25.00	0
	A		B		C		D
NEC	2X0	-2X9.74	-2X9.74	-2X1.05	-2X1.58	2X0	
FEC	-9.74	0	+1.05	-9.74	-0		
Total moments	-89.74	+60.53	-60.53	+28.16	-28.16	0	

Q.) Find the support moments for the frame shown in fig.



Soln:-

Fixed end moments

$M_{FAB} = -wl/8 = -\frac{30 \times 2}{8} = -7.5$ kNm

$M_{FBA} = +wl/8 = \frac{30 \times 2}{8} = +7.5$ kNm

$M_{FBC} = -wl^2/12 = -\frac{24 \times 4^2}{12} = -32$ kNm

$M_{FCB} = +wl^2/12 = \frac{24 \times 4^2}{12} = 32$ kNm

$M_{FBD} = 0$

$M_{FDB} = 0$

Rotation factor

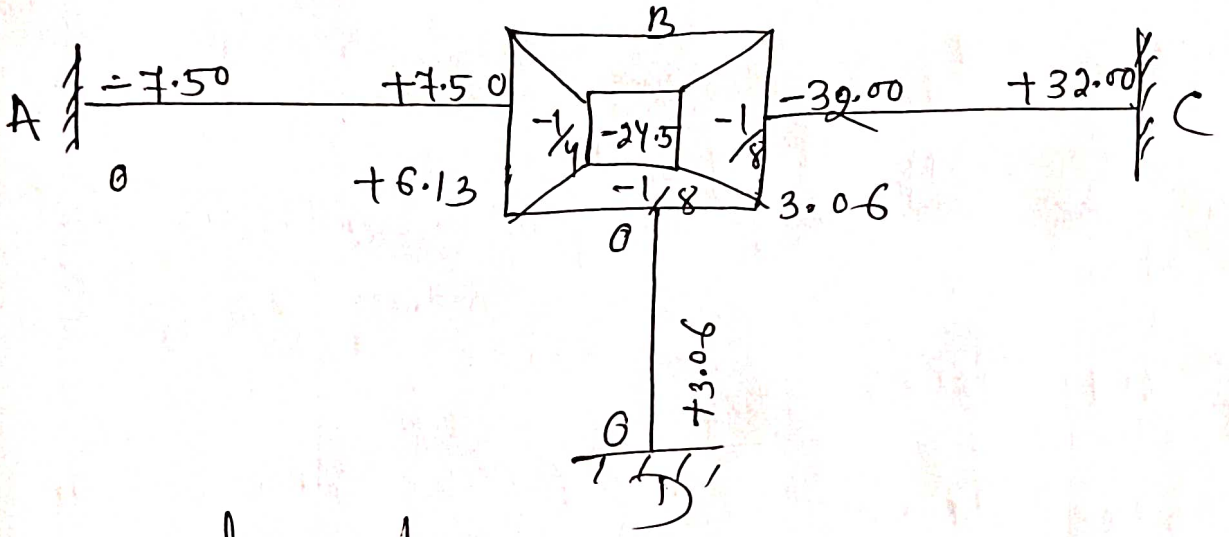
Joint	Member	K	ΣK	$\frac{K}{\Sigma K}$	$U = -\frac{1}{2} \times D.F$
B	BA	$I/2$	I	$1/2$	$-1/4$
	BC	$I/4$		$1/4$	$-1/8$
	BD	$I/4$		$1/4$	$-1/8$

Iteration table

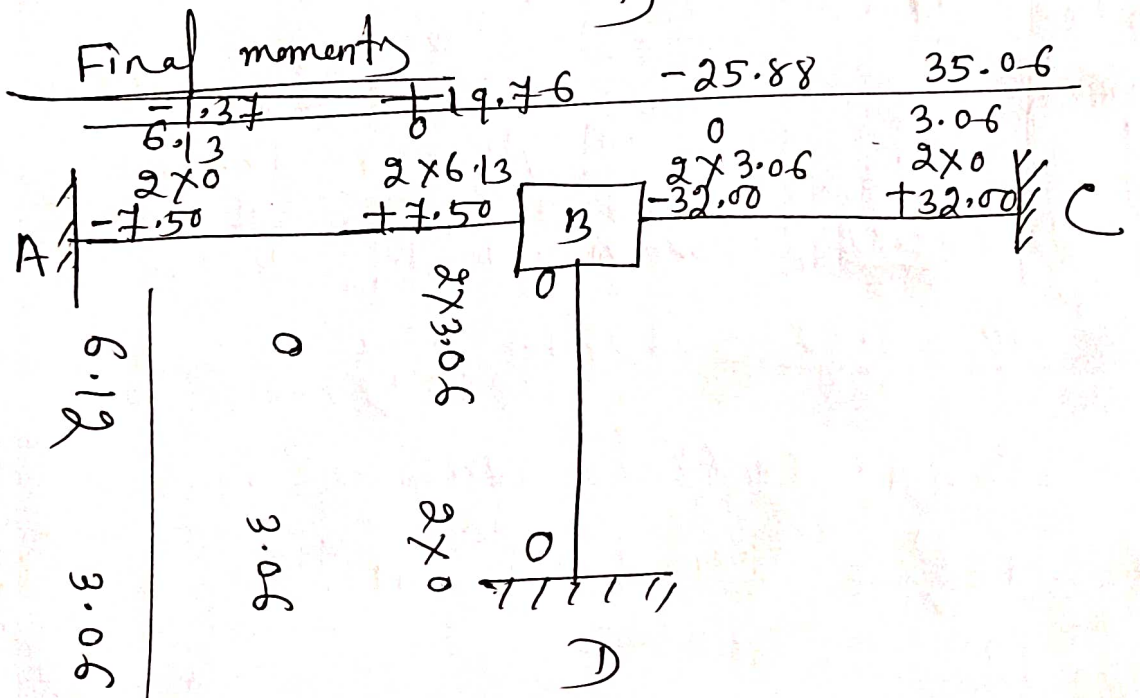
Sum of fixed end moments at 'B'.

$\Sigma M_B = M_{FBA} + M_{FBC} + M_{FBD}$

$= 7.5 - 32 + 0 = -24.5 \text{ KNm}$



Final moments



$M_{AB} = -7.50 + 2\times 0 + 6.13 = -1.37$ kNm

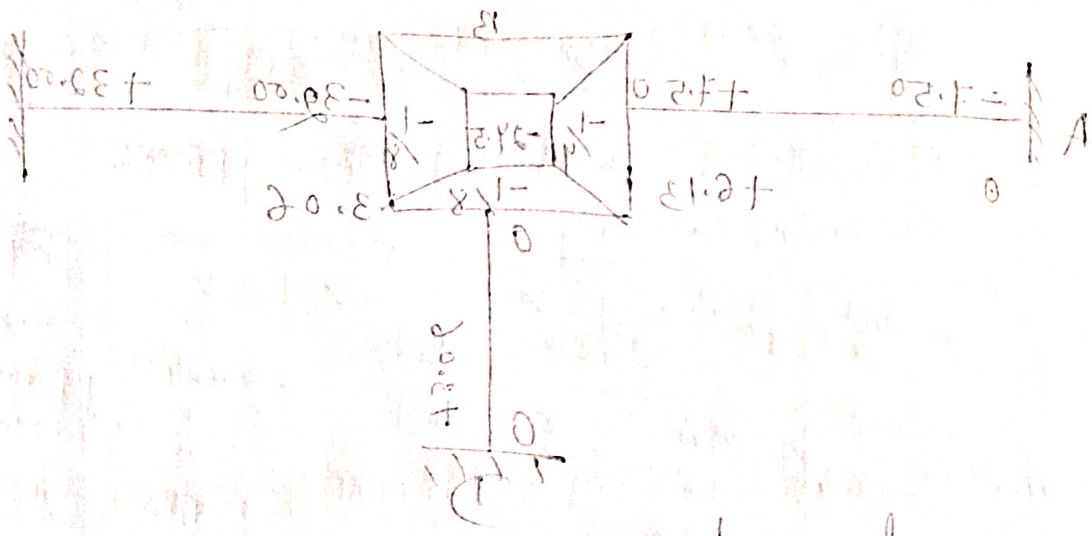
$M_{BA} = 7.50 + 2\times 6.13 + 0 = +19.76$ kNm

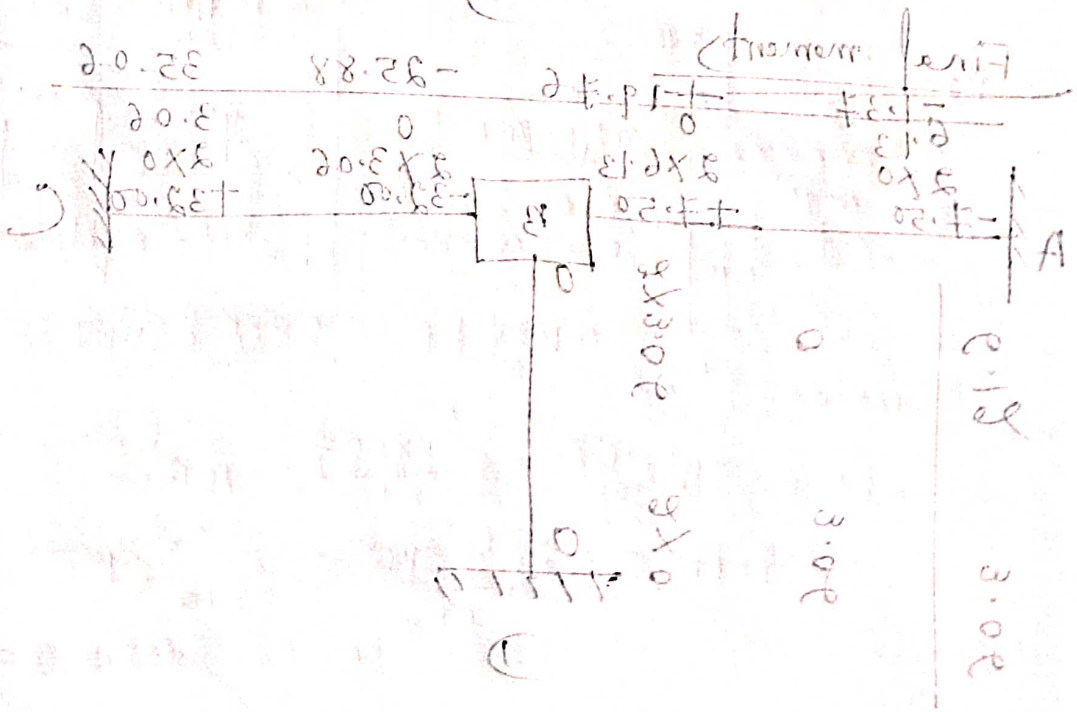
$M_{BC} = -32 + 2\times 3.06 + 0 = -25.88$ kNm

$M_{CB} = 32 + 2\times 0 + 3.06 = 35.06$ kNm

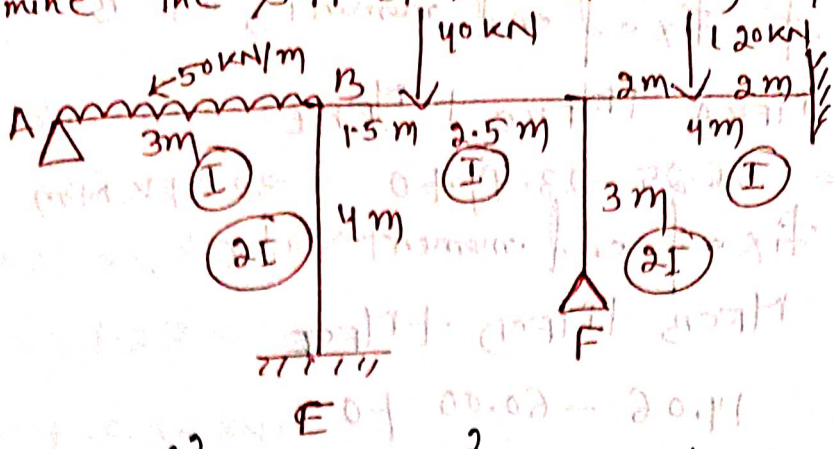
$M_{BD} = 0 + 2\times 3.06 + 0 = 6.12$ kNm

$M_{DB} = 0 + 2\times 0 + 3.06 = 3.06$ kNm.





Q) Determine the support moment shown in fig.



Sol?- $M_{FAB} = \frac{-wl^2}{12} = \frac{-50 \times 3^2}{12} = -37.50$ kNm

$M_{FBA} = \frac{+wl^2}{12} = \frac{50 \times 3^2}{12} = +37.50$ kNm

$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-40 \times 1.5 \times 2.5^2}{4^2} = -23.44$ kNm

$M_{FCB} = \frac{+wa^2b}{l^2} = \frac{40 \times 1.5^2 \times 2.5}{4^2} = +14.06$ kNm

$M_{FCD} = -wl/8 = \frac{-120 \times 4}{8} = -60.00$ kNm

$M_{FDC} = +wl/8 = \frac{120 \times 4}{8} = +60.00$ kNm

$M_{FBE} = 0$

$M_{FEB} = 0$

$M_{FCF} = 0$

$M_{FFC} = 0$

Rotation factor

Joint	Member	K	ΣK	K/ΣK	U = -1/2 × D - F
B	BA	3I/12 = I/4	I	1/4	-1/8
	BC	I/4		1/4	-1/8
	BE	2I/4 = I/2		1/2	-1/4
C	CB	I/4	I	1/4	-1/8
	CD	I/4		1/4	-1/8
	CF	6I/12 = I/2		1/4	-1/4

Iteration table

Sum of fixed end moments at B

$\Sigma M_B = M_{FBA} + M_{FBC} + M_{FBE}$

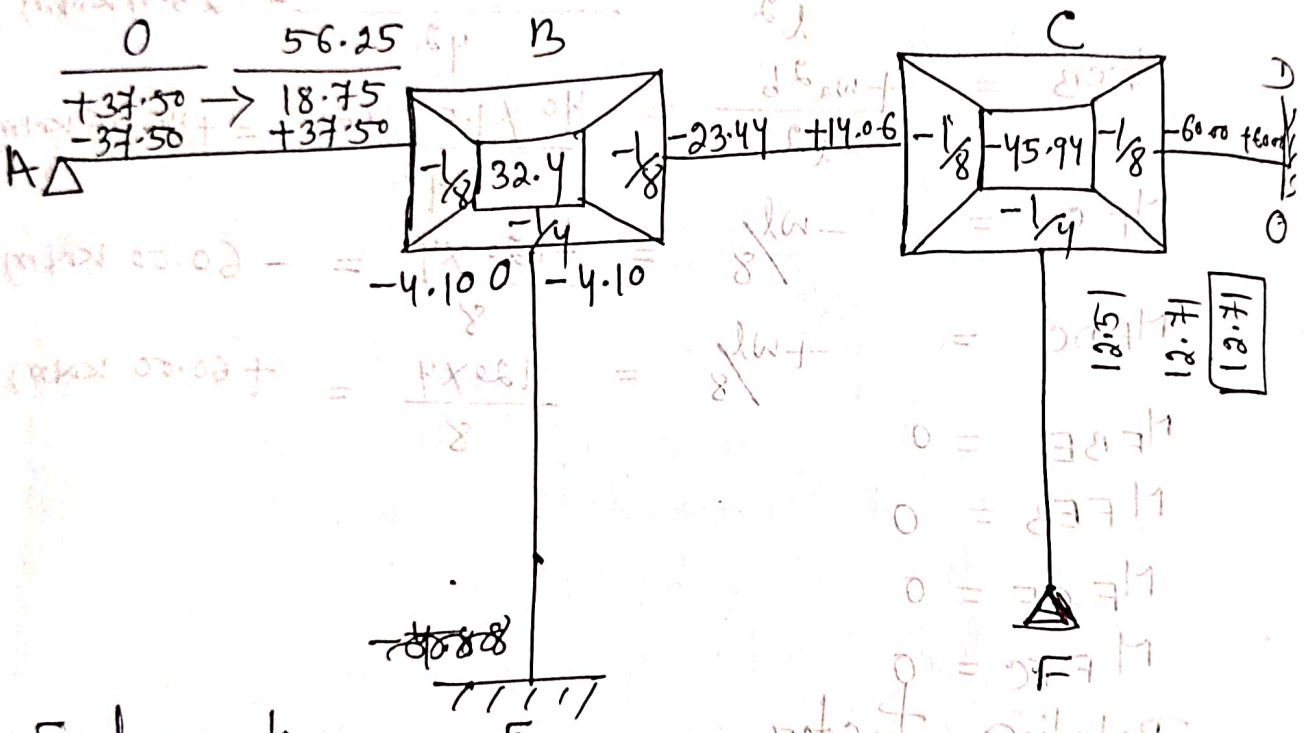
$= 56.25 - 23.44 + 0 = 32.81$ KNm

Sum of fixed end moments at 'C'.

$\Sigma M_C = M_{FCB} + M_{FCD} + M_{FCF}$

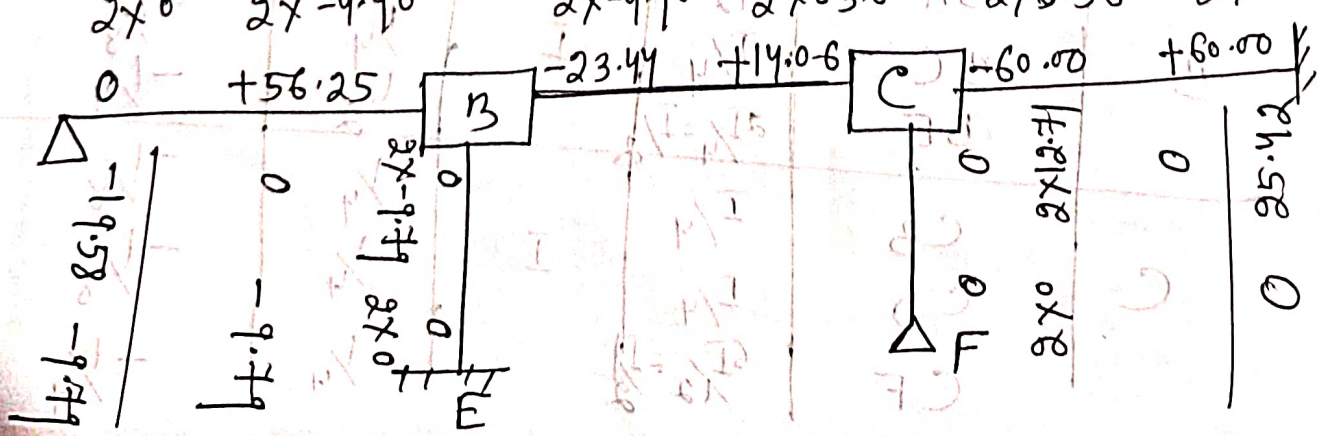
$= 14.06 - 60.00 + 0$

$= -45.94$ KNm



Final moments

Total moments	0	+46.45	-26.88	+21.88	-47.28	66.36
		0	6.34	-4.90	0	6.36
	2X0	2X-4.90	2X-4.90	2X6.36	2X6.36	2X0
	0	+56.25	-23.44	+14.06	-60.00	+60.00



Final moments

$M_{AB} = 0$

$M_{BA} = +46.45\ KNm$

$M_{BC} = +26.88\ KNm$

$M_{CB} = +21.88\ KNm$

$M_{CD} = -47.28\ KNm$

$M_{DC} = +66.36\ KNm$

$M_{FB} = -9.79\ KNm$

$M_{BF} = -19.58\ KNm$

$M_{FC} = +24.42\ KNm$

$M_{CF} = 0$